BOOK 2

Simply Good and Beautiful

MATH 6

COURSE BOOK

The Good and the Beautiful Curriculum

Book 2 Table of Contents

UNIT 3 OVERVIEW

LESSONS 61–90

Extra Supplies Needed

- colored pencils
- protractor
- ruler
- 2 standard dice
- coin
- scissors
- bowl

New Concepts Taught

- angle bisectors
- calculate discount and sale price
- check solutions to inequalities
- compare ratios
- compound interest
- compound probability with multiplication
- cross products with proportions
- dependent and independent variables
- distributive property with variables
- equations with square roots and cube roots
- equations with squared and cubed variables
- estimate square roots
- experimental probability
- factor expressions with variables
- fractions of a group—solve for the part, fraction, or whole
- graph inequalities on number lines
- graph lines on coordinate planes
- independent events with probability
- input/output tables for equations
- percent equation
- perpendicular bisectors
- predict outcomes of probability experiments
- proportions on a graph
- sample space
- simple interest
- solve one-step inequalities
- solve two-step equations
- solve two-step inequalities
- square roots of perfect squares greater than 225
- tax and total cost with tax
- theoretical probability
- total cost of discounted item with tax
- tree diagrams to record probability outcomes
- unit multipliers
- volume of prisms
- write an inequality from a graph

Concepts Reviewed and Expanded Upon

- add and subtract mass, length, and capacity
- check solutions to equations
- conversions—capacity
- conversions—length
- conversions—mass and weight
- inverse operations
- probability experiments
- probability outcomes
- proportional relationships
- proportions
- ratios
- simple probability
- US customary system and metric system
- volume of cubes, rectangular prisms, and cylinders

Lesson 63

ESTIMATING & FINDING SQUARE ROOTS

Watch the video lesson and/or read the mini lesson.

Warm-Up

Complete the problems below.

$4 \bullet$ _____ $= 400$

$\sqrt{4} =$ _____

$\sqrt{100} =$ _____

$\sqrt{4} \bullet \sqrt{100} =$ _____

$20^2 =$ _____

$\sqrt{400} =$ _____

$9 \bullet$ _____ $= 900$

$\sqrt{9} =$ _____

$\sqrt{100} =$ _____

$\sqrt{9} \bullet \sqrt{100} =$ _____

$30^2 =$ _____

$\sqrt{900} =$ _____

Video Lesson

Scan the QR code or watch the video lesson on goodandbeautiful.com/Math6.

$\sqrt{10}$

_____ < _____ < _____

_____ < _____ < _____

$\sqrt{10}$ is a decimal number between _____ and _____.

Mental Math Checkup

1. Count by $\frac{1}{4}$ from 5 to $7\frac{1}{4}$.

   ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___

2. Find each percent.

   What is 10% of 250? ___

   What is 10% of 120? ___

3. Multiply or divide.

   $400 \div 8 =$ ___

   $50 \bullet 11 =$ ___

Mini Lesson

The square root of a perfect square is a whole number. Examples:

$\sqrt{1}=1$	$\sqrt{16}=4$	$\sqrt{49}=7$	$\sqrt{100}=10$	$\sqrt{169}=13$
$\sqrt{4}=2$	$\sqrt{25}=5$	$\sqrt{64}=8$	$\sqrt{121}=11$	$\sqrt{196}=14$
$\sqrt{9}=3$	$\sqrt{36}=6$	$\sqrt{81}=9$	$\sqrt{144}=12$	$\sqrt{225}=15$

Estimating Square Roots

The square root of a number that is not a perfect square is not a whole number. Its value can be estimated by finding the two whole numbers that it is between.

Example: Find the approximate value of $\sqrt{30}$.

1. Find two consecutive square roots of perfect squares that $\sqrt{30}$ is between.

Consecutive numbers follow each other in order.

$\sqrt{25}=5$

$\sqrt{30}=?$

$\sqrt{36}=6$

$\sqrt{30}$ is between $\sqrt{25}$ and $\sqrt{36}$

$\sqrt{25}$ is less than $\sqrt{30}$ AND $\sqrt{30}$ is less than $\sqrt{36}$

$\sqrt{25} < \sqrt{30}$ $\qquad$ $\sqrt{30} < \sqrt{36}$

$\sqrt{25} < \sqrt{30} < \sqrt{36}$

Notice how those two inequalities can be combined to write a compound inequality.

- The smallest value is on the left.
- The common number is in the middle.
- The largest value is on the right.
- Less than symbols are between the numbers because the first number is less than the middle number, and the middle number is less than the last number.

2. Simplify the square roots that are perfect squares.

$\sqrt{25} < \sqrt{30} < \sqrt{36}$

$5 < \sqrt{30} < 6$

The value of $\sqrt{30}$ is a decimal number between 5 and 6.

Square Roots of Perfect Squares Greater Than 225

To find the square root of a perfect square greater than 225, use square roots of perfect squares that are multiples of 100 as benchmarks.

Examples: $\sqrt{100}=10$ $\quad$ $\sqrt{400}=20$ $\quad$ $\sqrt{900}=30$ $\quad$ $\sqrt{1,600}=40$

Notice the pattern!
$\sqrt{4}=2$
$\sqrt{400}=20$

Since a perfect square is a number multiplied by itself, the last digit of the perfect square can also be used as a clue.

Example: Simplify $\sqrt{784}$.

1. Find the first digit of the answer by using square roots that are multiples of 100 as benchmarks.

$\sqrt{400}=20$ and $\sqrt{900}=30$

$\sqrt{784}$ is between $\sqrt{400}$ and $\sqrt{900}$

$\sqrt{400}$ is less than $\sqrt{784}$ AND $\sqrt{784}$ is less than $\sqrt{900}$

$\sqrt{400} < \sqrt{784}$ $\qquad$ $\sqrt{784} < \sqrt{900}$

$\sqrt{400} < \sqrt{784} < \sqrt{900}$

$20 < \sqrt{784} < 30$

The value of $\sqrt{784}$ is between 20 and 30, so $\sqrt{784}$ is a number in the 20s. Two is the first digit.

$\sqrt{784} = 2\underline{?}$

2. Find the last digit of the answer by thinking of perfect squares. The last digit in 784 is 4. What single-digit number multiplied by itself could equal a number that ends in 4?

$2^2 = \underline{4}$ $\qquad$ $8^2 = 6\underline{4}$

The last digit is either 2 or 8.

Since 784 is closer to 900 than 400, the last digit is probably 8.

Does $\sqrt{784}$ equal 28?

Check to see if 28^2 is 784.

$\sqrt{784} = 28$ ✓

scratch work

$$\begin{array}{r} 28 \\ \times\ 28 \\ \hline 224 \\ +\ 560 \\ \hline 784 \end{array}$$

Practice

1. Write each square root in the blue box below between the consecutive square roots of perfect squares it is between. Then cross it off in the blue box. The first one is given as an example.

$\cancel{\sqrt{53}}$	$\sqrt{18}$	$\sqrt{44}$	$\sqrt{3}$	$\sqrt{172}$	$\sqrt{159}$
$\sqrt{6}$	$\sqrt{131}$	$\sqrt{70}$	$\sqrt{99}$	$\sqrt{10}$	$\sqrt{212}$

$\sqrt{1}$ ______ $\sqrt{4}$ ______ $\sqrt{9}$ ______ $\sqrt{16}$ ______ $\sqrt{25}$

$\sqrt{36}$ ______ $\sqrt{49}$ $\sqrt{53}$ $\sqrt{64}$ ______ $\sqrt{81}$ ______ $\sqrt{100}$

$\sqrt{121}$ ______ $\sqrt{144}$ ______ $\sqrt{169}$ ______ $\sqrt{196}$ ______ $\sqrt{225}$

2. Find the consecutive square roots of perfect squares that each square root listed is between. Then fill in the inequality symbols to write a compound inequality. The first one is given as an example.

$\sqrt{36} < \sqrt{45} < \sqrt{49}$ | ______ $\sqrt{59}$ ______

______ $\sqrt{28}$ ______ | ______ $\sqrt{2}$ ______

______ $\sqrt{183}$ ______ | ______ $\sqrt{134}$ ______

______ $\sqrt{109}$ ______ | ______ $\sqrt{200}$ ______

______ $\sqrt{11}$ ______ | ______ $\sqrt{48}$ ______

3. Using the information in the first column in Problem 2, fill in the blanks. The first one is given as an example.

$\sqrt{45}$ is a decimal number between 6 and 7.

$\sqrt{28}$ is a decimal number between _____ and _____.

$\sqrt{183}$ is a decimal number between _____ and _____.

$\sqrt{109}$ is a decimal number between _____ and _____.

$\sqrt{11}$ is a decimal number between _____ and _____.

4. Find the two whole numbers each square root is between. The first one is given as an example.

$\sqrt{93}$

$\sqrt{81} < \sqrt{93} < \sqrt{100}$

$9 < \sqrt{93} < 10$

$\sqrt{93}$ is between 9 and 10.

$\sqrt{54}$

$\sqrt{54}$ is between ____ and ____.

$\sqrt{205}$

$\sqrt{205}$ is between ____ and ____.

$\sqrt{70}$

$\sqrt{70}$ is between ____ and ____.

$\sqrt{129}$

$\sqrt{129}$ is between ____ and ____.

$\sqrt{8}$

$\sqrt{8}$ is between ____ and ____.

Practice

5. The numbers under the square root symbols below are perfect squares. Simplify each square root. The first one is given as an example.

$\sqrt{676}$

$\sqrt{400} < \sqrt{676} < \sqrt{900}$

$20 < \sqrt{676} < 30$

$\sqrt{676} = 2\underline{?}$

Try 4 or 6 for the second digit.

$24^2 = 576$

$26^2 = 676$ ✓

$\sqrt{676} = 26$

$\sqrt{625}$

$\sqrt{441}$

$\sqrt{289}$

Review

1. Find all solutions to the equations. Lessons 61 & 62

$x^3 = -343$

$x^3 = 216$

$x^2 = 225$

$\sqrt{x} = 11$

2. Write and solve an equation for each scenario. Lessons 54 & 55

A restaurant's seats are currently 40% full. There are 56 customers already seated. How many seats are in the restaurant?

There are 8 receivers on an American football team with 50 players. What percent of the team are receivers?

