LESSONS
1–30

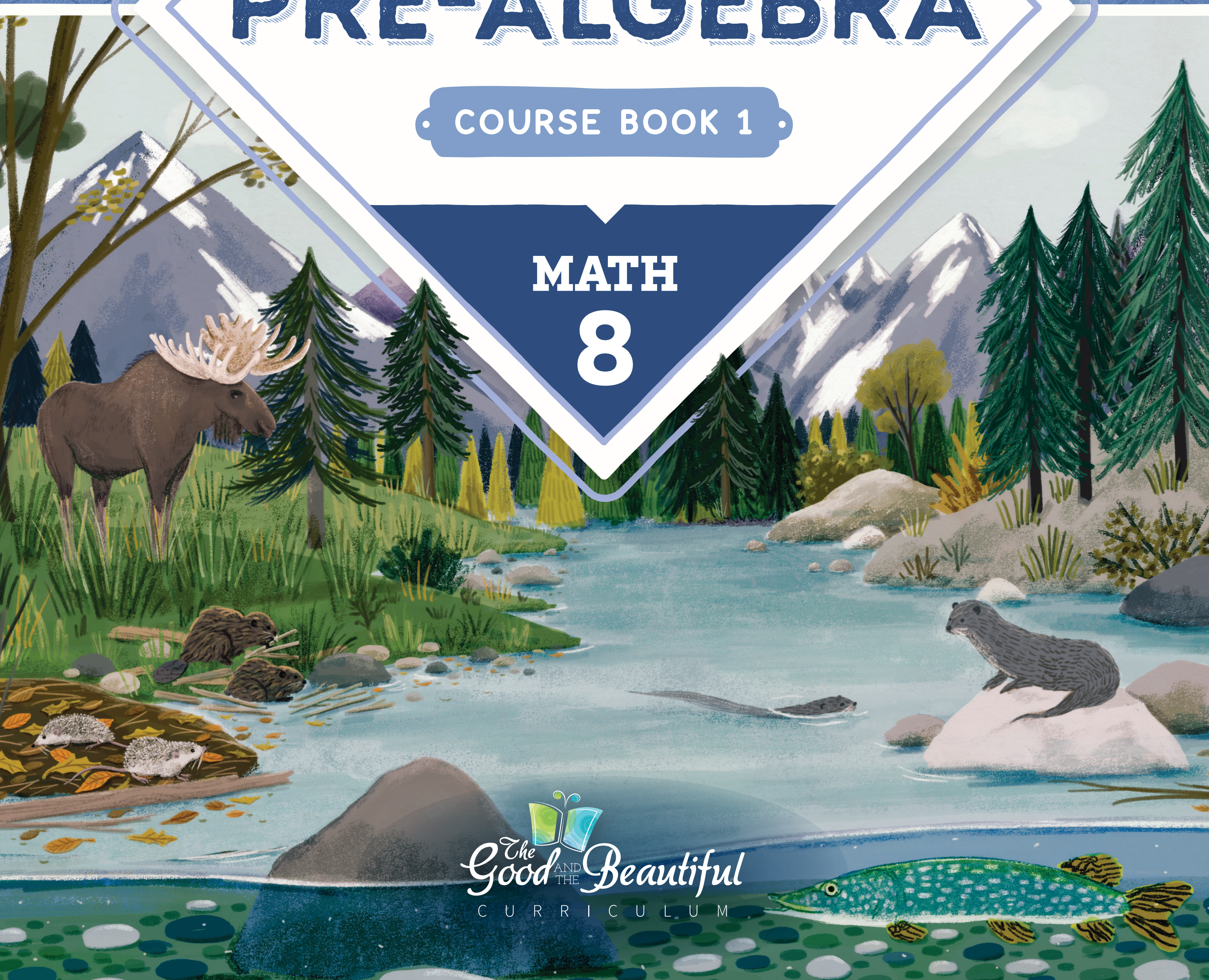
UNIT
1

Simply Good and Beautiful

PRE-ALGEBRA

COURSE BOOK 1

MATH
8



The Good and the Beautiful
CURRICULUM

COURSE BOOK 1

Table of Contents

About the Course

SUPPLIES NEEDED

- *Simply Good and Beautiful Pre-Algebra Course Books* 1, 2, 3, and 4
- *Simply Good and Beautiful Pre-Algebra Answers and Solutions*
- *Simply Good and Beautiful Math Scratch Pad* or other scratch paper
- Device to access videos
- Scientific calculator
- 2 standard dice
- Colored pencils
- Highlighter
- Protractor
- Ruler
- Tape or glue
- Compass
- Scissors

COURSE OVERVIEW

Pre-Algebra consists of Course Books 1, 2, 3, and 4. There are 120 total lessons divided into four units. Each unit ends with a unit review, assessment, and enrichment activity. The course is designed to be completed by the student independently, but the parent/teacher can choose to be as involved in the lessons as he or she would like.

GETTING STARTED

Simply open the first course book. The student may choose to watch the video lesson or just read the lesson overview if he or she feels confident in the lesson topic. Please note that videos may contain material not included in the written lesson overview. After completing the video and/or lesson overview, the student should complete the lesson practice and review sections.

The parent/teacher should check the student's work daily and provide immediate help and feedback. Students who struggle with the lesson practice should be encouraged to review the lesson overview or video for help.

Note: If printing at home, print pages at actual size.

LESSON DETAILS

Most lessons consist of a warm-up, video lesson, lesson overview, practice, and review.

WARM-UP: An activity that applies to the lesson topic or that reviews mental math skills.

VIDEO LESSON: Provides detailed teaching and interactive, guided practice of the lesson topic. Videos are about 12–15 minutes in length.

The Good and Beautiful Homeschool app can be used to access and watch the lesson videos. Use the QR code below to access app download information.

Alternatively, the videos can be accessed at goodandbeautiful.com/pre-algebra.

LESSON OVERVIEW: A concise written lesson on the topic.

PRACTICE: Practice that is dedicated to the lesson topic.

REVIEW: Daily review of topics from previous lessons.

A Reference Chart can be found at the back of each book.

Frequently Asked Questions

How many lessons should my student do each week?

There are 120 lessons in the course. If the student completes four lessons per week, he or she will complete the course in a standard school year with typical breaks for vacation or sickness.

How long do lessons take?

The average time to complete a lesson is 50–60 minutes. This includes time to watch the video and complete the course book sections.

What if my child does not do well on an assessment?

Each assessment question has a lesson number indicating where the content was first introduced. If your student misses an assessment question, he or she is encouraged to do one or more of the following:

- Reread the corresponding lesson overview.
- Rewatch the corresponding video.
- Complete the Extra Practice Worksheet for the corresponding lesson (available for purchase).
- Rework the problem given the answer. It can be helpful to know the answer when reworking a problem so mistakes can be found.

Do you include any specific doctrine?

No, the goal of our curriculum is not to teach doctrines specific to any particular Christian denomination but to teach general principles, such as honesty, hard work, and kindness. All Bible references in our curriculum are from the King James Version.

Does my student have to watch the videos?

The videos contain the bulk of the teaching and are highly recommended. However, if your student feels confident in the topic being taught, he or she can skip the video and read the lesson overview instead. A student who struggles with the lesson practice should be encouraged to go back and watch the video.

Some families prefer to have the parent/teacher facilitate the lesson using the lesson overview rather than have the child watch the video lesson independently.

Is Pre-Algebra completed independently by the child?

Yes, Pre-Algebra is designed for your student to complete independently, though at times the student may need parent/teacher assistance to understand a concept. The parent/teacher will need to check the student's work and should do so on a daily basis when possible, providing immediate feedback.

What if there isn't room to complete the work?

Pre-Algebra is designed to give students room to work in their course book. At times, additional paper may be needed. Students should always keep scratch paper on hand while completing the lessons. The *Simply Good and Beautiful Math Scratch Pad* is available for purchase.

Is a calculator used in Pre-Algebra? 🖩

This course is designed to be completed with a scientific calculator on hand for specific problems. Problems that allow the use of a calculator are marked with the calculator icon shown above. Any brand of scientific calculator is acceptable. Please note that calculators may vary, and your student is encouraged to read the manual for the specific calculator to understand how it functions.

Lesson Topics

UNIT 1

UNIT 2

UNIT 3

UNIT 4

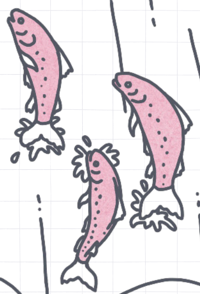
Unit 1 Overview

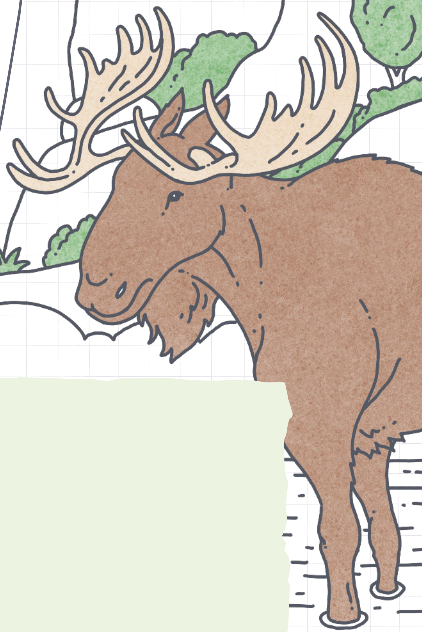
LESSONS 1–30

CONCEPTS COVERED

- Adding and subtracting decimals
- Adding and subtracting fractions
- Adding and subtracting in scientific notation
- Adding and subtracting integers
- Adding and subtracting roots
- Applications of properties of real numbers
- Combining like terms
- Commutative and associative properties
- Comparing and ordering fractions
- Converting decimals to fractions
- Converting fractions to decimals
- Distributive property
- Divisibility rules
- Estimating before performing operations
- Estimating cube roots
- Estimating square roots
- Evaluating expressions
- Evaluating expressions with roots
- Expanded notation
- Expanded notation with exponents
- Exponents
- Expressing unknowns in terms of the same variable
- Greatest common factors
- Identity and inverse properties
- Integer operations on a number line
- Inverse operations
- Least common multiples
- Multiplying and dividing in scientific notation
- Multiplying and dividing integers
- Multiplying and dividing signed fractions and decimals
- Multiplying roots
- Negative exponents
- Number sets
- Opposites and absolute value
- Order of operations
- Perfect squares and perfect cubes
- Place value
- Power of a product rule
- Power of a quotient rule
- Power rule for exponents
- Powers of 10
- Prime and composite numbers
- Prime factorization
- Principal square roots
- Product rule for exponents
- Quotient rule for exponents
- Radicals
- Rational and irrational numbers
- Reading and writing decimal numbers
- Reflexive property
- Relatively prime numbers
- Rounding to any place value
- Scientific notation
- Set notation
- Simplifying complex expressions
- Solving one-step equations
- Solving two-step equations
- Symmetric property
- Terms, constants, and coefficients
- Transitive property
- Upside down division
- Using inequalities to represent situations
- Venn diagrams
- Writing expressions and equations from word problems
- Zero product property

Exponents





SUPPLIES: item to use as a game pawn (button, cereal, bead, etc.)