3. Evaluate each expression. Lessons 19 & 20

$4.43 + \left(\frac{1}{2}\right)^3 \bullet 4^2$ ____________

$\frac{(-1.2)^2}{3} + 5^3$ ____________

Lesson 75

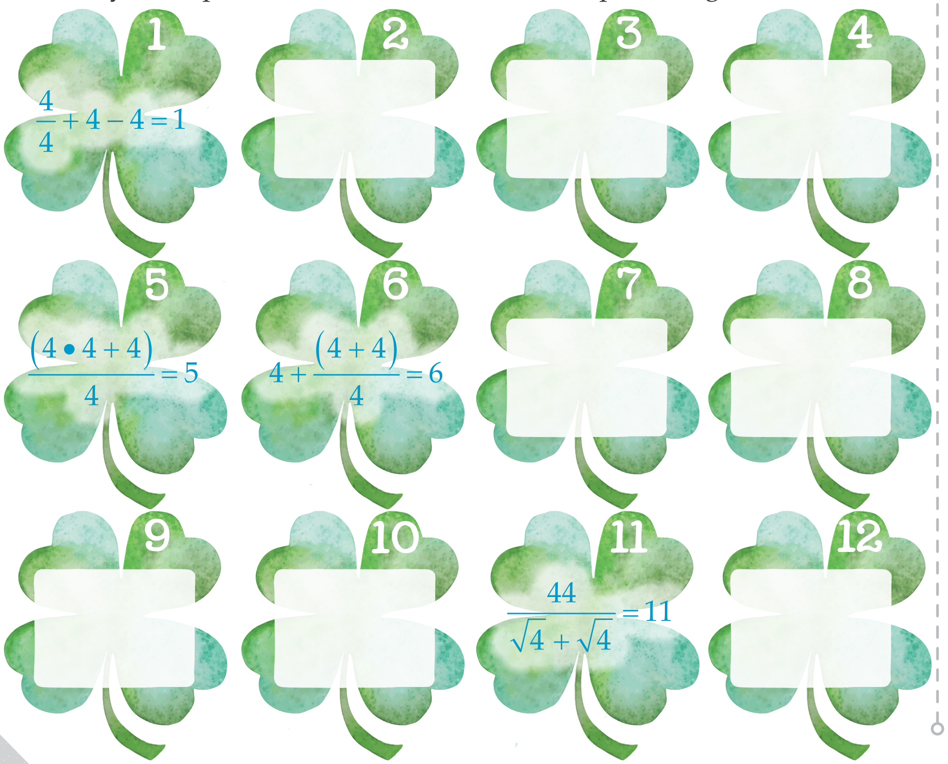
FOUR SEASONS GAMES

There is no video or review for this lesson.

Regardless of which season it is where you are right now, today you will participate in brain-stretching activities for all four seasons: winter, spring, summer, and fall.

Spring

Welcome to spring! Clover is one of the first plants to turn green and begin to thrive each spring. Clovers usually have three leaves, but sometimes a very rare four-leaf clover can be found. In the puzzles below, make each number in the list using exactly four 4s and different operations. You can add, subtract, multiply, divide, or use square roots. You may need parentheses as well. Four examples are given.

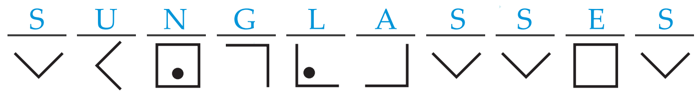


Summer

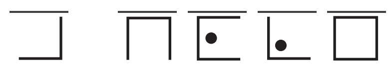
Use the cipher at the bottom of the page to answer the summer picnic riddles. Use the given answer to the first riddle to figure out how the cipher works.

Hint: Once you figure out the code for a letter, fill in all the blanks for that letter on the page.

What does the sun drink out of?

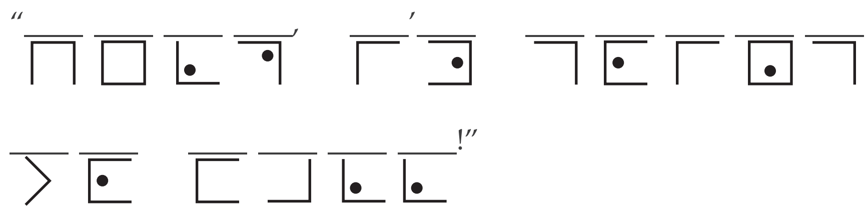


A large cooler of water weighs 40 lb.
What must you put in it to make it weigh 20 lb?

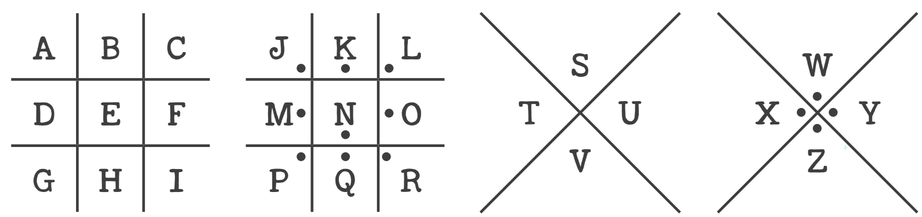


What fruit doesn't like to be alone?

What did summer say to spring?



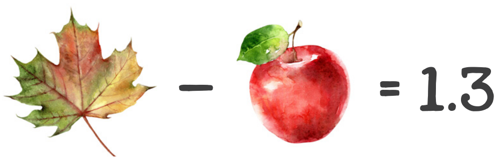
CIPHER



Fall

Find the numerical value of each of the four fall items: leaf, apple, pumpkin, and scarecrow.

Hint: Don't try to solve the equations in order. Only one equation can be solved first, and the solution to that equation will help you solve another equation.



leaf =

apple =

pumpkin =

scarecrow =

Winter

Four different families have planned a family ski trip to four different countries during four different winter months. Use the clues to figure out which family is going to which country during which month.

Hint: Once you know an answer, put a ✓ in that box and fill in the rest of the row and column of that 4 x 4 box with Xs. You may need to go through the clues more than once. You may use a map if needed.

		Month				Country			
		December	January	February	March	Switzerland	United States	France	Canada
Family	Schmidt								
	Noor								
	Chen								
	Lopez								
Country	Switzerland								
	United States								
	France								
	Canada								

1. The Chen family is not going to Europe.
2. The family who is traveling to France will go during the week of Christmas.
3. The Noor family is not traveling to North America.
4. The Schmidt family will travel three weeks after the family who is going to France and will visit the country north of the United States.
5. The Chen family will be traveling one month after the Schmidt family and one month before the Noor family.

Lesson 78

PROBABILITY EXPERIMENTS

Supplies
2 standard dice
1 coin
scissors
bowl

There is no video or review for this lesson.

Theoretical probability is the expected probability of an event occurring.

$$\frac{\text{number of desired outcomes}}{\text{number of possible outcomes}}$$

Experimental probability is the probability of an event occurring based on the results of an experiment.

$$\frac{\text{number of times an event happened in an experiment}}{\text{number of possible outcomes}}$$

The set of all possible outcomes of a probability experiment is called the ***sample space***. A tree diagram or chart can be used to show the sample space.

Dice Experiments

Complete the first chart to show the sample space for rolling two dice. Examples are given.

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	1, 1	1, 2	1, 3			
⚁	2, 1			2, 4		
⚂						
⚃						
⚄						
⚅						

Complete the chart to show the sample space for the **sum** of the numbers shown on the dice. Examples are given.

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4			
⚁	3			6		
⚂						
⚃						
⚄						
⚅						

Write the number of possible outcomes for rolling two dice and getting the sums below.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____ 6 ____

7 ____ 8 ____ 9 ____ 10 ____ 11 ____ 12 ____

Write the theoretical probability of rolling each sum. Examples are given.

1 $\underline{0}$ 2 ____ 3 $\frac{1}{18}$ 4 ____ 5 ____ 6 ____

7 ____ 8 ____ 9 ____ 10 ____ 11 ____ 12 ____

When rolling two dice, which sum is most likely to be rolled? (Write the sum that has the highest theoretical probability.) _____

What is the theoretical probability of rolling doubles (the same number on both dice)? _____

Roll two standard dice and use tally marks to record the outcomes. Repeat for a total of 36 dice rolls. Then write the frequency (the number of tally marks) for each outcome.

Sum of Numbers on Dice	Tally Marks	Frequency	Frequency Expected
2			1
3			2
4			3
5			4
6			5
7			6
8			5
9			4
10			3
11			2
12			1

Which outcome(s) occurred the most? ____________________

Which outcome(s) occurred the least? ____________________

How do the results compare to the expected frequencies? __________

____

____

____

Why do you think the results are different from the expected frequencies?

____

____

____

Based on your results, what is the experimental probability of rolling each sum?

1 ____ 2 ____ 3 ____ 4 ____ 5 ____ 6 ____

7 ____ 8 ____ 9 ____ 10 ____ 11 ____ 12 ____

This space is intentionally left blank for double-sided printing purposes.

Drawing Names Experiment

A homeschool co-op was performing probability experiments. Students' names were written on tickets and put in a bowl. Some names were written more than once. The tickets are shown below. Answer the questions below first. Next, cut the tickets out, fold each one in half to hide the name, and put the tickets in a bowl. Then follow the instructions on the right side of this page.

How many tickets were in the bowl? _____

Which name has the highest theoretical probability of being drawn?

____

What is the theoretical probability of drawing that name? ______

Which two names have the second-highest probability of being drawn? ____________________ and ____________________

Octavio	Octavio	Merrill
Merrill	Merrill	Natalia
Keith	Keith	Angelica
Angelica	James	Fredrik
Fredrik	Krista	Krista
Krista	Krista	Krista
Emilia	Emilia	Ryan
Ryan	Drake	Arthur
Arthur	Arthur	Adrian
Jamie	Jamie	Preston

Draw one ticket from the bowl. Which name did you draw?

____

Put the ticket back in the bowl and mix them up. Draw another ticket. Which name did you draw? __________________

Because Krista's name is on $\frac{1}{6}$ of the tickets, she would theoretically have her name drawn 1 time out of every 6 draws. Draw a ticket 6 times, returning the drawn ticket and mixing up the tickets each time. How many times did you draw Krista's name? _____

Return all tickets to the bowl and mix them up. Draw a ticket 10 times, putting the ticket back in the bowl after each draw, and record the results (names) below.

1. __________ 2. __________ 3. __________ 4. __________

5. __________ 6. __________ 7. __________ 8. __________

9. __________ 10. __________

Based on your experiment, what is the experimental probability of drawing . . .

Merrill's name? ______

Krista's name? _____

Keith's name? ______

Arthur's name? ______

Drake's name? ______

Who has the highest experimental probability of winning the drawing? _____________

How could Drake increase the theoretical probability of his name being drawn? ____

____

Coin Experiments

What is the theoretical probability of flipping heads on a coin? ______

What is the theoretical probability of flipping tails on a coin? ______

If you flip a coin 10 times, how many times do you expect it to land on heads? _____ tails? _____

Now flip a coin 10 times. Each time write a tally mark on the chart to record the outcome. Then write the frequency for each outcome.

Heads	Tails
Frequency of heads:	Frequency of tails:

Did you get exactly 5 heads and 5 tails? _____

In 50 tosses, how many times would you expect the coin to land on heads? ____ tails? ____

Now toss the coin 50 times. Record the results on the chart below.

Heads	Tails
Frequency of heads:	Frequency of tails:

What is the theoretical probability of getting heads in 50 coin tosses? _____ tails? _____

Based on the results of 50 tosses, what is the experimental probability of tossing heads? _____ tails? _____

Are the results closer to the theoretical probability of $\frac{1}{2}$ heads and $\frac{1}{2}$ tails with 10 tosses or 50 tosses? _____________

Using a computer-generated program, a coin was tossed 50,000 times. The results were 25,025 heads and 24,975 tails. Then a coin was tossed 100,000 times, and there were 50,007 heads and 49,993 tails. What do you notice happens as the number of tosses increases?

____

____

____

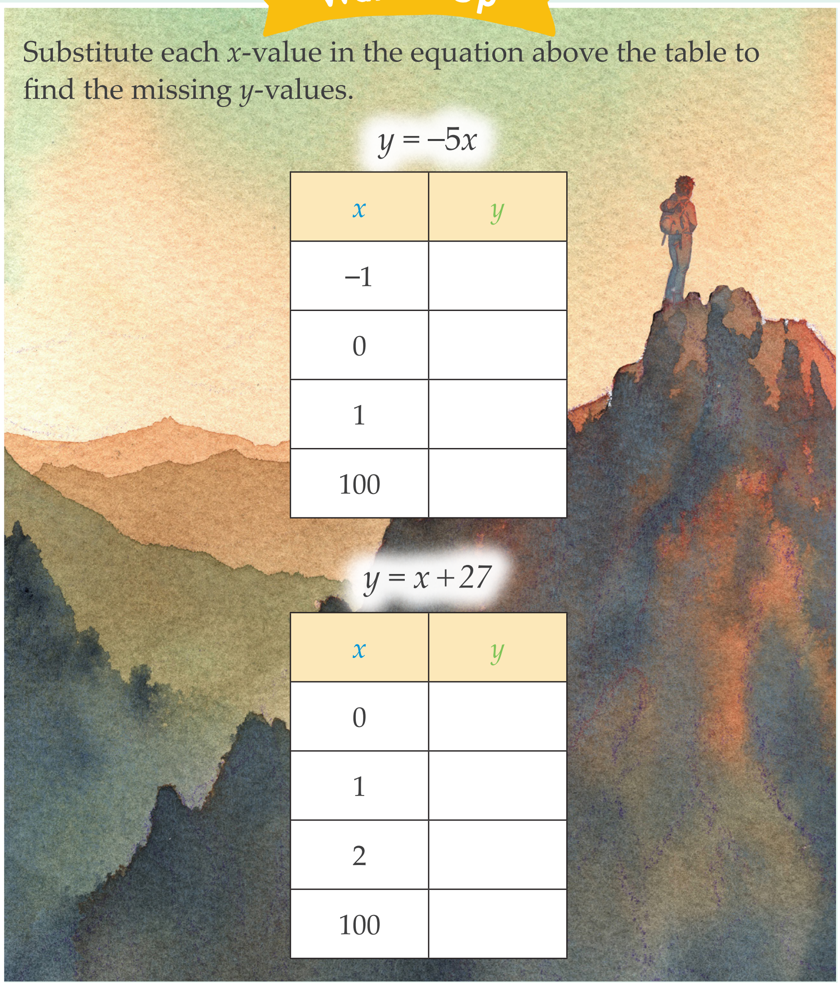
The more experiments that are performed, the closer the experimental probability gets to the theoretical probability!

Lesson 83

GRAPHING LINES

☐ Watch the video lesson and/or read the mini lesson.

Warm-Up



Substitute each x-value in the equation above the table to find the missing y-values.

$y = -5x$

x	y
–1	
0	
1	
100	

$y = x + 27$

x	y
0	
1	
2	
100	

Video Lesson

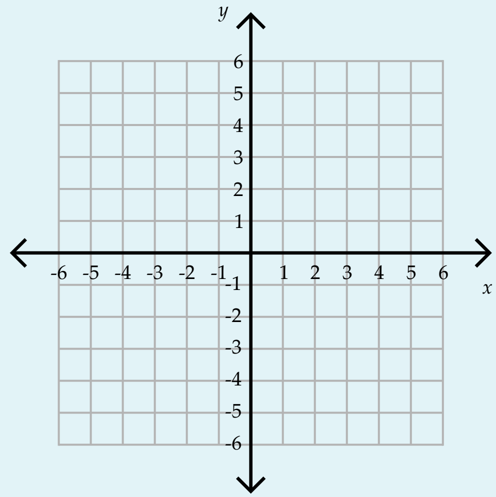
Scan the QR code or watch the video lesson on goodandbeautiful.com/Math6.

x	y
–1	
0	
1	
2	

ordered pairs:

________ ________

________ ________



Mental Math Checkup

1. Count down by $\frac{1}{4}$ from $8\frac{1}{4}$ to 6.

☐, ☐, ☐, ☐, ☐, ☐, ☐, ☐, ☐, ☐

2. Find each percent.

What is 300% of 5? ☐

What is 300% of 50? ☐

3. Multiply or divide.

$1{,}000 \div 50 =$ ☐

$2{,}100 \bullet 4 =$ ☐

Mini Lesson

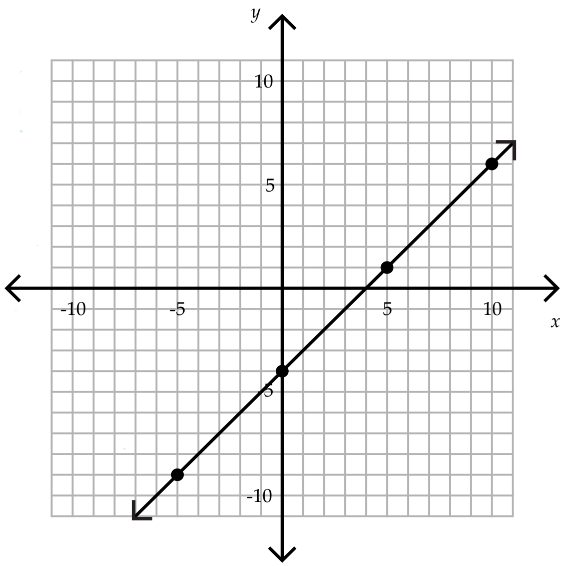
Tables and graphs can be used to show the relationship between independent and dependent variables.

In an equation, table, ordered pair, or graph, the independent variable is the x-value, and the dependent variable is the y-value. On the right is an equation and input-output table from Mini Lesson 82.

$y = x - 4$

x	y
–5	–9
0	–4
5	1
10	6

Each row of the table represents an ordered pair: $(-5, -9)$, $(0, -4)$, $(5, 1)$, $(10, 6)$. The ordered pairs can be plotted on a coordinate plane and connected to form a line. The line is a visual representation of the equation. The graph below shows the line $y = x - 4$.



Note: Arrows are drawn at the ends of the line to show that the relationship continues.

Graphing Equations

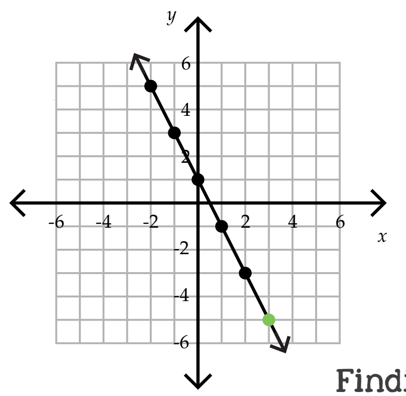
To graph an equation, make a table of values by substituting each x-value into the equation and finding the corresponding y-value. Then create ordered pairs from the table. Plot and connect the ordered pairs to form a line.

Example: Graph the equation $y = -2x + 1$.

Substitute the given x-values into the equation to find the y-values.

x	y
–2	$-2(-2)+1=5$
–1	$-2(-1)+1=3$
0	$-2(0)+1=1$
1	$-2(1)+1=-1$
2	$-2(2)+1=-3$

Create ordered pairs from the table.
$(-2, 5)$, $(-1, 3)$, $(0, 1)$, $(1, -1)$, $(2, -3)$



Plot the ordered pairs. Connect the points with a line.

This graph shows the line $y = -2x + 1$.

Finding Missing Values

Missing values can be found using an equation or a graph. To use the equation, substitute the value of the independent variable, x, in the equation to find the missing value for the dependent variable, y. To use the graph, go across to the x-value and find the y-value on the line.

Example: For the equation $y = -2x + 1$ graphed above, find the y-value when x is 3.

- In the equation, substitute 3 in place of x. $y = -2(3) + 1 = -5$
- On the graph, go over 3, and then go down to the line (green point above). The y-value at that point is –5. When x is 3, y is –5.

Practice

1. Complete each input-output table. Then list the ordered pairs from each table on the lines below the table.

$y = x - 4$

x	y
–1	
0	
1	
2	

_______ _______

_______ _______

$y = 3x$

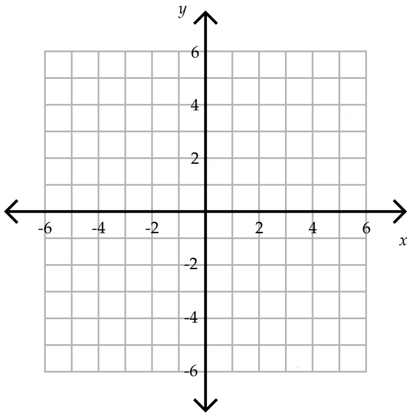
x	y
–1	
0	
1	
2	

_______ _______

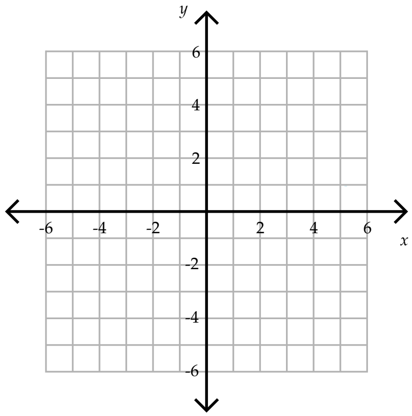
_______ _______

2. Graph the equations from Problem 1 on the coordinate planes below.

$y = x - 4$



$y = 3x$



3. Use the equation $y = -3x - 5$ to answer the questions.

What is the y-value when x is 8? _____

What is the y-value when x is –3? _____

4. Complete each input-output table. For each table, plot the ordered pairs on the graph on the next page and connect the points from the table to form a line. Use a ruler to extend each line to the edge of the graph. Each line will go through one of the triangles, which represent the tallest mountain summit on each continent in the world. Using the key at the bottom, write the name of the mountain on the line under each table. Once the names are written in the blanks, the mountains will be in order from tallest to shortest.

1. $y = 5x - 37$

x	y
6	
7	
8	

Mt. _______

2. $y = -4x + 21$

x	y
3	
4	
6	

Mt. _______

3. $y = x$

x	y
–5	
0	
5	

Mt. _______

4. $y = 0.5x + 4$

x	y
–8	
0	
4	

Mt. _______

5. $y = 3x - 9$

x	y
0	
2	
4	

Mt. _______

6. $y = -x$

x	y
–8	
0	
8	

Mt. _______

7. $y = -3x + 6$

x	y
0	
2	
4	

Mt. _______

- ▲ Aconcagua
- ▲ Vinson
- ▲ Kosciuszko
- ▲ Kilimanjaro
- ▲ Everest
- ▲ Denali
- ▲ Elbrus

LESSONS 89–90

UNIT ASSESSMENT

Instructions

- Unit assessments give you practice with the math concepts learned in this unit without having you overpractice concepts that you have mastered. These assessments also give you practice working on math problems for an extended period of time. This helps you to extend focus and attention span and to be better prepared for any type of testing you will have to do in the future. Here are some tips: First, always read the instructions carefully. Sometimes you can get answers wrong simply because you did not understand the instructions. Second, do not rush through exercises you think you already know. Instead, do your work carefully. Sometimes you can get answers wrong, even though you understand the concept, just because you rushed. Finally, if you feel you are having trouble focusing, take a quick break to do something else, like ten jumping jacks, and then come back. There are no videos, mini lessons, or practice problems for Lessons 89–90.
- **For Lesson 89,** complete all the exercises with purple headers only. You may cover the additional practice sections or fold the page to concentrate only on the purple sections. Have your parent or teacher correct the work. If there are mistakes in a section, your parent or teacher will check the orange "Additional Practice" checkbox for that section.
- **For Lesson 90,** complete all the orange sections that are checked. If you still make multiple mistakes, review those sections. All the principles will be reviewed again in upcoming units. If you have only a few or no orange sections to practice, you may move on to the next lesson.
- Parents/teachers may determine if the student may use the Reference Chart for the assessment. It is recommended that the student first try the assessment without the Reference Chart and then refer to it if needed.

Student

SOLVING EQUATIONS WITH ROOTS AND SQUARED & CUBED VARIABLES (LESSONS 61 & 62)

Solve each equation.