WARM-UP

Use the distributive property to rewrite each expression.

a. $-3\left(a+4b-17\right)$ b. $j\left(k+km-5n\right)$

________________ ________________

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

When multiplication is repeated, it is helpful and efficient to have a notation for representing repeated multiplication. This is done using powers.

POWERS

A power is written with an exponent and a base. The ***base*** is the number that is multiplied by itself when using an exponent. The ***exponent*** is the number showing how many times to multiply the base number by itself.

In 5^3, 5 is the base and 3 is the exponent. The whole expression is referred to as a power and is read "5 to the third power" or "5 cubed." It can be written in factored form as $5 \bullet 5 \bullet 5$, which can be evaluated by multiplying the fives: $5^3 = 5 \bullet 5 \bullet 5 = 25 \bullet 5 = 125$.

Example 1: Evaluate 1.2^2.

1.2^2 Multiply 1.2 by itself.

$= 1.2 \bullet 1.2$

$= 1.44$

A number raised to the second power is referred to as "squared."

Example 2: Evaluate $\left(\frac{4}{5}\right)^3$.

$\left(\frac{4}{5}\right)^3$ Multiply the fraction three times.

$= \frac{4}{5} \bullet \frac{4}{5} \bullet \frac{4}{5}$

$= \frac{64}{125}$

Example 3: Evaluate 1^{10}.

1^{10}

$= 1 \bullet 1 \bullet 1 \bullet 1 \bullet 1 \bullet 1 \bullet 1 \bullet 1 \bullet 1 \bullet 1$

$= 1$ Note: 1 raised to any power is just 1.

Example 4: Evaluate $(-2)^4$.

$(-2)^4$ Multiply –2 four times.

$= (-2)(-2)(-2)(-2)$

$= 16$

Notice that the answer to Example 4 is positive. The problem $(-2)^4$ is not the same as -2^4, which has a negative answer. In -2^4, the exponent only applies to the 2, not the negative sign. In $(-2)^4$, the number –2 is multiplied four times. In -2^4, the number 2 is multiplied four times, and then the result is made negative. The exponent applies to whatever it is right next to. When the exponent is right next to a number, it applies only to that number. When the exponent is right next to a parenthesis, it applies to whatever is inside the parentheses.

$$-2^4 = -(2 \bullet 2 \bullet 2 \bullet 2) = -16$$

Any number (except zero) raised to the zero power is 1.

$8^0 = 1$ $(-11)^0 = 1$

Variables can also be written as the base with an exponent.

$a^7 = a \bullet a \bullet a \bullet a \bullet a \bullet a \bullet a$

$z^0 = 1$ for $z \neq 0$

Why can't $0^0 = 1$?

Any multiple of zero will always be zero, so 0^0 can't follow the rule above.

Since 0^0 can't equal both zero and one, 0^0 is *undefined*.

Expressions with repeated multiplication can be rewritten with exponents. An expression may contain different bases. For example, the expression below is a product involving three different bases. Each base can be written with an exponent.

$$2 \bullet 2 \bullet 2 \bullet 5 \bullet 5 \bullet 8 \bullet 8 \bullet 8 = 2^3 \bullet 5^2 \bullet 8^3$$

Example 5: Rewrite the expression with exponents.

$x \bullet x \bullet y \bullet y \bullet x \bullet z$ Because of the commutative property, variables can be rearranged so all common variables are together.

$= x^3 y^2 z$

EXPONENTS AND POWERS OF 10

Positive powers of 10 can be evaluated quickly by looking at the exponent. The exponent tells how many zeros are in the final answer.

For example, the power 10^3 can be evaluated by multiplying 10 three times. Notice that there are three zeros in the answer.

$$10^3 = 10 \bullet 10 \bullet 10 = 1000$$

Knowing the pattern for powers of 10 helps with mental math.

Similarly, when a number is multiplied by a positive power of 10, the exponent tells how many places to move the decimal point to the right. Zeros are written at the end of the number when necessary.

Example 6: Evaluate $21.378 \bullet 10^5$.

$21.378 \bullet 10^5$ The decimal point is moved five places to the right.

$= 2{,}137{,}800$ Two zeros must be added to move a total of five places.

EXPANDED NOTATION WITH EXPONENTS

Numbers in expanded notation can be written using exponents. Look at the expanded notation for 12,689 below.

$$12{,}689 = (1 \bullet 10{,}000) + (2 \bullet 1000) + (6 \bullet 100) + (8 \bullet 10) + (9 \bullet 1)$$

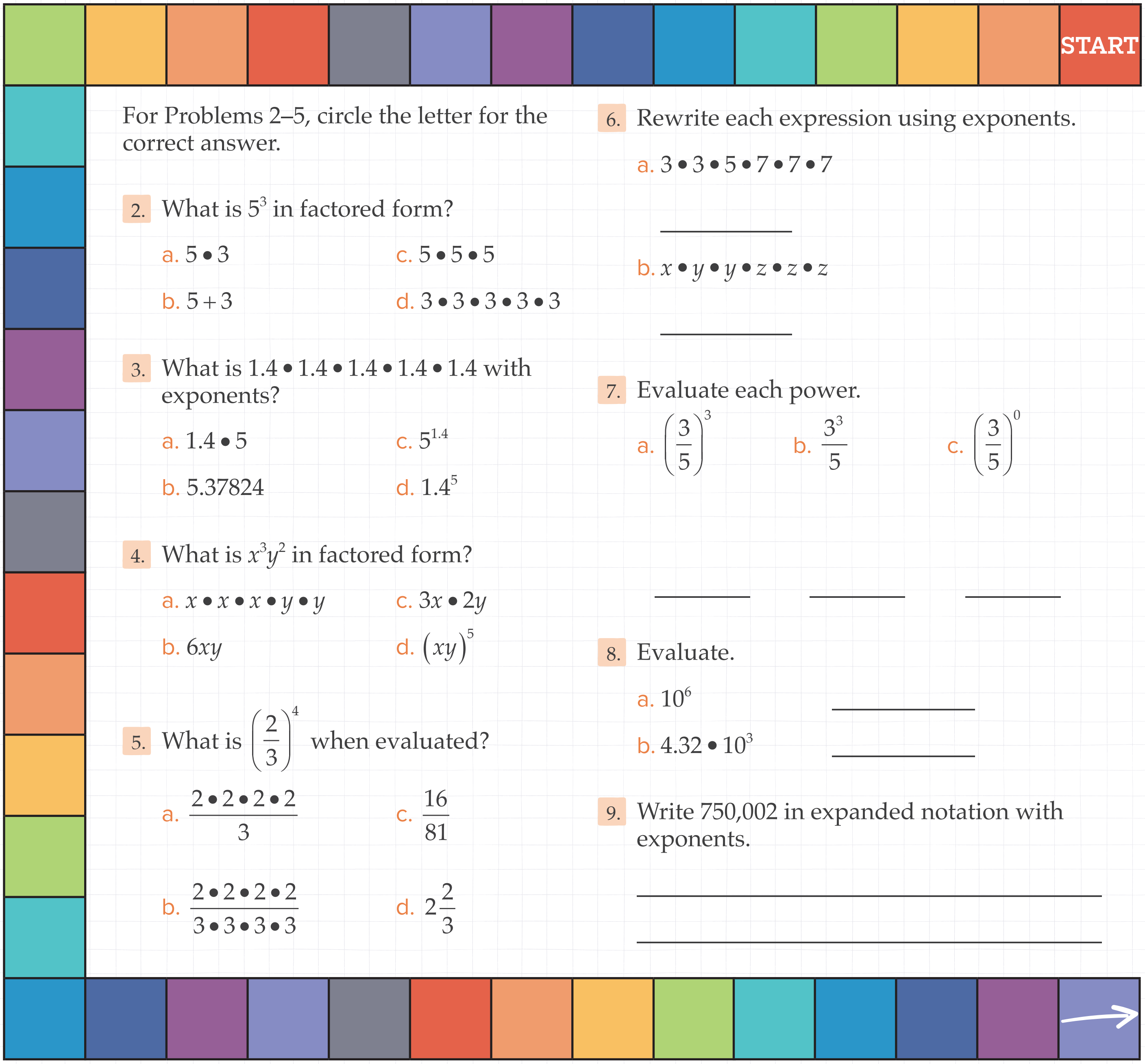
The powers of 10 representing the place value of each digit can be written using exponents as shown below. Remember that $10^0 = 1$, so the last number in parentheses can be written with an exponent as well.

$$12{,}689 = (1 \bullet 10^4) + (2 \bullet 10^3) + (6 \bullet 10^2) + (8 \bullet 10^1) + (9 \bullet 10^0)$$

PRACTICE

1. Choose a power to evaluate from the box below. Move a game pawn the number of spaces on the track based on the answer. For example, if the answer is 81, move the pawn 81 spaces. Any negative answer moves the pawn backward. Continue choosing powers and see how quickly you can make it to the end of the track. Play at least twice to see if you can make it to the end by completing fewer problems.
✦ Hint: Not everything with a negative sign ends up being negative! To make counting spaces easier, the same colored squares are always spaced 10 spaces apart. For example, if you have to move 34 spaces and are currently on red, move to the next red three times, and then move four extra spaces.

-3^4 $\quad(-3)^4$ $\quad(-4)^3$ $\quad 4^3$ $\quad 145^0$ $\quad 2^6$ $\quad 6^2$ $\quad 10^2$ $\quad(-2)^6$ $\quad(-2)^5$ $\quad -2^4$ $\quad 2^7$ $\quad 7^2$ $\quad 5^3$ $\quad 3^5$ $\quad 37^1$



For Problems 2–5, circle the letter for the correct answer.