$\sqrt{x} = 7$ __________ $\sqrt[3]{x} = -4$ __________

$x^2 = 64$ __________ $x^3 = -125$ __________

ESTIMATING SQUARE ROOTS (LESSON 63)

Find the two whole numbers each square root is between.

$\sqrt{60}$ is between _____ and _____.

$\sqrt{141}$ is between _____ and _____.

Additional Practice

SOLVING EQUATIONS WITH ROOTS AND SQUARED & CUBED VARIABLES

variable under a square/cube root sign: square/cube both sides
squared/cubed variable: take the square/cube root of both sides
An equation with a squared variable has two solutions.

$\sqrt{x} = 9$ __________ $\sqrt[3]{x} = 3$ __________

$x^2 = 49$ __________ $x^3 = -216$ __________

Additional Practice

ESTIMATING SQUARE ROOTS

Find and simplify the consecutive roots of perfect squares the given square root is between.

$\sqrt{75}$ is between _____ and _____.

$\sqrt{157}$ is between _____ and _____.

DISTRIBUTIVE PROPERTY & FACTORING WITH VARIABLES (LESSON 71)

Use the distributive property to simplify each expression.

$7(2+4a-3) =$ ________________

$\frac{1}{4}(12c+48) =$ ________________

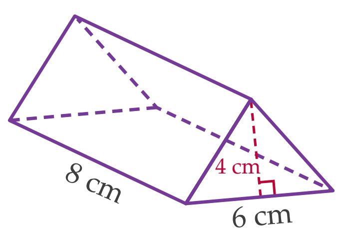
Factor each expression.

$16+40t =$ ________________

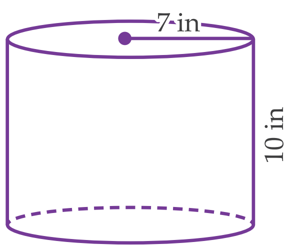
$35b+15 =$ ________________

VOLUME OF PRISMS & CYLINDERS (LESSON 73)

Find the volume of each solid.



$V =$ ______



$V \approx$ __________

Additional Practice

DISTRIBUTIVE PROPERTY & FACTORING WITH VARIABLES

Distribute the value outside the parentheses to each value inside the parentheses. Use the distributive property to simplify each expression.

$4(5d+12-3d) =$ ________________

$\frac{1}{3}(9f-24) =$ ________________

Use the template ___(___ + ___) to factor an addition expression with two terms. The GCF gets written on the first blank. Factor each expression.

$14+49n =$ ________________

$144s+60 =$ ________________

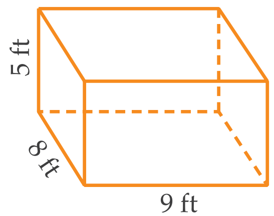
Additional Practice

VOLUME OF PRISMS & CYLINDERS

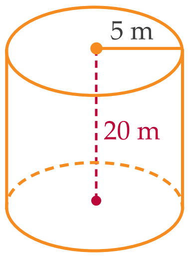
To find the volume of a prism or cylinder, multiply the area of the base by the height. $V =$ *area of base* • *height*

See Reference Chart for area formulas.

Find the volume of each solid.



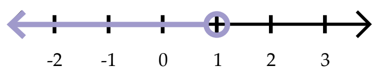
$V =$ ______

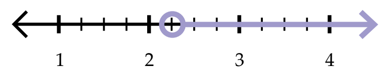


$V \approx$ __________

GRAPHING INEQUALITIES ON NUMBER LINES (LESSON 79)

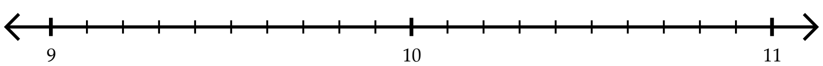
Write the inequality represented by each graph.





____________ ____________

Graph the inequality $x \geq 10.6$.



SOLVING ONE-STEP INEQUALITIES (LESSON 80)

Solve each inequality.

$5 + j > -2$ ____________

$\frac{k}{3} \leq 4$ ____________

$72 \geq 12m$ ____________

SOLVING TWO-STEP INEQUALITIES (LESSON 81)

Solve each inequality.

$11n - 4 < 73$ ____________

$-3 \leq \frac{p}{5} - 13$ ____________

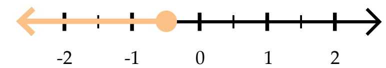
Additional Practice

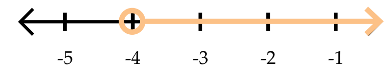
GRAPHING INEQUALITIES ON NUMBER LINES

$<$ less than (open circle) $\leq$ less than or equal to (closed circle)

$>$ greater than (open circle) $\geq$ greater than or equal to (closed circle)

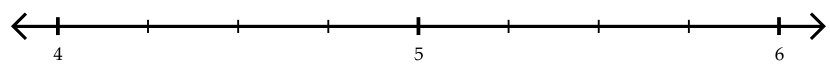
Write the inequality represented by each graph.





____________ ____________

Graph the inequality $x < 5.25$.



Additional Practice

SOLVING ONE-STEP INEQUALITIES

Rules for Solving Inequalities

- The variable must be isolated on one side.
- Inverse operations are used to isolate the variable.
- Whatever is done to one side must be done to the other.
- When a variable is moved from one side of an inequality to the other, the direction of the inequality sign must be reversed.

Solve each inequality.

$q - 8 < 6$ $7r > 42$ $-10 \leq \frac{s}{2}$

____________ ____________ ____________

Additional Practice

SOLVING TWO-STEP INEQUALITIES

Steps to Solve a Two-Step Inequality

- Add or subtract on both sides of the inequality to isolate the term with the variable.
- Multiply or divide on both sides of the inequality to isolate the variable.

Solve each inequality.

$67 > 9t + 4$ ____________ $\frac{v}{8} - 1 \leq 3$ ____________

DEPENDENT & INDEPENDENT VARIABLES (LESSON 82)

Write I for the independent variable and D for the dependent variable.

days since the lawn was mowed ____ length of grass ____

Complete the input-output table using the equation given.

$b = \frac{a}{7}$

a	b
7	
3	
49	
−14	

Fill in the missing values in the table and write the equation.

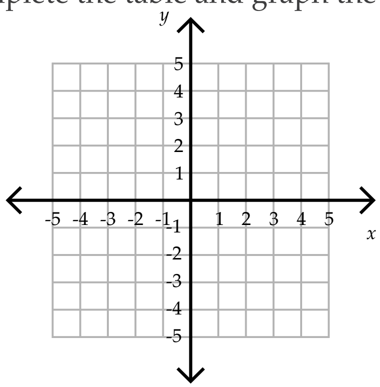
c	d
0	−5
15	10
−3	
5	

____
(equation)

GRAPHING LINES (LESSON 83)

Use the equation $y = 2x - 1$ to complete the table and graph the line.

x	y
−2	
−1	
1	
2	



When x is 0, what is y? ______

Additional Practice

DEPENDENT & INDEPENDENT VARIABLES

A change in the independent variable (input) causes a change in the dependent variable (output).

Write I for the independent variable and D for the dependent variable.

time to bake a cake ____ temperature of the oven ____

Complete the input-output table using the equation given.

$g = 8f$

f	g
1	
−6	
11	
−2	

Determine the rule and fill in the missing values. Use the variables and the rule to write the equation.

equation: ___________

j	k
−4	2
2	8
20	
6	

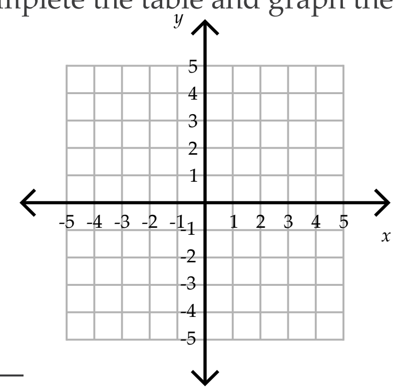
Additional Practice

GRAPHING LINES

Substitute each x-value into the equation and find the corresponding y-value. Each row is an ordered pair. Plot and connect the ordered pairs. Substitute an x-value in the equation or find the x-value on the graph to find a missing y-value.

Use the equation $y = -x + 3$ to complete the table and graph the line.

x	y
−1	
0	
1	
3	



When x is 2, what is y? ______

UNIT 4 OVERVIEW

LESSONS 91–120

Extra Supplies Needed

- colored pencils
- ruler
- scissors
- glue or tape
- protractor
- paper
- tape measure

New Concepts Taught

- solve proportions given part to whole ratios
- solve proportions given part to part ratios
- percent problems with proportions
- unit rates
- corresponding parts of congruent and similar figures
- missing side lengths in similar figures
- parallel lines cut by a transversal
- corresponding angles, alternate interior angles, alternate exterior angles
- polyhedrons and Platonic solids
- statistical questions and surveys
- qualitative and quantitative data
- create circle graphs
- create and interpret line plots with decimal values
- convert square units of area
- conversions using multiple unit multipliers
- calculate measures of central tendency
- interpret and choose measures of central tendency
- create and interpret box plots
- identify first, second, and third quartiles in box plots
- interpret and analyze data displayed graphically
- base 2
- scientific notation
- understand calculator displays and errors on calculators

Concepts Reviewed and Expanded Upon

- scale drawings and map scales
- congruent figures
- similar figures
- regular and irregular polygons
- data, population, sample
- closed-ended and open-ended questions
- bias in statistics
- pictographs
- create and interpret bar graphs
- create and interpret line graphs
- interpret circle graphs
- converting between Fahrenheit and Celsius
- create and interpret histograms
- create and interpret stem and leaf plots
- convert between Fahrenheit and Celsius
- strategies for solving word problems

Lesson 96

SIMILAR & CONGRUENT FIGURES

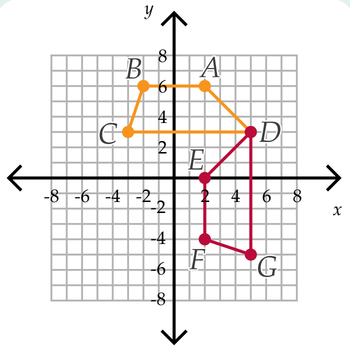
Supplies

8 different colored pencils

Watch the video lesson and/or read the mini lesson.

Warm-Up

Identify each type of transformation shown below. Write the transformation name on the lines. Then write the lengths of the segments.



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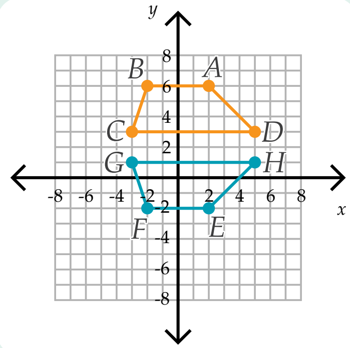

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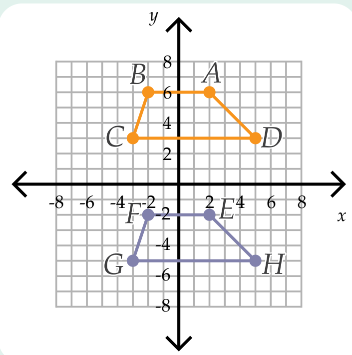


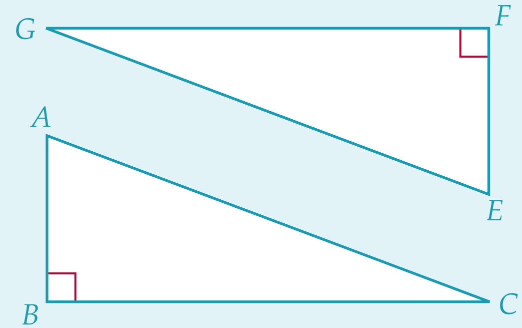
____

$\overline{AB}$ = ___ units

$\overline{EF}$ = ___ units

Video Lesson

Scan the QR code or watch the video lesson on goodandbeautiful.com/Math6.



$\triangle ABC \cong$ _____

$\overline{GF} \cong$ _____

$\angle ACB \cong$ _____

Mental Math Checkup

1. Convert each percent or decimal to a fraction.

   $61\% =$ ☐ $0.625 =$ ☐ $0.\overline{33} =$ ☐

2. Convert each improper fraction to a mixed number or whole number.

   $\frac{32}{9} =$ ☐ $\frac{300}{10} =$ ☐ $\frac{90}{11} =$ ☐

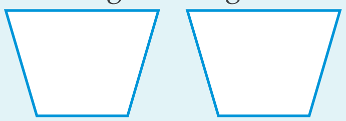
3. Simplify using the order of operations.

   $26 + 4 - 15 =$ ☐ $15 \bullet 3 \div 9 =$ ☐

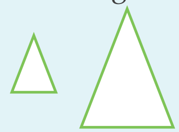
Mini Lesson

Congruent figures have the exact same shape and size. ***Similar*** figures have the exact same shape but not necessarily the same size; similar figures are proportional.

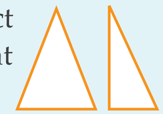
Congruent Figures



Similar Figures

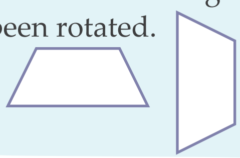


Figures that are the same type of shape but not the exact same shape are not similar. For example, the triangles at the right are not similar.



Figures can be transformed and still be similar or congruent.

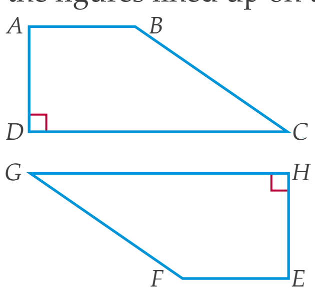
These trapezoids are still congruent even though one has been rotated.



These parallelograms are still similar even though one has been reflected.

Identifying & Naming Corresponding Parts of Congruent Figures

Congruent figures have corresponding sides and angles. To determine corresponding sides/angles in congruent figures, picture the figures lined up on top of each other.



Corresponding vertices for the trapezoids:
A corresponds to *E*, *B* corresponds to *F*, *C* corresponds to *G*, *D* corresponds to *H*

Corresponding figures, sides, and angles must be named in order of corresponding vertices. The symbol $\cong$ is used to show congruence.

trapezoid $ABCD \cong$ trapezoid $EFGH$

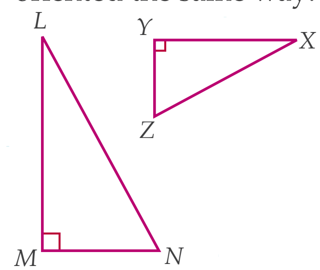
Note: It is incorrect to say trapezoid *ABCD* is congruent to trapezoid *GHEF* because the order of the vertices does not correspond.

Corresponding parts of congruent figures are congruent. The corresponding sides have the same length. The corresponding angles have the same degree measure.

$\overline{AB} \cong \overline{EF}$	$\overline{CD} \cong \overline{GH}$	$\angle DAB \cong \angle HEF$	$\angle BCD \cong \angle FGH$
$\overline{BC} \cong \overline{FG}$	$\overline{DA} \cong \overline{HE}$	$\angle ABC \cong \angle EFG$	$\angle CDA \cong \angle GHE$

Identifying & Naming Corresponding Parts of Similar Figures

Similar figures have corresponding sides and angles. To determine corresponding sides/angles in similar figures, picture the figures oriented the same way.



Corresponding vertices for the triangles:
L corresponds to *X*, *M* corresponds to *Y*, *N* corresponds to *Z*

The symbol $\sim$ is used to show similarity.

$\triangle LMN \sim \triangle XYZ$

Corresponding angles of similar figures are congruent.

$\angle LMN \cong \angle XYZ$ $\angle MNL \cong \angle YZX$ $\angle NLM \cong \angle ZXY$

Corresponding sides of similar figures are not congruent if the figures are different sizes.

Finding Corresponding Parts Without a Picture

When a picture is not given, identify corresponding vertices based on their position in the given name. For example, if $\triangle RST$ and $\triangle UVW$ are congruent, then *R* corresponds to *U*, *S* corresponds to *V*, and *T* corresponds to *W*.

$\overline{RS} \cong \overline{UV}$, $\overline{ST} \cong \overline{VW}$, $\overline{TR} \cong \overline{WU}$

$\angle RST \cong \angle UVW$, $\angle STR \cong \angle VWU$, $\angle TRS \cong \angle WUV$

Missing angle measures and side lengths can also be found.

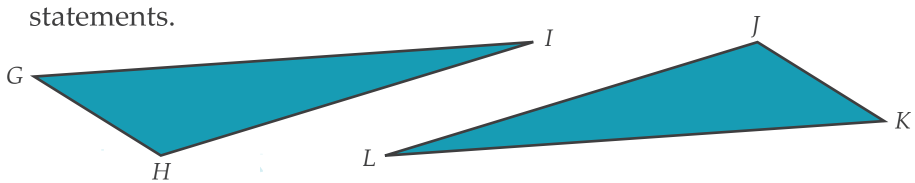
Example: If $m\angle RST$ is 45°, what is $m\angle UVW$?
Since $\angle RST \cong \angle UVW$, $m\angle UVW = m\angle RST$, so $m\angle UVW$ is 45°.

Practice

1. Determine whether each statement is always, sometimes, or never true and put a check mark in that box. An example is given.

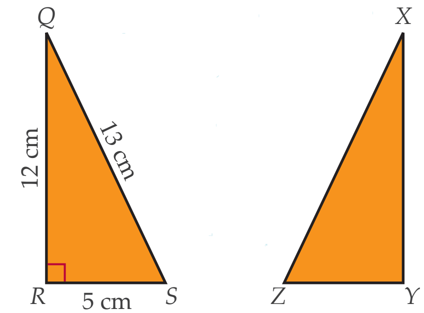
	always true	sometimes true	never true
Two squares are congruent.		✓	
Two squares are similar.			
Two rectangles are similar.			
Two similar triangles are congruent.			
Two congruent triangles are similar.			
Two similar triangles will have congruent angles.			
An equilateral triangle and a square are similar.			
An acute triangle and a right triangle are similar.			
Two right triangles are similar.			

2. The triangles below are congruent. Complete the congruence statements.



$\triangle GHI \cong$ ______ $\overline{JK} \cong$ ____ $\angle GIH \cong$ ______

3. The triangles below are congruent. Complete the congruence statements and write the measurements.



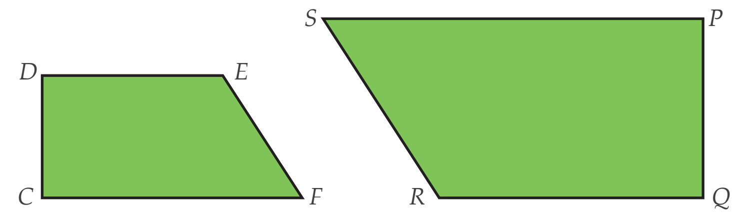
$\triangle QRS \cong$ ______

$\angle ZXY \cong$ ______

$\overline{ZY} =$ ______

$m\angle ZYX =$ ______

4. The trapezoids below are similar. Complete the similarity statement. Then list 4 pairs of congruent angles.



trapezoid $CDEF \sim$ trapezoid ______

______ $\cong$ ______ ______ $\cong$ ______

______ $\cong$ ______ ______ $\cong$ ______

5. $\triangle ABC$ and $\triangle MNO$ are congruent. Use the information below to find each angle measure.

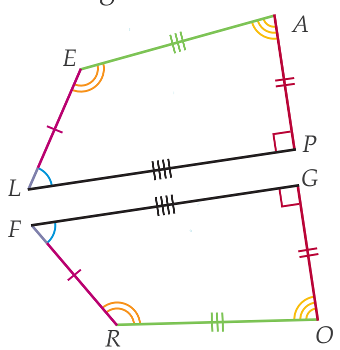
$m\angle ABC = 110°$ $m\angle NOM = 45°$

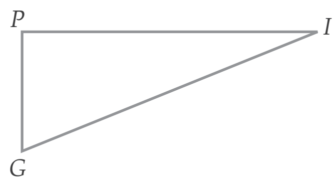
$m\angle MNO =$ ______ $m\angle BCA =$ ______

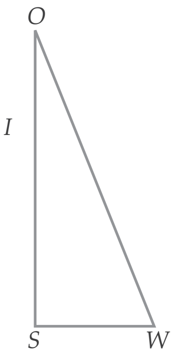
$m\angle CAB =$ ______ $m\angle OMN =$ ______

Practice

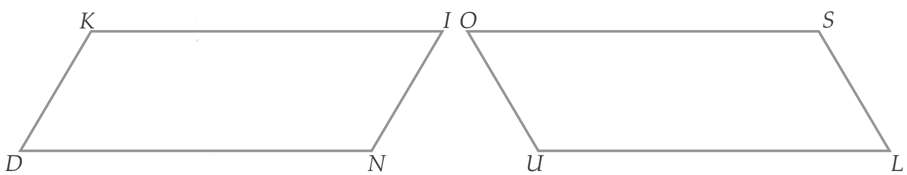
6. Each pair of figures below is either congruent or similar. Use colored pencils to show the corresponding parts of each pair. Then name the figures and make a statement about their congruence or similarity. An example is given.



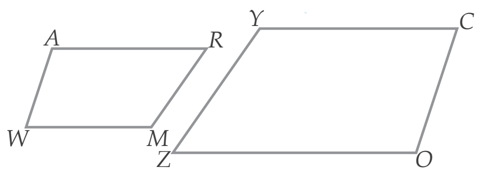


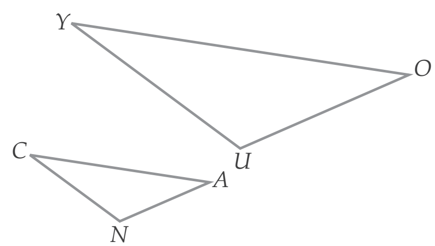


trapezoid *LEAP* ≅ trapezoid *FROG* ____________________



____





____________________ ____________________

Review

1. Create a unit rate using the given information. Then use it to answer the question. Lesson 95

There are 52 weeks in one year. How many weeks are in 4 years?

unit rate ________ answer ________

There are 60 minutes in one hour. How many minutes are in 7 hours?

unit rate ________ answer ________

There are 24 hours in one day. How many days are in 144 hours?

unit rate ________ answer ________

2. Evaluate each expression for the values given and write the answer on the line. Lesson 35

$b^2 + c^3 + 9$ $b = 7$ $c = 2$ _____

$5d - 8e - 17$ $d = 8$ $e = 5$ _____

3. Simplify each expression. Lesson 46

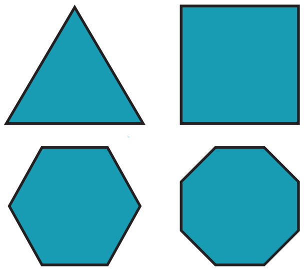
$8h - 11g + 19h + 2g$ ________ $-16q - 8p - 10q + 12p$ ________

$-3 + 9t + 21 - 4t$ ________ $14u - 9v + 25$ ________

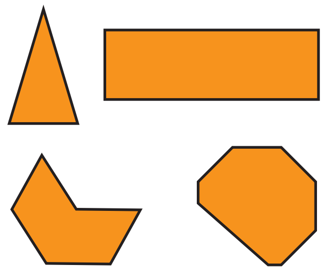
Mini Lesson

A ***polygon*** is a two-dimensional, closed shape with straight sides. Examples of polygons are triangles, rectangles, hexagons, and octagons. A ***regular polygon*** is a polygon that has all sides of equal length and all angles of equal measure. Polygons that are not regular are called irregular polygons.