2. What is 5^3 in factored form?

a. $5 \bullet 3$ c. $5 \bullet 5 \bullet 5$

b. $5 + 3$ d. $3 \bullet 3 \bullet 3 \bullet 3 \bullet 3$

3. What is $1.4 \bullet 1.4 \bullet 1.4 \bullet 1.4 \bullet 1.4$ with exponents?

a. $1.4 \bullet 5$ c. $5^{1.4}$

b. 5.37824 d. 1.4^5

4. What is x^3y^2 in factored form?

a. $x \bullet x \bullet x \bullet y \bullet y$ c. $3x \bullet 2y$

b. $6xy$ d. $(xy)^5$

5. What is $\left(\frac{2}{3}\right)^4$ when evaluated?

a. $\frac{2 \bullet 2 \bullet 2 \bullet 2}{3}$ c. $\frac{16}{81}$

b. $\frac{2 \bullet 2 \bullet 2 \bullet 2}{3 \bullet 3 \bullet 3 \bullet 3}$ d. $2\frac{2}{3}$

6. Rewrite each expression using exponents.

a. $3 \bullet 3 \bullet 5 \bullet 7 \bullet 7 \bullet 7$

____

b. $x \bullet y \bullet y \bullet z \bullet z \bullet z$

____

7. Evaluate each power.

a. $\left(\frac{3}{5}\right)^3$ b. $\frac{3^3}{5}$ c. $\left(\frac{3}{5}\right)^0$

________ ________ ________

8. Evaluate.

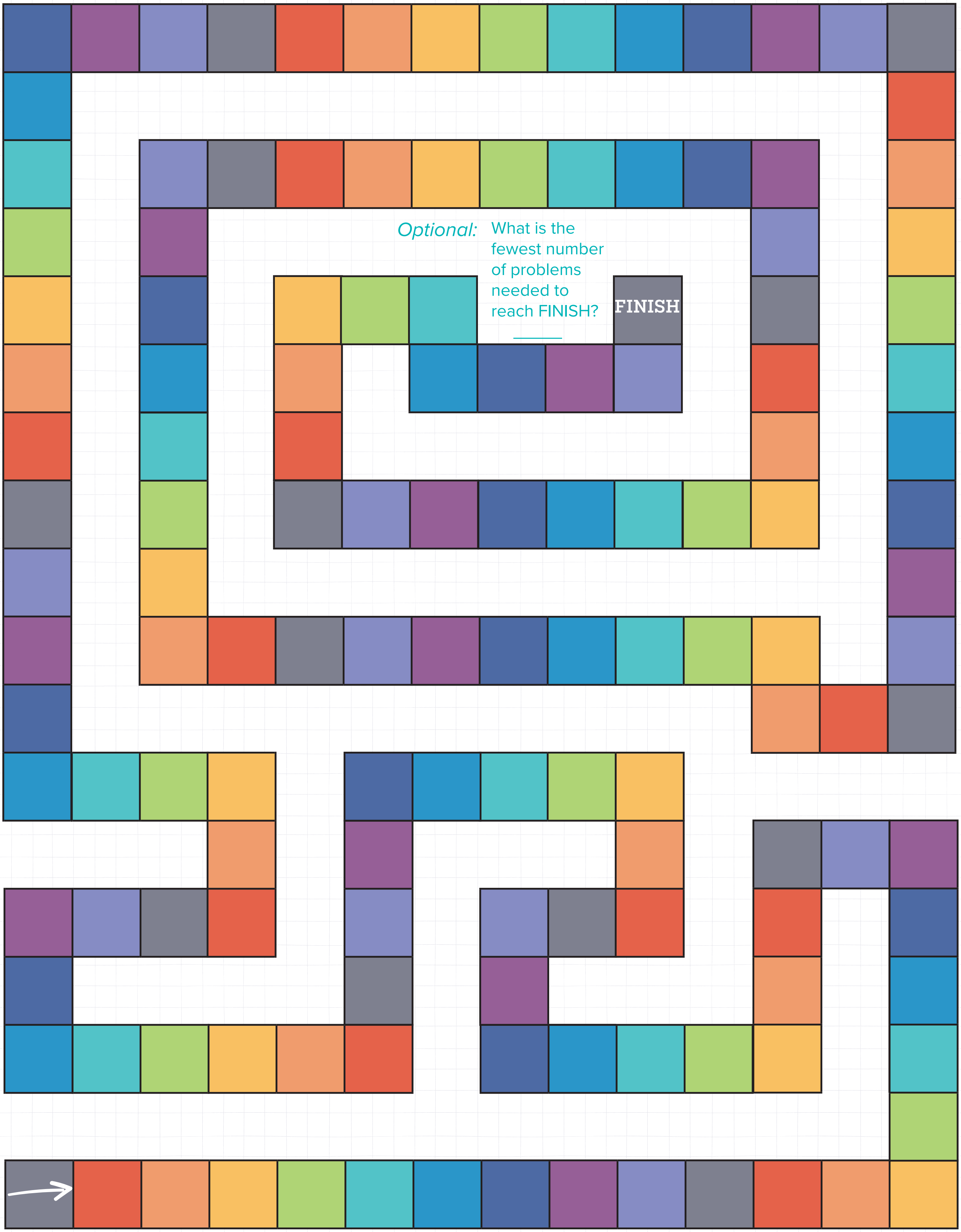
a. 10^6 __________

b. $4.32 \bullet 10^3$ __________

9. Write 750,002 in expanded notation with exponents.

____

____



Optional: What is the fewest number of problems needed to reach FINISH? ______

REVIEW

1. Use expanded form and the distributive property to multiply 7 by 545. L8

____

2. Find the value of the variable in each equation using inverse operations and fact families. L8

a. $g - 135 = 45$

$g =$ _______

b. $10 \bullet d = 125$

$d =$ _______

3. One strategy for subtracting mentally when the number being subtracted (the subtrahend) is near a multiple of 10 is to round the subtrahend to the multiple of 10 that it is close to and then compensate. For example, to find 72 – 19, subtract 20 from 72 to get 52. Then compensate for taking away too many (20 instead of 19) by adding one to 52 to get 53. Use this strategy to complete each problem mentally.

a. 154 – 29 _______

b. 87 – 38 _______

c. 295 – 69 _______

d. 351 – 99 _______

4. Write S next to statements that are sometimes true, A next to statements that are always true, and N next to statements that are never true. L3

a. ____ The sum of a number and its opposite is zero.

b. ____ The sum of a negative number and a positive number is negative.

c. ____ The opposite of a number is positive.

d. ____ The absolute value of a positive number is a negative number.

e. ____ Subtracting is the same as adding the opposite of the subtrahend.

5. Write three numbers not equal to 6.5 that round to 6.5. L1

_________ _________ _________

Combining Like Terms

WARM-UP

Simplify.

$$\frac{\left(10+(7-12)^3\right)}{|45-53|\bullet 5}$$

______

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

A fundamental skill in algebra is knowing how to simplify and evaluate expressions. An ***expression*** is a number, variable, or combination of numbers and variables joined by operations. An example of an expression is $3x^2 + 4xy + 5$. A ***term*** is one part of an expression, which may be a number, a variable, or a product of numbers and variables. Terms are separated by plus or minus signs. In the expression example, $3x^2$ is a term, $4xy$ is a term, and 5 is a term. A ***constant*** is a term with no variable, so 5 is a constant in that expression. A ***coefficient*** is a number that is multiplied by a variable or product of variables. The coefficient of x^2 is 3, and the coefficient of xy is 4. The chart below shows two additional expressions and their parts.

Expression	Number of Terms	Coefficients	Constants
$6ax^2 + by^2 - 18$	3	6, 1	–18
$65m^3 - 9k^2 + 7j + 34$	4	65, –9, 7	34

SIMPLIFYING EXPRESSIONS

Like terms are terms with the same variables raised to the same power. Below are some examples of like terms.

$7ab$ and $-12ab$ $243z^2$ and z^2 $35xy^3$ and $26xy^3$

Simplifying an expression involves combining all like terms and performing any possible operations. When combining like terms, add coefficients and keep the same variable.

$$3a + 2a = a + a + a + a + a = 5a$$

Example 1: Simplify the expression $83xy + 21x - 45xy + 18x$.

$83xy + 21x - 45xy + 18x$ — $83xy$ and $-45xy$ are like terms. $21x$ and $18x$ are like terms.

$= 83xy - 45xy + 21x + 18x$ — Rearrange to write like terms next to each other using the commutative property of addition.

$= 38xy + 39x$ — Combine like terms by adding or subtracting the coefficients. The expression is simplified.

Note: The remaining two terms are not like terms because they do not have the exact same variable part.

Example 2: Simplify the expression $4.78t^3 + 3.21st - 0.5t^3 + 2.9 + 5.49st$.

$4.78t^3 + 3.21st - 0.5t^3 + 2.9 + 5.49st$ — The like terms are in matching colors.

$= 4.78t^3 - 0.5t^3 + 3.21st + 5.49st + 2.9$ — Rearrange to write like terms next to each other.

$= 4.28t^3 + 8.7st + 2.9$ — Combine like terms.

EVALUATING EXPRESSIONS

An expression can be evaluated when values are given for the variables. Substitute the value provided for each variable, and simplify using the order of operations. Substituting values for variables is often referred to as “plugging in” values. When plugging in values, it can help to write parentheses around the numbers that are substituted into the expression.

Example 3: Evaluate the expression $6a+4b$ when $a=5$ and $b=-3$.

$6a+4b$	Substitute the values of a and b into the expression.
$6(5)+4(-3)$	Multiply.
$=30+(-12)$	Add.
$=18$	

Example 4: Evaluate the expression $3f-2g^2$ when $f=2$ and $g=-2$.

$3f-2g^2$	Substitute the values of f and g into the expression.
$3(2)-2(-2)^2$	Evaluate the exponent.
$=3(2)-2(4)$	Multiply.
$=6-8$	Subtract.
$=-2$	

Example 5: Evaluate the expression $12z-24w+15zw$ when $z=\frac{1}{2}$ and $w=\frac{2}{3}$.

$12z-24w+15zw$	Substitute the values of z and w into the expression.
$12\left(\frac{1}{2}\right)-24\left(\frac{2}{3}\right)+15\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)$	Multiply.
$=6-16+5$	Subtract.
$=-10+5$	Add.
$=-5$	

Example 6: Evaluate the expression $cd+de$ when $c=1.2$, $d=2.3$, and $e=3.4$.

$cd+de$	Substitute the values of c and d and e into the expression.
$(1.2)(2.3)+(2.3)(3.4)$	Multiply.
$=2.76+7.82$	Add.
$=10.58$	

When an expression is written as a fraction, substitute given values and simplify the numerator and denominator separately. Then divide.

Example 7: Evaluate the expression below when $x=-2$, $y=4$, and $z=7$.

$$\frac{xy+yz}{xz}$$

Substitute the values of x and y and z into the expression.

$$\frac{(-2)(4)+(4)(7)}{(-2)(7)}$$

Multiply in the numerator and denominator.

$$=\frac{-8+28}{-14}$$

Simplify the numerator.

$$=\frac{20}{-14}=-1\frac{6}{14}=-1\frac{3}{7}$$

Write as a mixed number and simplify.

PRACTICE

Complete each problem below. Find the answer in the table on the next page and cross off the phrase next to it. Once all problems have been completed, write the remaining phrases, from top to bottom, at the bottom of the page to discover a neat fact about God's creation!

For Problems 1–6, simplify the expressions by combining like terms. Tip: Highlighting like terms in the same color can help when combining like terms!

1. $3x+5-4x$

____

2. $2ab-ab+3a+2a$

____

3. $4c^2+3c-7c^2-c$

____

4. $-2.5p-q+1.3p-1.1q$

____

5. $\frac{1}{2}u^2v-\frac{2}{3}u+u-\frac{1}{6}u^2v$

____

6. $j^2l-(-5j)-2j^2l+j$

____

For Problems 7–12, evaluate the expressions for the given values.

7. $t+2s-1$ Values: $t=3$, $s=1$

____

8. $ab+b^2$ Values: $a=2$, $b=-1$

____

9. $3e + 4f - 1.5g^3$ Values: $e = 1.5$, $f = 2.5$, $g = -2$

10. $\dfrac{nm - m}{n^2 + m^2}$ Values: $n = 2$, $m = 4$

11. $\dfrac{-w - (-2x)}{w}$ Values: $w = 3$, $x = \dfrac{1}{2}$

12. $\dfrac{q + 1.4p}{-r}$ Values: $p = 2$, $q = 6$, $r = -1$

8.8	More than 71% of
$\frac{2}{3}$	Although only about 3% of
4	Less than 36% of
$-3c^2 + 2c$	Only about 7% of
$-1.2p - 2.1q$	the earth's water is underground,
-1	the earth's water is salt water,
1.45	the earth's water is fresh water,
$-\frac{2}{3}$	the earth's surface is water,
1	more than 100,000 species of plants and animals
$-x + 5$	more than 40,000 species of plants and animals
$-j^2l + 6j$	fewer than 20,000 species of plants and animals
$\frac{1}{5}$	fewer than 10,000 species of plants and animals
$\frac{1}{3}u^2v + \frac{1}{3}u$	have their homes in saltwater habitats.
26.5	have their homes in underground habitats.
$ab + 5a$	have their homes in aquatic habitats.
$x - 7$	have their homes in freshwater habitats.

Fact:

UNIT 1 | LESSON 15

Logic Lesson 1

A-MAZE-ing Mazes

In the United States, England, and other countries, mazes made by cutting paths through fields of tall cornstalks are a traditional fall destination. Corn maze venues often include hayrides, pumpkin picking, petting zoos, and other farm experiences. Some corn mazes have puzzles or riddles to solve along the way in order to receive clues about how to get out of the maze. Complete the corn maze logic puzzles in this lesson. This lesson has no video or review problems.

Tips for Solving Logic Puzzles:

Logic puzzles can be approached in different ways. The following tips can help when solving logic puzzles.

- When there are several options, assume something is true and continue until it doesn't work. Then start over, assuming something different is true.
- Guess and check and try various combinations.
- Find a pattern.
- Make a list showing all options.
- Draw a diagram or picture.
- Do not work until the point of frustration. Take a break or move to another problem. It's OK to look at the solutions and gain understanding for logic puzzles in this way.

PIE PARTNERS

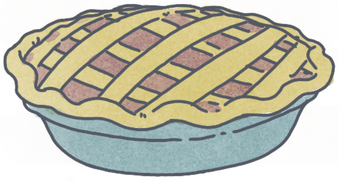
Blake and Nellie sell slices of homemade pies at a corn maze. At the end of a day, they had $\frac{1}{4}$ of a blueberry pie, $\frac{1}{3}$ of a pumpkin pie, $\frac{1}{2}$ of an apple pie, $\frac{2}{3}$ of a cherry pie, $\frac{3}{4}$ of a banana cream pie, and 1 whole chocolate pie left over. Find a way to divide the leftover pies, without making any more cuts, so that Blake and Nellie take equal amounts of pie home to share with their families. On the lines below, list which pies each partner should take.

Blake: ______________________________

____

Nellie: ______________________________

____



KETTLE CORN

Mae bought a bag of kettle corn for $1 at a corn maze concession stand. She paid using exactly 50 coins. Which coins, and how many of each, did Mae use to pay for the kettle corn?

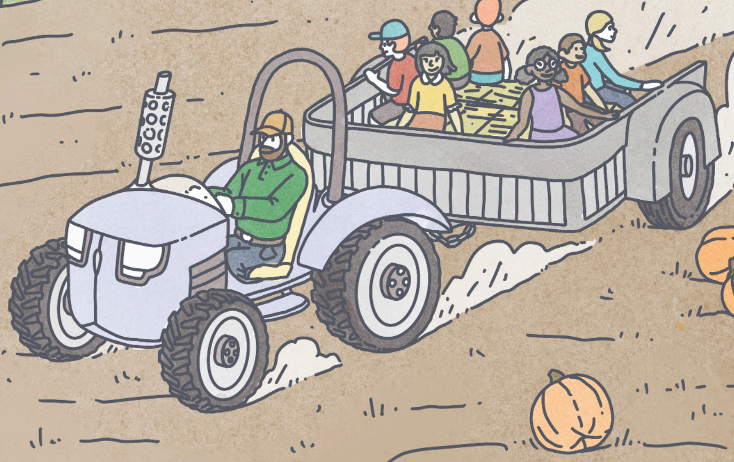
____

____

HELPFUL HINT

Hattie and Henry were in a corn maze when they met a worker who said he'd tell them which way they needed to turn next if they correctly solved the following puzzle: "A spider is building a web. Every 10 minutes, the web doubles in size. If the web is completely finished in 50 minutes, when was the web 25% complete?" Write the answer to the puzzle on the line below.

____

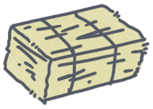


HUMDINGER HAYRIDES

Ree and Piper offer hayrides at a corn maze. During one ride, they noticed that if 5 of Ree's passengers moved to Piper's wagon, each would have the same number of passengers, but if 5 of Piper's passengers moved to Ree's wagon, Ree would have twice as many passengers as Piper. How many passengers did each have on that ride?

Ree: ______ passengers

Piper: ______ passengers



DAY DEBATE

Five siblings work at their family-owned corn maze and are debating about which day of the week it is during their lunch break. The maze is closed on Sundays. If only one of the following statements is true, what day of the week is it?

Hank: I know for sure we're in the last half of the work week.

Joe: No, we're not because yesterday was Monday.

Cathy: No, the day before yesterday was Monday.

Olive: All I know is that tomorrow is not Friday.

Max: Everything has gone wrong for me today, so today is definitely Monday.

____

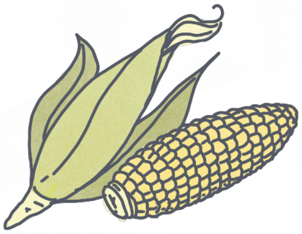
SPEEDY SHUCKING

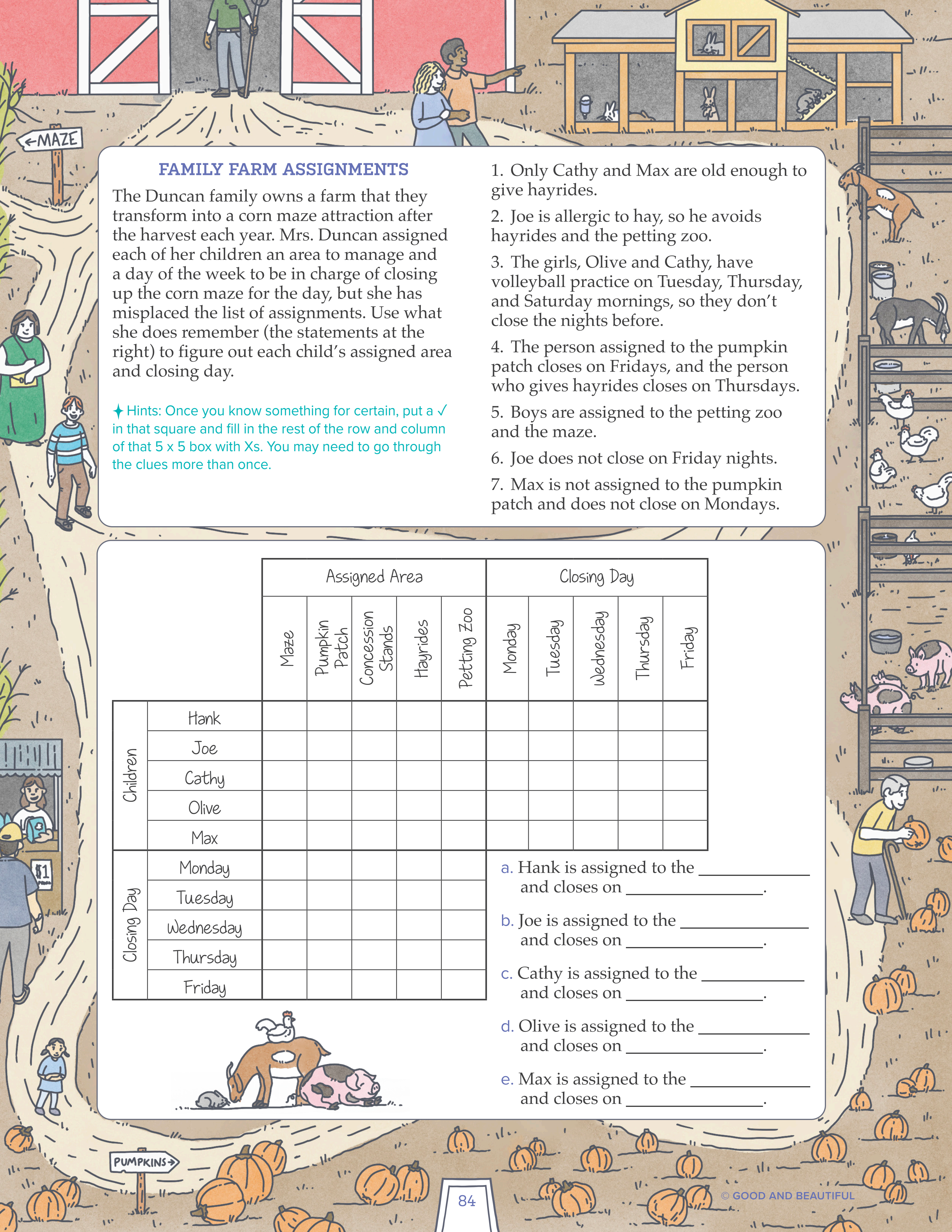
Removing the husks from ears of corn is called husking or shucking the corn.