Regular Polygons



Irregular Polygons

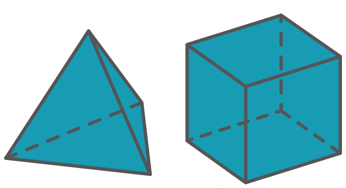


Some regular polygons have special names. For example, a regular triangle is called an equilateral triangle. A regular quadrilateral is called a square.

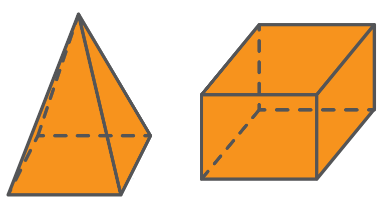
A ***polyhedron*** is a three-dimensional figure with polygons as faces. Examples of polyhedrons are prisms and pyramids; their faces are made up of polygons. Since polygons do not have curved edges, polyhedrons do not have curved surfaces. Spheres and cylinders are not polyhedrons because they have curved surfaces.

Just as there are regular polygons, there are also regular polyhedrons. A regular polyhedron is a polyhedron that has congruent regular polygons as faces.

Regular Polyhedrons



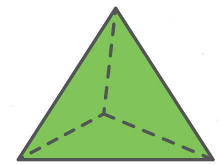
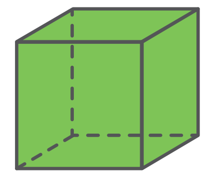
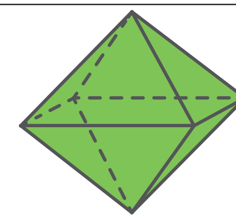
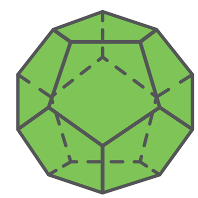
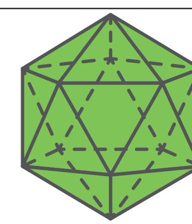
Irregular Polyhedrons



The five regular polyhedrons below are referred to as Platonic solids.

Platonic solids are named for the Greek philosopher Plato.

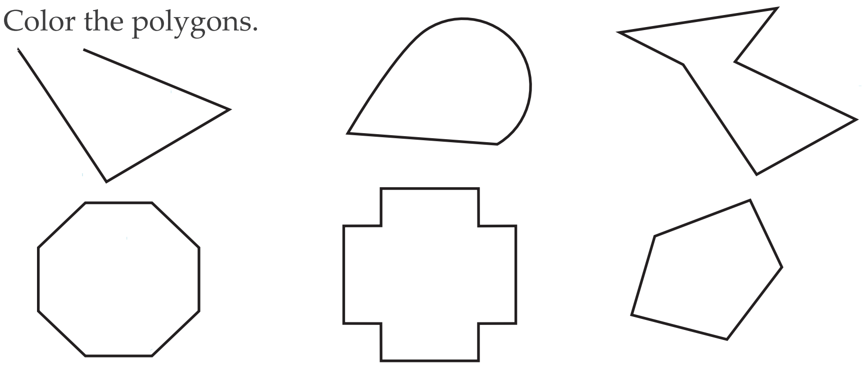
Platonic Solids

Tetrahedron *All faces are equilateral triangles.* *There are 4 faces.*	
Cube *All faces are squares.* *There are 6 faces.*	
Octahedron *All faces are equilateral triangles.* *There are 8 faces.*	
Dodecahedron *All faces are regular pentagons.* *There are 12 faces.*	
Icosahedron *All faces are equilateral triangles.* *There are 20 faces.*	

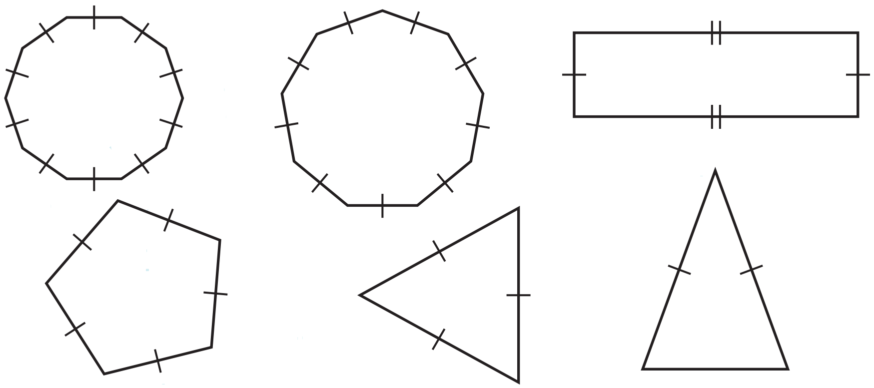
Practice

Hint: Use a marker to mark vertices and edges that you have already counted.

1. Color the polygons.



2. Color the regular polygons green and the irregular polygons red.



3. Write *T* next to statements that are true and *F* next to statements that are false.

____ All polyhedrons are Platonic solids.

____ All Platonic solids are polyhedrons.

____ A cylinder is a polyhedron but not a Platonic solid.

____ Square pyramids are Platonic solids.

____ Octahedrons are polyhedrons.

4. Cut out the two templates for Platonic solids on the next page. Be careful not to cut off the tabs. Then fold on the black lines and glue the tabs to create a tetrahedron and a dodecahedron. Once they are assembled, use the Platonic solids that you created to fill in the chart and answer the questions below the chart.

	number of vertices	number of edges	number of faces
tetrahedron			
*cube			
*octahedron			
dodecahedron			

**The rows for cube and octahedron are completed during the video for this lesson.*

Circle the answer:

All of the numbers in the table are *even* / *odd* numbers.

Which two Platonic solids have the same number of edges?

____________________ and ____________________

For each Platonic solid, take the number of vertices (*V*), subtract the number of edges (*E*), and then add the number of faces (*F*). You should get the same answer every time. Write the number in the blank to complete the formula.

$$V - E + F = \_\_\_\_\_\_$$

An icosahedron has 20 faces and 12 vertices. Using the formula you just discovered, find the number of edges in an icosahedron:

Number of edges: ______

Lesson 103

HISTOGRAMS

Supplies

ruler

Watch the video lesson and/or read the mini lesson.

Warm-Up

Below are the distances (in meters) jumped by female skiers in the first round of the 2018 Olympics ski jumping event. Complete the frequency table from the given data.

56	77.5	84.5	86.5	89.5	94	101.5
71.5	78	85	86.5	91.5	97	102.5
72	80	85	88	93	98.5	103.5
74.5	80.5	86	88	93	99	105.5
77	83	86.5	88.5	93.5	101	106.5

Distance (m)	Frequency
50–60	
60–70	
70–80	
80–90	
90–100	
100–110	

When a data point is on the edge of an interval, it goes in the *upper* interval.

Video Lesson

Scan the QR code or watch the video lesson on goodandbeautiful.com/Math6.

Time (sec)	Frequency
30–35	
35–40	
40–45	
45–50	
50–55	
55–60	
60–65	
65–70	
70–75	

Time (sec)	Frequency
30–37	
37–44	
44–51	
51–58	
58–65	
65–72	

Mental Math Checkup

1. Convert each percent or decimal to a fraction.

45% = ☐ 0.24 = ☐ 67% = ☐

2. Convert each improper fraction to a mixed number or whole number.

$\frac{124}{10}$ = ☐ $\frac{89}{11}$ = ☐ $\frac{300}{15}$ = ☐

3. Simplify using the order of operations.

$5 \bullet 12 \div 6 =$ ☐ $22 - 6 \bullet 2 =$ ☐

Mini Lesson

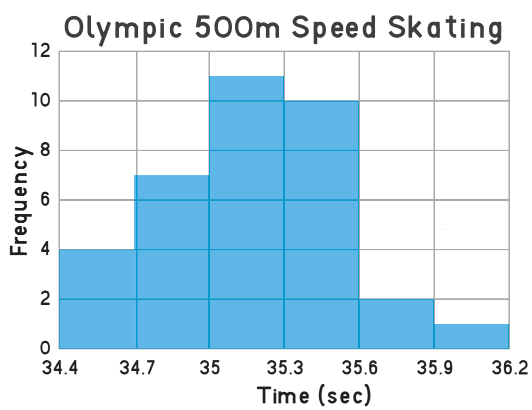
A ***histogram*** is a bar graph that displays data in intervals of equal size with no space between the bars. Creating a frequency table can help to organize the data needed to create a histogram. Each interval of a frequency table shows a range of values and corresponds to one bar on the histogram.

Example: Create a frequency table and histogram for the data below.

Here are the top 35 times from the 2018 men's Olympic speed skating 500 meter race (in seconds):

34.410	34.420	34.650	34.680	34.780	34.831	34.839
34.890	34.900	34.930	34.934	35.010	35.020	35.080
35.130	35.154	35.158	35.160	35.192	35.196	35.220
35.230	35.310	35.330	35.340	35.380	35.410	35.500
35.506	35.510	35.545	35.546	35.640	35.860	35.920

Time (sec)	Frequency
34.4–34.7	||||
34.7–35.0	𝍸 ||
35.0–35.3	𝍸 𝍸 |
35.3–35.6	𝍸 𝍸
35.6–35.9	||
35.9–36.2	|

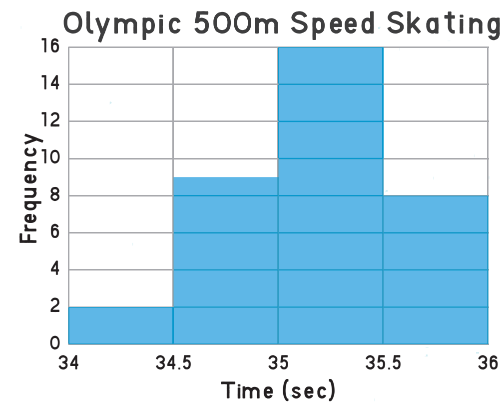


This histogram and frequency table are organized with intervals of size 0.3 seconds. Notice that the minimum value for the intervals is 34.4 seconds, and the maximum is 36.2 seconds.

When making a frequency table and histogram, it is important to pick intervals that allow all the data to be seen easily. The same data set is shown in the next two graphs but with different interval sizes.

This frequency table and histogram use intervals of size 0.5 seconds. The minimum value for the intervals is 34.0 seconds, and the maximum is 36.0 seconds.

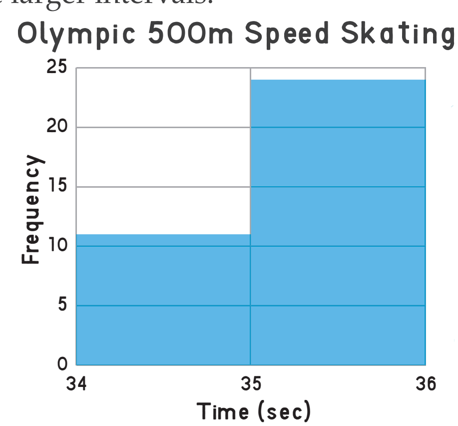
Time (sec)	Frequency
34.0–34.5	||
34.5–35.0	𝍸 ||||
35.0–35.5	𝍸 𝍸 𝍸 |
35.5–36.0	𝍸 |||



Notice how this histogram only has four bars of data instead of six bars. This is because the intervals are larger, so one bar represents a greater range of time, and fewer bars are needed to cover the same data.

Here is the same data again but this time with intervals of size 1.0 seconds. The minimum value for the intervals is still 34.0 seconds, and the maximum is 36.0 seconds. Once again, there are fewer bars for this histogram because of the larger intervals.

Time (sec)	Frequency
34.0–35.0	𝍸 𝍸 |
35.0–36.0	𝍸 𝍸 𝍸 𝍸 ||||



The first graph with six bars makes it easiest to see all the data because of the smaller intervals.

Practice

Use the data below for all problems on this page.

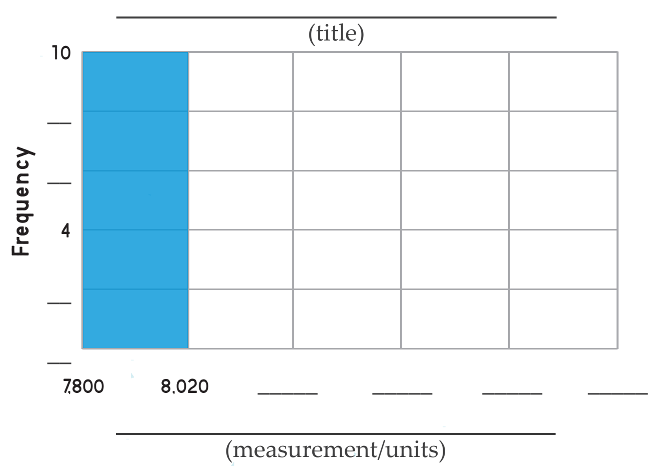
The heights of some of the world's highest mountain peaks (rounded to the nearest meter) are listed below:

8,848	8,611	8,188	7,893	8,516	8,126
8,051	7,884	8,586	8,080	7,821	7,871
8,167	7,952	8,163	8,091	7,823	7,932
8,035	7,804	8,485	8,027	7,937	7,816

1. Using a minimum value of 7,800 m and a maximum value of 8,900 m for the intervals, create a frequency table for the given data with intervals of 220 m. An example is given.

Height (m)	Frequency
7,800–8,020	𝍸 𝍸

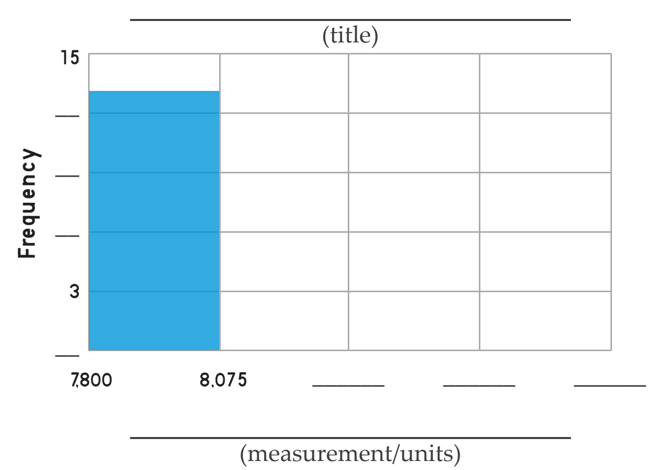
Create a histogram for this data based on the intervals you used above. Be sure to label and title your graph.



2. Using the same data set and minimum and maximum values for the intervals, create a frequency table for the data with intervals of 275 m. An example is given.

Height (m)	Frequency
7,800–8,075	𝍸 𝍸 |||

Create a histogram for this data based on the intervals you used above. Be sure to label and title your graph.



3. How are these two histograms similar? ____________________

____

How are these two histograms different? ____________________

____

Do you think one of them represents this data set better than the other? Why or why not?

____

LITERARY Lengths

Using a ruler, measure the length of 20 different books around your home. You may use any units you wish, but you should use the same units for all measurements. Record your measurements here:

1. ________ 2. ________ 3. ________ 4. ________ 5. ________

6. ________ 7. ________ 8. ________ 9. ________ 10. ________

11. ________ 12. ________ 13. ________ 14. ________ 15. ________

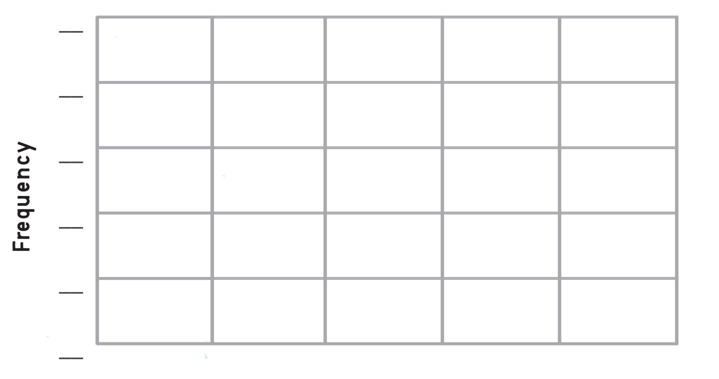
16. ________ 17. ________ 18. ________ 19. ________ 20. ________

Choose a minimum and maximum value for your intervals that will include all of your measurements, and divide this range into 5 equal intervals. Use these intervals to create a frequency table for your data. Make sure you fill in the units for length in parentheses.

Length ( )					
Frequency					

Now create a histogram for this data based on the intervals you used above. Be sure to label and title your graph.

____
(title)



____
(measurement/units)

Review

1. Write *C* next to closed-ended questions and *O* next to open-ended questions. Lesson 100

___ What is your favorite month?

___ What time do you get up in the morning?

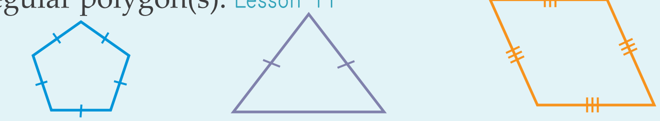
___ Do you prefer Mexican or Chinese food?

___ Do you like to sing?

___ Do you floss before or after brushing?

___ What's your favorite sport?

2. Write *R* inside the regular polygon(s) and *I* inside the irregular polygon(s). Lesson 99



3. Use the information below to fill in the blanks. Lesson 66

principal amount: $2,400 compound interest rate: 2%

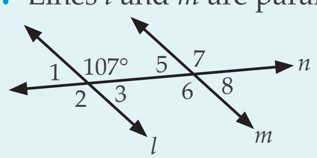
interest earned during year 1: __________

total amount after one year: __________

interest earned during year 2: __________

total amount after two years: __________

4. Lines *l* and *m* are parallel. Write the answers on the lines. Lesson 98



m∠3 = _____ m∠6 = _____

Which angle corresponds with the given angle? _______

Lesson 110

WORD PROBLEM STRATEGIES

There is no video for this lesson.

Today you get to imagine that you are a secret math agent with an upcoming mission in an undisclosed location! Each word problem you solve has a letter or number underneath it. On the last page, write the letter or number from each problem on the line above its answer. When you are finished with the problems, unscramble the letters and/or numbers in each box to learn about your next mission.

Word Problem Strategies

Sometimes word problems can seem intimidating at first, but now that you're a secret math agent, you should possess some strategies for tackling them without fear! The "secret" to solving word problems is to determine what is being asked, what information is given, and what information is needed to answer the question. Some word problems include extra information that is not needed.

These strategies may be helpful when solving word problems:

- Draw a picture or diagram.
- Make a list, table, or chart.
- Write and solve an equation.
- Find a pattern.
- Use a smaller or simpler case.
- Guess, check, and revise.

In this lesson you will be introduced to some of these strategies, and then you will practice using them to solve problems.

Determining What Is Asked, Given & Needed

Example: Five friends are going to a movie. Movie tickets cost $9 per person. The theater can seat 300 people. How much will the five friends pay for tickets?

What is being asked? How much will five movie tickets cost?
What is given? 5 friends, $9 per ticket, 300 seats
What is needed? the cost per ticket

It may be helpful to cross out information that is not needed:

Five friends are going to a movie. Movie tickets cost $9 per person. ~~The theater can seat 300 people.~~ How much will the five friends pay for tickets?

Now answer the question. Use a problem-solving strategy if desired. For example, drawing a picture or diagram might be helpful:

$9	$9	$9	$9	$9

$9 + $9 + $9 + $9 + $9 = 5 • $9 = $45. The friends will pay $45.

Determine what is being asked, what is given, and what is needed in the word problem below. Then solve the problem.

1. Bentley is inviting 11 friends over for a barbecue. He will need 24 hot dogs. There are 8 hot dogs in a package, and there are 10 packages of hot dogs in a case. How many packages of hot dogs should Bentley buy?

   What is being asked? ______________________

   What is given? ______________________

   ____

   What is needed? ______________________

Cross out any information in the word problem that isn't needed.

Answer: ____ packages

S

As you practice each problem-solving strategy, continue to consider what is being asked, what is given, and what is needed.

Draw a Picture or Diagram

Example: Cambree is cutting a 6-foot-long board into pieces that are $\frac{1}{3}$ of a foot long. How many pieces will there be?

<table>
<tr><td colspan="3">1 ft</td><td colspan="3">1 ft</td><td colspan="3">1 ft</td><td colspan="3">1 ft</td><td colspan="3">1 ft</td><td colspan="3">1 ft</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

The diagram shows each foot of the 6-foot board. The smaller pieces are $\frac{1}{3}$ of a foot. There are 18 of them, so there will be 18 pieces.

Solve the problems on this page by drawing a picture or a diagram.

1. Josh is lining up his toy boats and toy cars next to each other. The boats are 10 cm long, and the cars are 6 cm long. When will the two lines of toys first be the same length?

at ____ cm
R

2. Cadence is laying tile in her foyer, which is a square with an area of 81 ft^2. How many 18-inch by 18-inch square tiles will she need to cover the foyer?

____ tiles
C

3. Robert is planting a garden. One-third of the garden will be corn, and $\frac{1}{3}$ will be potatoes. He will split the remaining area into three parts, two of which will be onions, and one of which will be radishes. What fraction of the garden will be onions, and what fraction will be radishes?

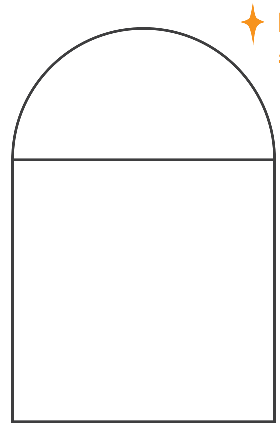
____ onions ____ radishes
O E

4. The perimeter of a rectangle is 58 meters. One side measures 11 meters. What is the area of the rectangle?

____ m^2
M

5. Elias is making a stained glass window that is a square with a semicircle on top of it. The diameter of the semicircle is 6 ft. What is the area of the window?

Hint: A picture is given. Find the area of the square and the semicircle and add them together.



____ ft^2
2

Make a List, Table, or Chart

Example: If a recipe that makes 30 cookies uses 2 eggs, how many eggs are needed to make 75 cookies?

Make a table. Fill in the given values, and then find the missing values. Continue the table until the desired value is reached.

Eggs	1	2	3	4	5
Cookies	15	30	45	60	75

Five eggs are needed to make 75 cookies.