Dan and Delaney competed in a timed corn-shucking contest. Together, they shucked 75 ears of corn. Of all the contestants, Dan was randomly selected to get a 3-minute head start. He shucked 4 ears of corn per minute. Three minutes later, Delaney began, and she shucked at a rate of 5 ears per minute. When they finished, how many ears of corn had Dan and Delaney each shucked?

Dan: ______ ears of corn

Delaney: ______ ears of corn





FAMILY FARM ASSIGNMENTS

The Duncan family owns a farm that they transform into a corn maze attraction after the harvest each year. Mrs. Duncan assigned each of her children an area to manage and a day of the week to be in charge of closing up the corn maze for the day, but she has misplaced the list of assignments. Use what she does remember (the statements at the right) to figure out each child's assigned area and closing day.

✦ Hints: Once you know something for certain, put a ✓ in that square and fill in the rest of the row and column of that 5 x 5 box with Xs. You may need to go through the clues more than once.

1. Only Cathy and Max are old enough to give hayrides.
2. Joe is allergic to hay, so he avoids hayrides and the petting zoo.
3. The girls, Olive and Cathy, have volleyball practice on Tuesday, Thursday, and Saturday mornings, so they don't close the nights before.
4. The person assigned to the pumpkin patch closes on Fridays, and the person who gives hayrides closes on Thursdays.
5. Boys are assigned to the petting zoo and the maze.
6. Joe does not close on Friday nights.
7. Max is not assigned to the pumpkin patch and does not close on Mondays.

		Assigned Area					Closing Day				
		Maze	Pumpkin Patch	Concession Stands	Hayrides	Petting Zoo	Monday	Tuesday	Wednesday	Thursday	Friday
Children	Hank										
	Joe										
	Cathy										
	Olive										
	Max										
Closing Day	Monday										
	Tuesday										
	Wednesday										
	Thursday										
	Friday										

a. Hank is assigned to the ____________ and closes on ______________.

b. Joe is assigned to the ____________ and closes on ______________.

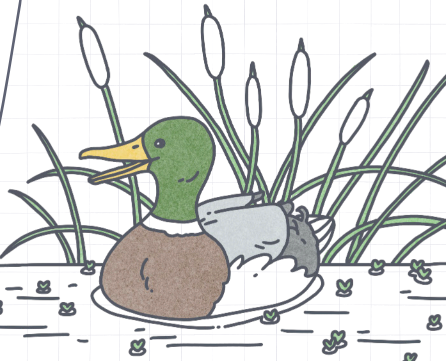
c. Cathy is assigned to the ____________ and closes on ______________.

d. Olive is assigned to the ____________ and closes on ______________.

e. Max is assigned to the ____________ and closes on ______________.

UNIT 1 | LESSON 21

Simplifying Complex Expressions

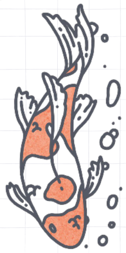


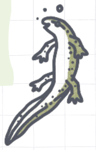
WARM-UP

Simplify the expression.

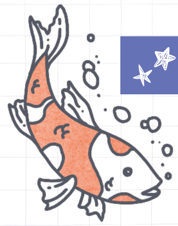
$$3\sqrt{2} \bullet \sqrt{\frac{64}{4}} + \sqrt{2}$$

______





LESSON



Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

Complex expressions can be simplified using a combination of rules learned in previous lessons. Expressions are unique, and each expression must be examined to determine the methods of simplifying that apply. Remember to follow the order of operations when simplifying any expression.

Roots are related to exponents and are evaluated in the exponent step in the order of operations. Review the examples below to see how some complex expressions are simplified.

Tip for Simplifying Expressions:

Complete only one step at a time and rewrite the rest of the expression exactly as it is.

Example 1: Simplify the expression.

$$6\left(\left(\frac{1}{5}r\right)^2+5r-\frac{7}{3}\right)+4\left(r^2-\frac{5}{4}\right)$$

Within parentheses, evaluate exponents. Use the power of a product rule.

$$=6\left(\frac{1}{25}r^2+5r-\frac{7}{3}\right)+4\left(r^2-\frac{5}{4}\right)$$

Distribute.

$$=\frac{6}{25}r^2+30r-14+4r^2-5$$

Combine like terms.

$$=4\frac{6}{25}r^2+30r-19$$

Example 2: Simplify the expression.

$$\left(\sqrt{400}+13\right)\div\left(5\left|\sqrt[3]{-27}\right|+18\right)$$

Within parentheses, evaluate roots.

$$=(20+13)\div(5|-3|+18)$$

Evaluate the absolute value and simplify in the parentheses.

$$=33\div(5\bullet 3+18)$$

Multiply and add in parentheses.

$$=33\div(15+18)$$

$$=33\div 33$$

$$=1$$

Example 3: Simplify the expression.

$$\left(\frac{a^3b^{-8}}{c^5}\right)^2$$

Apply the power of a product and power of a quotient rules.

$$=\frac{a^6b^{-16}}{c^{10}}$$

Rewrite the expression with positive exponents.

$$=\frac{a^6}{c^{10}}\bullet\frac{1}{b^{16}}$$

Multiply.

$$=\frac{a^6}{c^{10}b^{16}}$$

Note: Exponents should all be positive in simplified expressions. This fraction is considered simplified.

Example 4: Simplify the expression.

$$\frac{3\left(c^{3}\right)^{4}}{\left(2c^{7}\right)^{5}}$$

Apply the power rule and the power of a product rule.

$$=\frac{3c^{12}}{32c^{35}}$$

Simplify using the quotient rule.

$$=\frac{3}{32}c^{12-35}$$

$$=\frac{3}{32}c^{-23}$$

Rewrite the expression with positive exponents.

$$=\frac{3}{32}\bullet\frac{1}{c^{23}}$$

Multiply.

$$=\frac{3}{32c^{23}}$$

Example 5: Simplify the expression.

$$\frac{m^{2}p^{3}n^{3}m^{4}p}{m^{4}n^{8}p^{4}n^{3}}$$

Multiply. Use the product rule.

$$=\frac{m^{6}p^{4}n^{3}}{m^{4}n^{11}p^{4}}$$

Simplify using the quotient rule.

$$=m^{6-4}\bullet n^{3-11}\bullet p^{4-4}$$

$$=m^{2}\bullet n^{-8}\bullet p^{0}$$

Rewrite the expression with positive exponents.

$$=m^{2}\bullet\frac{1}{n^{8}}\bullet 1$$

Multiply.

$$=\frac{m^{2}}{n^{8}}$$

Expressions within a radical should be simplified before the square root is taken. Apply the order of operations under the radical in the numerator and denominator.

Example 6: Simplify the expression.

$$\sqrt{\frac{200-2\bullet 28}{9+4^{2}\bullet 2-5}}$$

Multiply in the numerator.
Evaluate the exponent in the denominator.

$$=\sqrt{\frac{200-56}{9+16\bullet 2-5}}$$

Subtract in the numerator.
Multiply in the denominator.

$$=\sqrt{\frac{144}{9+32-5}}$$

Add and subtract from left to right in the denominator.

$$=\sqrt{\frac{144}{36}}$$

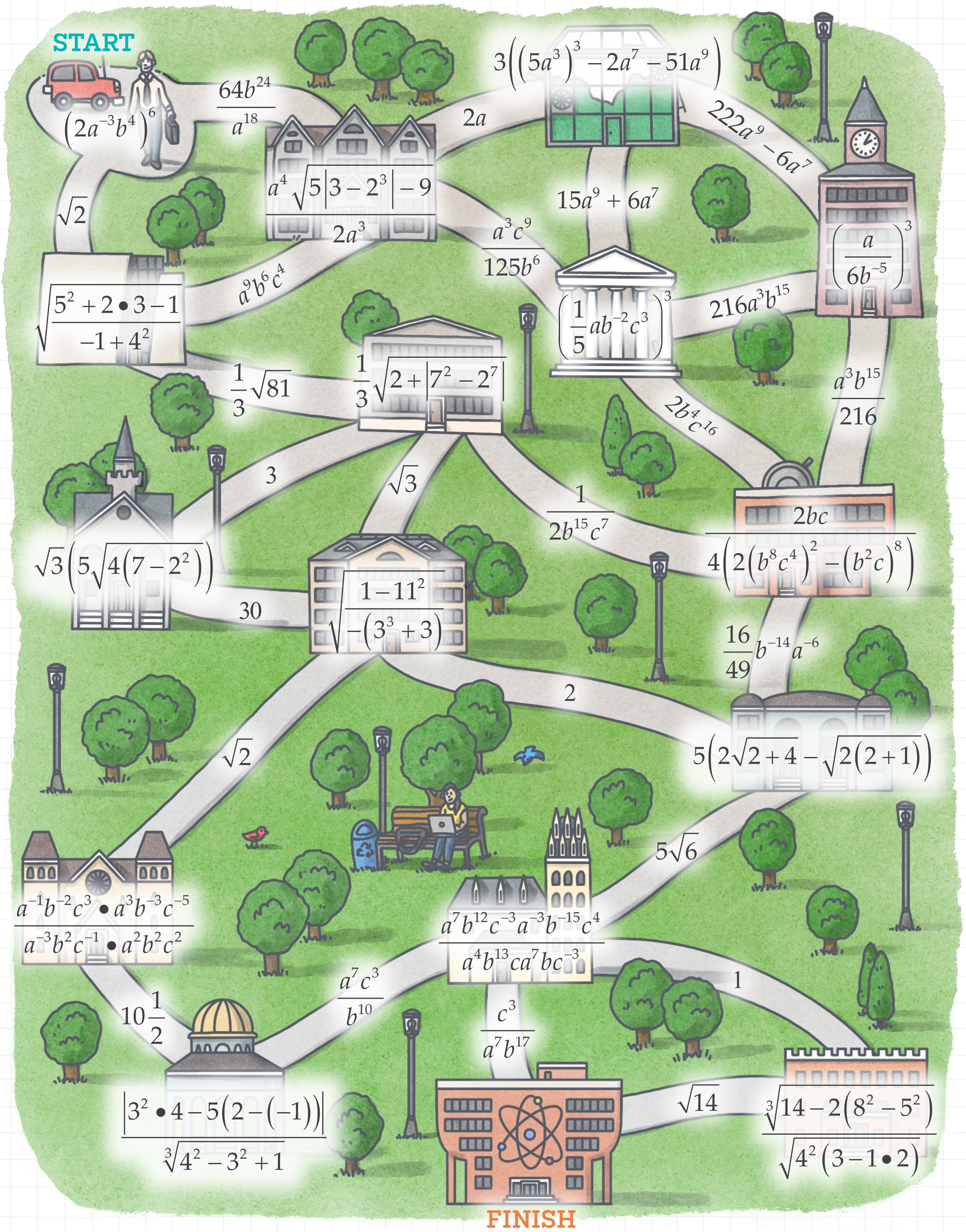
Divide.