Solve the problems below by making a list, table, or chart.

1. If two large pizzas cost $17, how much will five large pizzas cost at the same unit price?

$ ____
N

2. How many outfits can be made with three shirts, three pairs of pants, and two pairs of boots? Hint: Name the shirts S1, S2, and S3; the pants P1, P2, and P3; and the boots B1 and B2. Consider drawing a tree diagram. Develop an organized way to create a list of possible outfits. Then count the outfits on the list.

____ outfits
L

Write and Solve an Equation

Example: Six out of 40 science club members are also in a book club. What percent of science club members are in the book club?

Think of what is being asked. ⟶ What percent of 40 is 6?

Write an equation. Use a variable for the unknown. Use multiplication for "of" and equals for "is."

$$x \cdot 40 = 6$$

Divide both sides by 40 to get $x = 0.15$. Because the question asked for a percent, convert the decimal to a percent. $x = 15\%$

15% of science club members are in the book club.

Solve the problems below by writing and solving an equation.

1. Out of 90 restaurant customers, 36 chose roasted asparagus as a side dish. What percent of customers chose the asparagus?

____ %
K

2. If you triple Martin's age and add 4, you get his mom's age. Martin's mom is 43. How old is Martin?

____ years old
6

3. Marilyn divides the money in her purse evenly among her 6 grandchildren. If each grandchild receives $4.62, how much money was in Marilyn's purse?

$ ____
Y

Use a Smaller or Simpler Case

Example: What number is multiplied by 55 to get 4,015?

If you're not sure what operation to use to get the answer, think of a smaller or simpler case. For example, what number is multiplied by 5 to get 30? It's 6. And 30 divided by 5 is 6. So to find the answer to the question, divide.

$$4{,}015 \div 55 = 73$$

Solve the problems on this side of the page by thinking of a smaller or simpler case.

1. What number is 19 multiplied by to get 1,558?

   A

2. What number do you divide 777.6 by to get 9?

   Hint: Think of a smaller case, such as "What number do I have to divide 27 by to get 9?" and think of the relationships between the numbers.

   T

3. What do you have to add to –14 to get 95?

   Hint: What do you have to add to 90 to get 95? What is the mathematical relationship between the numbers?

   U

4. What do you have to add to 23.78 to get 32?

   J

Congratulations, secret math agent! You have proven that you are ready for more mathematical missions. You are to report to the Palace Museum in Beijing, China. Unscramble the letters to reveal more details about the meeting.

___ ___ ___ ___ ___ ___

$\frac{2}{9}$ 36 36 3 18 40

Meet at the Gallery of ___ ___ ___ ___ ___ ___

___ ___ ___ ___ ___ ___

109 13 27.72 50.13 18 8.22

Date: ___ ___ ___ ___ ___ ___

___ ___ ___ ___ ___ ___ ___ ___

82 3 $\frac{1}{9}$ 86.4 $\frac{1}{9}$ 30 86.4 $\frac{2}{9}$

Your Code Name (a living thing): ___ ___ ___ ___ ___ ___ ___ ___

___ ___ ___ ___ ___ ___

$\frac{2}{9}$ 30 82 42.50 198 $\frac{2}{9}$

Your Mission Leader's Code Name (a color): ___ ___ ___ ___ ___ ___

Note: There is no review for this lesson.

Lesson 118

COURSE REVIEW

Supplies

ruler

There are approximately 18,000 species of butterflies in the world. They come in a variety of colors and sizes and can be found almost all over the world, even in parts of the arctic tundra. You'll learn more about these magnificent insects as you review concepts from Math 6.

Note: Although Lesson 117 was an introduction to calculators, calculators should not be used on this review.

Factors & Factoring

Lessons 2, 3, 48 & 71

Viceroy and monarch butterflies have a lot in common. Their patterns and coloring are almost the same. Throughout this course you have learned to find and use the *greatest common factor* of numbers.

Write the prime factorization of each number. Then find the GCF of the numbers.

140: ________________

GCF: ______

98: ________________

Factor each expression.

$98v + 140$ ______________

$15 + 60m$ ______________

Use the distributive property to simplify each expression.

$11(12 + 7b + 4)$ ______________

$\frac{2}{3}(39f + 15)$ ______________

Operations with Fractions, Decimals & Integers

Lessons 9, 12–13, 17, 25, 31–32 & 56

Butterflies have a four-stage life cycle. In the problems below, perform the four basic operations with fractions, decimals, and integers.

Simplify.

$\dfrac{\frac{2}{3}}{\frac{4}{15}} =$ _____

$\frac{5}{8} + \frac{3}{16} =$ _____

$\frac{5}{8} \div \frac{3}{16} =$ _____

$5\frac{1}{2} - 2\frac{2}{3} =$ _____

$22.1 - 1.58 =$ _____

$163 + 98.6 =$ _____

$68.04 \div 5.4 =$ _____

$19.2 \bullet 8.5 =$ _____

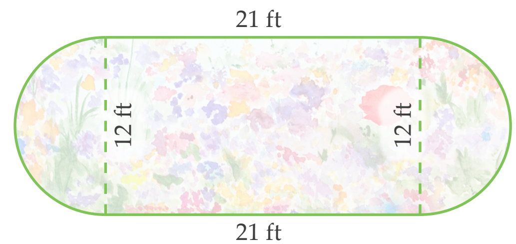
$29 + (-56) =$ _____

$-23 \bullet 15 =$ _____

Area, Perimeter & Volume

Lessons 22, 42, 73 & 107

Butterflies are pollinators. Suppose a flower garden with the outline shown below was planted to attract butterflies. Find the area and perimeter of the garden. Use 3.14 for π.



$A \approx$ ________ ft^2

$P \approx$ ________ ft

Convert the area to square yards.

$A \approx$ ________ yd^2

Suppose the butterfly conservatory at a botanical garden is built in the shape of a rectangular prism. The area of the base is 9,200 ft^2, and it is 20 feet tall. Find the volume of the conservatory in cubic feet.

$V =$ __________ ft^3

Converting Units & Scientific Notation

Lessons 67–69, 106, 109 & 116

Suppose that a pupa (chrysalis) took two weeks to transform into a butterfly. Convert two weeks to hours.

2 weeks = ________ hours

A female Queen Alexandra's birdwing has a wingspan of up to 28 cm and can weigh up to 12 g. Convert 28 cm to kilometers, and then rewrite the answer in scientific notation. Convert 12 g to milligrams, and then rewrite the answer in scientific notation.

28 cm = _________ km = ____ × ________ km

12 g = __________ mg = ____ × ________ mg

Butterflies are cold-blooded, and most are unable to fly when the temperature is below 60 °F. Convert 60 °F to degrees Celsius. Round to the nearest tenth.

60 °F ≈ ______ °C

Suppose a watering can at a botanical garden holds 1.5 gallons. How many cups does it hold?

_____ c

Solving Equations & Inequalities

Lessons 49–50, 61–62, 72 & 81

Solve each equation or inequality.

$7.2b = 64.8$

$b =$ _____

$89 = 8u - 15$

$u =$ _____

$t^2 = 64$

$t =$ _____, $t =$ _____

$e + 15 = -2$

$e =$ _____

$\sqrt[3]{r} = -4$

$r =$ _____

$\frac{f}{8} - 41 = 11$

$f =$ _____

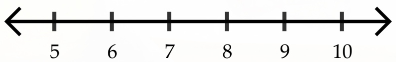
$12 \geq 3l + 9$

____

Solve the inequality and graph the solution on the number line below.

$8y - 47 > 17$

____



Percents, Ratios & Proportions

Lessons 52–55, 85 & 91–93

Some butterflies use camouflage to avoid predators. Some species eat toxins as caterpillars and become poisonous. Other species have coloring similar to poisonous species, so predators avoid them as well.

Just as there is more than one way for butterflies to avoid predators, you have learned more than one way to solve problems involving percents. Solve each problem below using the method of your choice.

What is 15% of 114? _____

30 is 24% of what number? _____

84 is what percent of 120? _____

The ratio of children to adults at a butterfly conservatory is 2 : 5. There are 203 people at the conservatory.

How many of them are children? _____

Three out of every 16 butterflies in a butterfly conservatory are monarchs. There are 546 butterflies that are NOT monarchs.

How many butterflies are in the conservatory? _____

Percent Discounts & Tax

Lessons 64 & 65

Alek plans to buy a pair of binoculars for observing butterflies. Find each amount below. Round to the nearest cent.

original price: $18.98

percent discount: 10%

tax rate: 6%

amount of discount: _________

sale price: _________

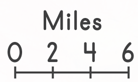
amount of tax: __________

total cost: __________

Scale Drawings

Lesson 94

Brian and Arlene are using a city map to find a butterfly conservatory. Use a ruler (centimeter side) to find the map scale.



____________ : ____________

Arlene measures the distance from their hotel to the conservatory on the map. The measured distance is 0.75 cm.

What is the actual distance? ____________

Proportions on a Graph

Lessons 82 & 86

Create a ratio table and answer the questions for the scenario below. Then graph the equation.

If 4 tickets to a butterfly conservatory cost $10, how much will 6 tickets cost? ______

x	y
2	
4	10
6	
8	

Write an equation to represent the scenario.

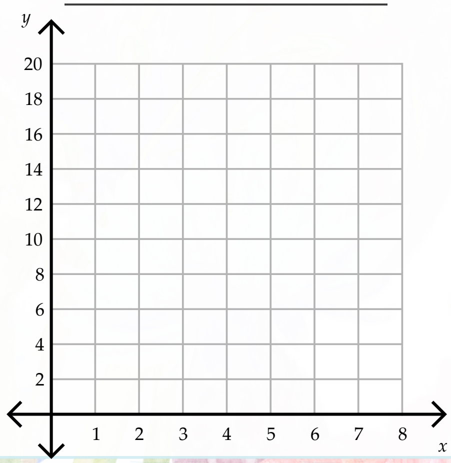
____

What is the independent variable?

____

What is the dependent variable?

____



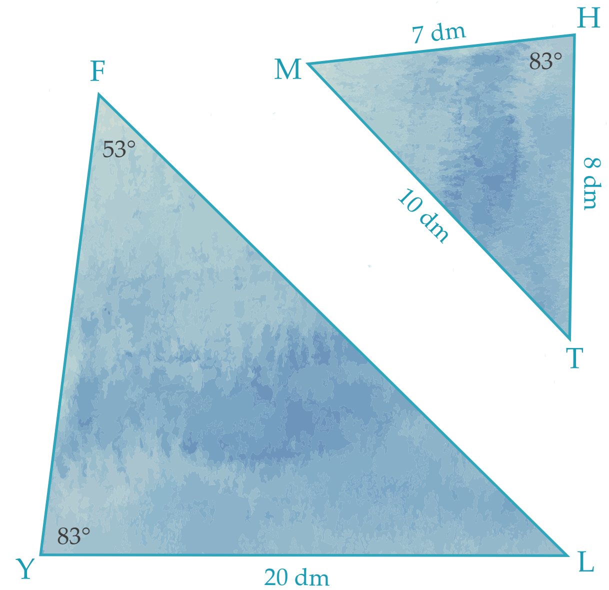
Similar Figures

Lessons 6, 96 & 97

Butterflies and moths are genetically *similar*.

Complete the statements below for the *similar* triangles.

$\triangle MTH \sim \triangle FLY$



$m\angle FLY =$ _____

$m\angle TMH =$ _____

$m\angle MTH =$ _____

$FY =$ ________

$FL =$ ________

$\angle YFL \cong$ _______