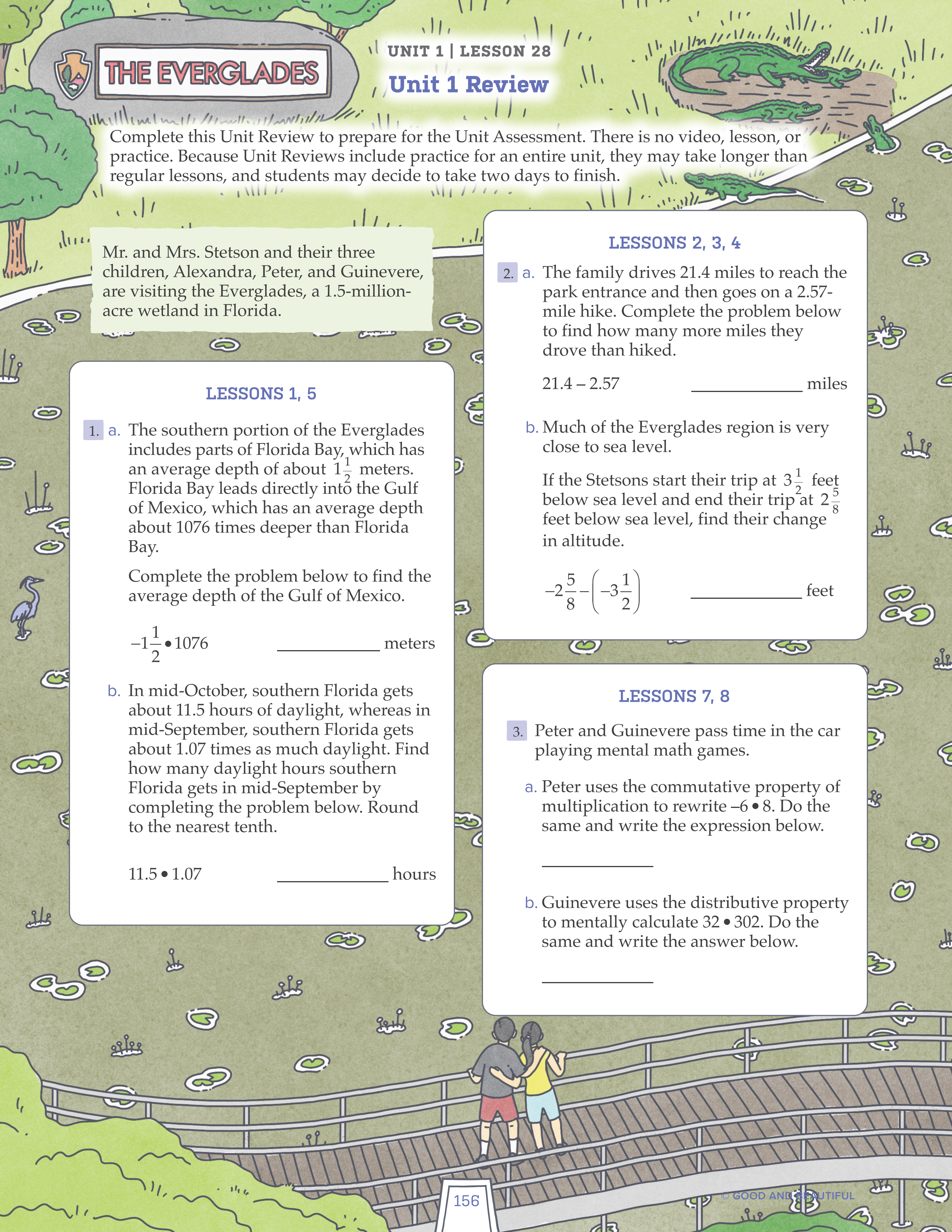
$$=\sqrt{4}=2$$

Take the square root.

PRACTICE

Simplify the expressions to determine which path to follow in order to get the physics professor to his physics lab. Begin at START.





Unit 1 Review

Complete this Unit Review to prepare for the Unit Assessment. There is no video, lesson, or practice. Because Unit Reviews include practice for an entire unit, they may take longer than regular lessons, and students may decide to take two days to finish.

Mr. and Mrs. Stetson and their three children, Alexandra, Peter, and Guinevere, are visiting the Everglades, a 1.5-million-acre wetland in Florida.

LESSONS 1, 5

1. a. The southern portion of the Everglades includes parts of Florida Bay, which has an average depth of about $1\frac{1}{2}$ meters. Florida Bay leads directly into the Gulf of Mexico, which has an average depth about 1076 times deeper than Florida Bay.

Complete the problem below to find the average depth of the Gulf of Mexico.

$-1\frac{1}{2} \bullet 1076$ ____________ meters

b. In mid-October, southern Florida gets about 11.5 hours of daylight, whereas in mid-September, southern Florida gets about 1.07 times as much daylight. Find how many daylight hours southern Florida gets in mid-September by completing the problem below. Round to the nearest tenth.

$11.5 \bullet 1.07$ ____________ hours

LESSONS 2, 3, 4

2. a. The family drives 21.4 miles to reach the park entrance and then goes on a 2.57-mile hike. Complete the problem below to find how many more miles they drove than hiked.

$21.4 - 2.57$ ____________ miles

b. Much of the Everglades region is very close to sea level.

If the Stetsons start their trip at $3\frac{1}{2}$ feet below sea level and end their trip at $2\frac{5}{8}$ feet below sea level, find their change in altitude.

$-2\frac{5}{8} - \left(-3\frac{1}{2}\right)$ ____________ feet

LESSONS 7, 8

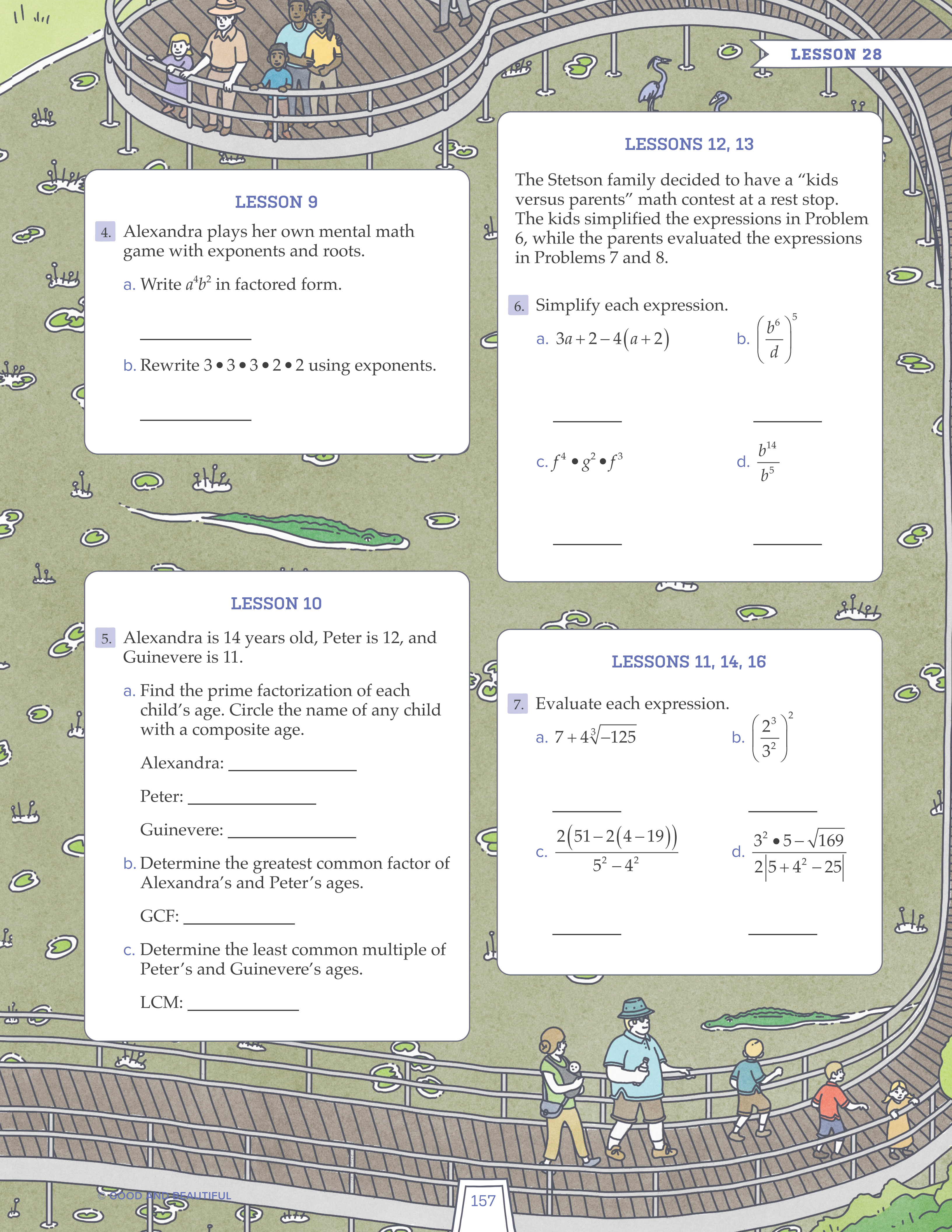
3. Peter and Guinevere pass time in the car playing mental math games.

a. Peter uses the commutative property of multiplication to rewrite $-6 \bullet 8$. Do the same and write the expression below.

____

b. Guinevere uses the distributive property to mentally calculate $32 \bullet 302$. Do the same and write the answer below.

____



LESSON 9

4. Alexandra plays her own mental math game with exponents and roots.

a. Write a^4b^2 in factored form.

____

b. Rewrite $3 \bullet 3 \bullet 3 \bullet 2 \bullet 2$ using exponents.

____

LESSON 10

5. Alexandra is 14 years old, Peter is 12, and Guinevere is 11.

a. Find the prime factorization of each child's age. Circle the name of any child with a composite age.

Alexandra: ______________

Peter: ______________

Guinevere: ______________

b. Determine the greatest common factor of Alexandra's and Peter's ages.

GCF: ____________

c. Determine the least common multiple of Peter's and Guinevere's ages.

LCM: ____________

LESSONS 12, 13

The Stetson family decided to have a "kids versus parents" math contest at a rest stop. The kids simplified the expressions in Problem 6, while the parents evaluated the expressions in Problems 7 and 8.

6. Simplify each expression.

a. $3a+2-4(a+2)$

____

b. $\left(\dfrac{b^6}{d}\right)^5$

____

c. $f^4 \bullet g^2 \bullet f^3$

____

d. $\dfrac{b^{14}}{b^5}$

____

LESSONS 11, 14, 16

7. Evaluate each expression.

a. $7+4\sqrt[3]{-125}$

____

b. $\left(\dfrac{2^3}{3^2}\right)^2$

____

c. $\dfrac{2(51-2(4-19))}{5^2-4^2}$

____

d. $\dfrac{3^2 \bullet 5-\sqrt{169}}{2\left|5+4^2-25\right|}$

____

Unit 1 Assessment

○ This assessment covers concepts taught in Unit 1. Problems are designed to assess multiple skills. Read the instructions carefully and do not rush through the problems.

○ You may use the Reference Chart for the assessment. Calculators should only be used when noted. Lesson numbers are given by each problem so you can review lessons for any answers that are incorrect.

1. Write the number below using numerals. L1

five thousand, forty-three
and eighty-two hundredths

____

2. Draw lines to connect equivalent values. L2

$1\frac{2}{5}$	$3.\overline{8}$
$3\frac{7}{8}$	3.875
$1\frac{5}{11}$	1.4
$3\frac{8}{9}$	$1.\overline{45}$

3. Find the distance between the two numbers on the number line. L3

–15 and 42 __________

4. Perform the indicated operation. L4–6

a. $5\frac{7}{8} - 2\frac{1}{4}$

____

b. $7.485 + 2.956$

____

c. $-\frac{8}{5} \bullet 1\frac{3}{4}$

____

d. $-3.8 \bullet (-4.53)$

____

e. $-4.2 \div 0.06$

____

f. $\dfrac{3\frac{2}{5}}{7\frac{1}{2}}$

____

12. Simplify each expression. L14

a. $\left(c^2ba^3\right)^5$

______

b. $\left(\frac{r^7}{ts^3}\right)^2$

______

13. Evaluate the expression. L16

$$\frac{\sqrt{225}}{\sqrt[3]{64}+\sqrt{64}}$$

______

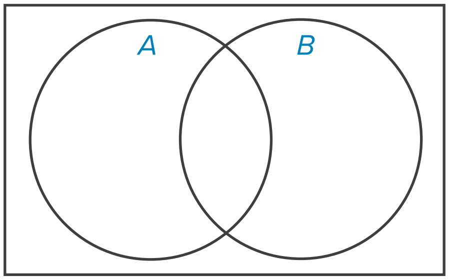
14. a. Determine which perfect squares are on either side of 123. L17

______ and ______

b. Determine which integers are on either side of $\sqrt{123}$. Circle the integer that is closer to $\sqrt{123}$.

______ and ______

15. Let set *A* be the set of three-letter number words from “one” to “ten.” Let set *B* be the set of number words from “one” to “ten” that start with “t.” Fill in the Venn diagram. L18



16. Rewrite each expression with a positive exponent. L13 & L19

a. b^{-5}

______

b. $\frac{1}{c^{-3}}$

______

c. $q^{-3} \bullet q$

______

17. Simplify each expression. L20

a. $\sqrt{25} \bullet \sqrt{4}$

______

b. $3\sqrt{5}+7\sqrt{5}$

______

18. Evaluate the expression. L21

$$\frac{3^2\sqrt{|-25-11|}}{\sqrt[3]{27}}$$

______

LESSONS
31–60

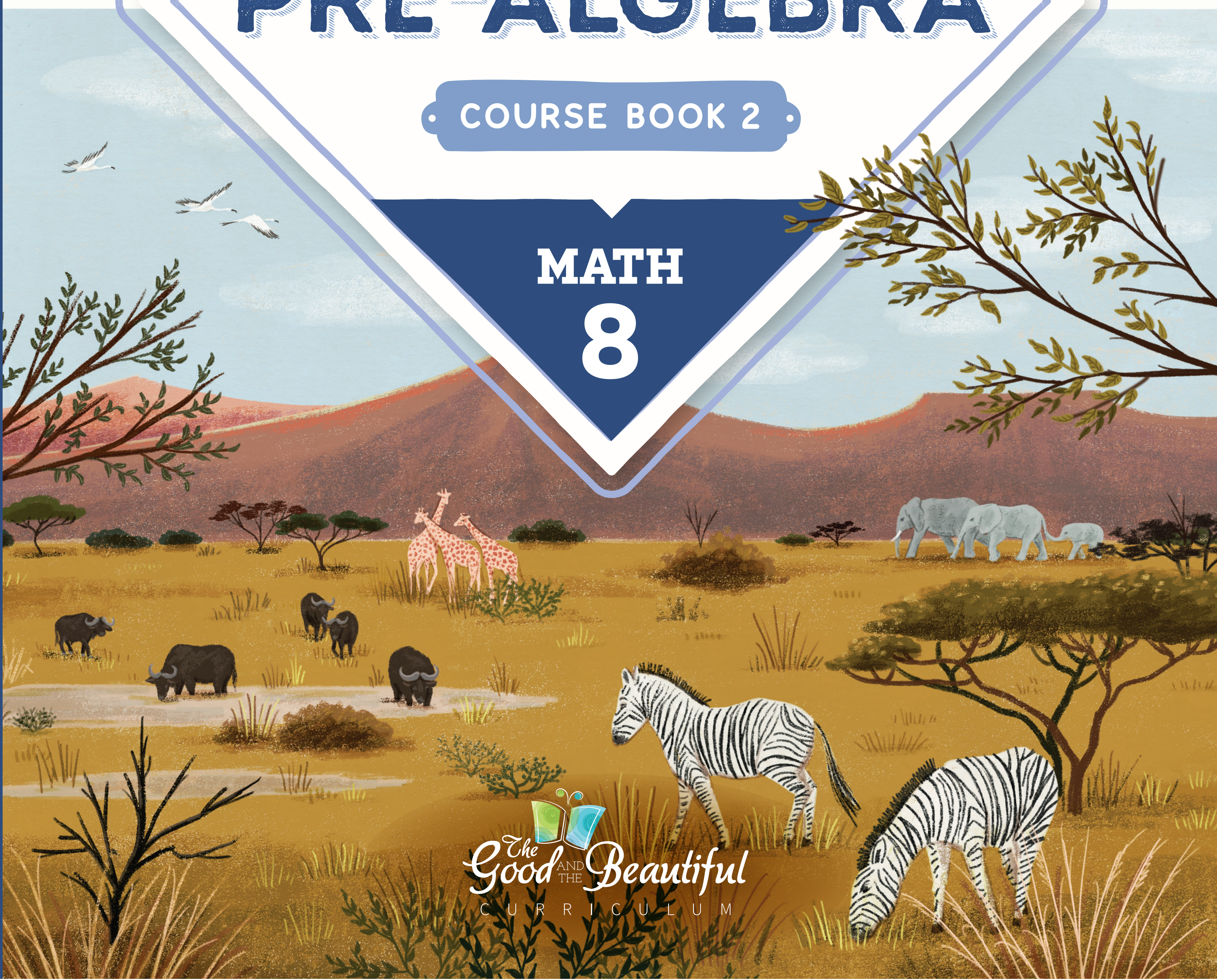
UNIT
2

Simply Good and Beautiful

PRE-ALGEBRA

COURSE BOOK 2

MATH
8



The Good and the Beautiful
CURRICULUM

COURSE BOOK 2

Table of Contents

Unit 2 Overview

LESSONS 31–60

CONCEPTS COVERED

- Calculating slope using the formula
- Comparing linear functions
- Comparing linear representations
- Constant of proportionality
- Converse of Pythagorean theorem
- Converting between forms of a linear equation
- Converting between fractions, decimals, and percents
- Distance formula
- Domain and range
- Equations of horizontal and vertical lines
- Finding a fraction given a whole and part
- Finding a percent given a whole and part
- Finding a whole given a fraction and part
- Finding a whole given a percent and part
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- Finding missing output values
- Finding missing sides on right triangles
- Finding percents of numbers
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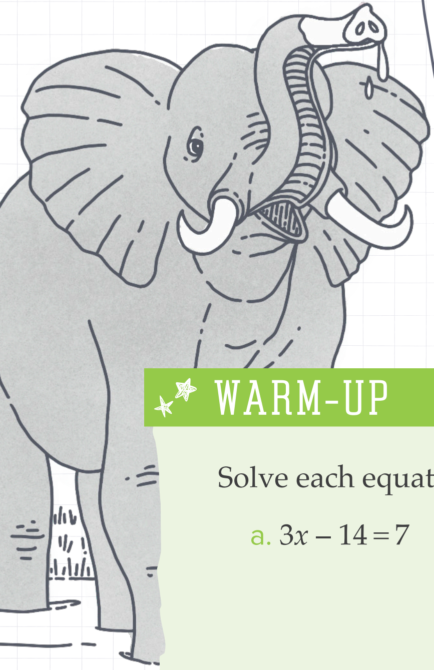
- Writing an equation from a graph
- Writing equations from tables
- Writing linear equations from a table
- Writing the equation of a line in slope-intercept form and point-slope form given multiple points
- x- and y-intercepts



UNIT 2 | LESSON 32

Modeling Real-World Situations with Equations

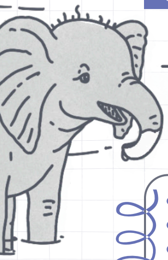
WARM-UP

Solve each equation.

a. $3x - 14 = 7$

b. $5(x + 6) = 40$

LESSON



Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

Mathematical calculations are frequently needed in everyday situations. Often, by analyzing the situation, an equation that fits the situation can be written. It is helpful when writing an equation to think of and write two separate expressions that represent the same amount. Then put an equal sign between the expressions to make an equation. Several real-world situations are detailed below.

Example 1: A box of chalk comes with t pieces. The Larsen family used three boxes plus nine pieces of chalk from another box. The Hopper family used four boxes and had two pieces of chalk left over. The Hoppers and the Larsens used the same number of pieces of chalk. How many pieces of chalk come in a box?

Write an expression to represent how much chalk each family used.

Larsen Family: $3t + 9$

Hopper Family: $4t - 2$

Since both families used the same number of chalk pieces, these expressions are equivalent. Write an equation by setting the expressions equal to each other.

$3t + 9 = 4t - 2$	Subtract 3t from each side.
$3t + 9 - 3t = 4t - 2 - 3t$	
$9 = t - 2$	Add 2 to each side.
$9 + 2 = t - 2 + 2$	
$11 = t$	Each box has 11 pieces of chalk.

Example 2: Nathan and Preston are on the swim team, and each of them swims l laps as part of his warm-up. Preston swims the backstroke, and Nathan swims the butterfly. Nathan takes six minutes to complete one lap, and he stretches for two minutes. Preston takes three minutes to complete one lap, and he stretches for eight minutes. Preston and Nathan warm up for the same amount of time. How many laps do they each swim?

Write an expression to represent the amount of time each boy spent warming up.

Nathan: $6l + 2$

Preston: $3l + 8$

Since both boys spend the same amount of time warming up, these expressions are equivalent. Write an equation by setting the expressions equal to each other.

$6l + 2 = 3l + 8$	Subtract 3l from each side.
$6l + 2 - 3l = 3l + 8 - 3l$	
$3l + 2 = 8$	Subtract 2 from each side.
$3l + 2 - 2 = 8 - 2$	
$3l = 6$	Divide both sides by 3.
$\frac{3l}{3} = \frac{6}{3}$	
$l = 2$	They each swim 2 laps.

Example 3: Christine and Holly went to a fair and bought homemade stickers. Each sticker cost x dollars. Christine bought 15 stickers and split the cost equally with her four siblings. Holly went to the fair three times, and every time she bought 11 stickers. Each time she went to the fair, she received a $5 discount on her stickers. Christine and Holly spent the same amount on stickers. How much did each sticker cost?

Write an expression to represent how much money each girl spent on stickers.
Note: Since Christine split the cost equally with four siblings, the cost is split among five people.

Christine: $\frac{15x}{5}$

Holly: $3(11x-5)$

Since the girls spent the same amount on stickers, these expressions are equivalent. Write an equation by setting the expressions equal to each other.

$$\frac{15x}{5}=3(11x-5)$$

On the left, simplify the expression. On the right, distribute the 3.

$$3x=33x-15$$

Subtract $3x$ from both sides.

$$3x-3x=33x-15-3x$$

$$0=30x-15$$

Add 15 to both sides.

$$0+15=30x-15+15$$

$$15=30x$$

Divide both sides by 30.

$$\frac{15}{30}=\frac{30x}{30}$$

$$\frac{1}{2}=x$$

Each sticker cost $\frac{1}{2}$ of a dollar, or $0.50.

Example 4: Peter enjoys 3D printing with his dad. Together they design and print mini race cars to donate to local preschools. In one week Peter and his dad printed z cars and separated those cars plus eight others equally into 13 donation boxes. The next week, Peter and his dad printed z more cars. Unfortunately, four cars were printed incorrectly. The cars that were printed successfully were put into another donation box. If each donation box ended up with the same number of cars, how many cars did Peter and his dad make each week?

Write an expression to represent the number of cars in each donation box each week.

First Week: $\frac{z+8}{13}$

Second Week: $z-4$

Since each box ended up with the same number of cars, the expressions are equivalent. Write an equation by setting the expressions equal to each other.

$$\frac{z+8}{13}=z-4$$

Multiply both sides by 13.

$$13\bullet\frac{z+8}{13}=(z-4)\bullet 13$$

Distribute 13 to both terms on the right side.

$$z+8=13z-52$$

Subtract z from both sides.

$$z+8-z=13z-52-z$$

$8 = 12z - 52$ Add 52 to both sides.

$8 + 52 = 12z - 52 + 52$

$60 = 12z$ Divide both sides by 12.

$$\frac{60}{12} = \frac{12z}{12}$$

$5 = z$ They made 5 cars each week.

PRACTICE

1. Harley and Dennis each set up a lemonade stand in their neighborhood and agree to each charge d dollars per cup of lemonade. Harley spends \$10 on supplies and sells 44 cups of lemonade, while Dennis spends \$8 on supplies and sells 36 cups of lemonade.

   a. Write an expression that represents Dennis's profit.

   ✦ Hint: Profit is the amount of money made after subtracting expenses.

   ____

   b. Write an expression that represents Harley's profit.

   ____

   c. Write and solve an equation to find how much they charged for each cup of lemonade if Dennis and Harley made the same amount of profit.

   ________ per cup

2. A party package comes with b bottles of bubbles. At a party, the children use six packages of bubbles and then use 12 more bottles that they find on a table. The adults grab eight packages of bubbles but end up having four bottles left over.

   a. Write an expression that shows how many bottles of bubbles the children used.

   ____

   b. Write an expression that shows how many bottles of bubbles the adults used.

   ____

   c. Suppose the children and adults used the same amount of bubbles. Write and solve an equation to find how many bottles are in a package.

   ________ bottles per package

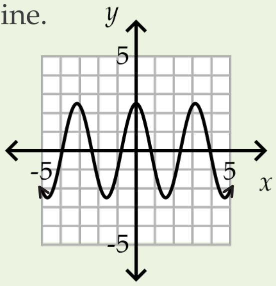
UNIT 2 | LESSON 38

Linear Functions

WARM-UP

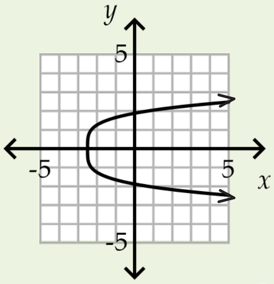
Use the vertical line test to determine if each graph represents a function. Write "yes" or "no" on the line.

a.



____

b.



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LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

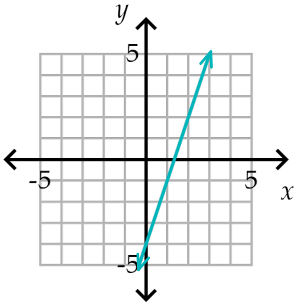
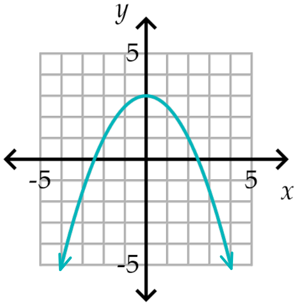
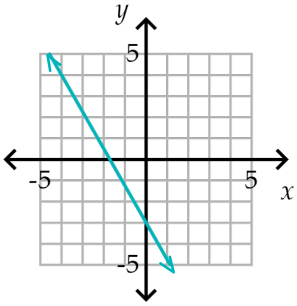
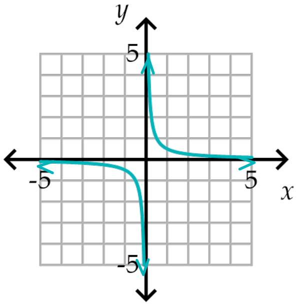
VIDEO PRACTICE

LESSON OVERVIEW

The most basic type of function is a linear function. A ***linear function*** is a function that forms a straight line when graphed.

LINEAR GRAPHS

The table below shows some examples of linear and nonlinear functions. Functions can form many different kinds of graphs that each have their own unique shape. Notice that each linear graph is a straight line.

Graph of Function	Linear?
	Yes The graph is a straight line.
	No The graph is a curve.
	Yes The graph is a straight line.
	No The graph contains curves.

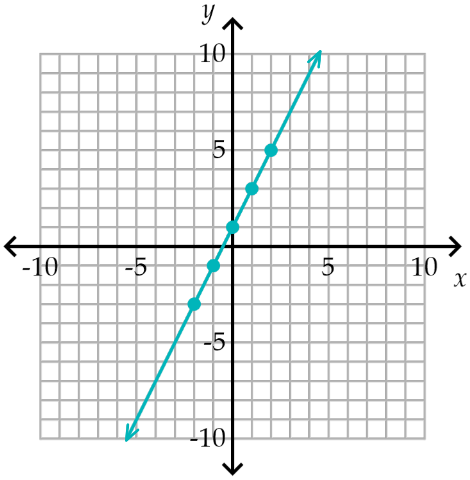
EQUATIONS OF LINEAR FUNCTIONS

Graphing an equation from a table can help determine if an equation is linear or nonlinear. When making a table, any input value that works in the equation can be chosen.

To graph the equation $y = 2x + 1$, a table is created with input values of –2, –1, 0, 1, and 2. The corresponding y-values are found by substituting the x-values into the equation. Then the points formed by the input/output pairs are plotted. The points are connected. It is clear from the graph that the points form a straight line. Therefore, $y = 2x + 1$ is a linear function.

x	y
–2	–3
–1	–1
0	1
1	3
2	5

Tip: Choose input values that are negative, positive, and zero.

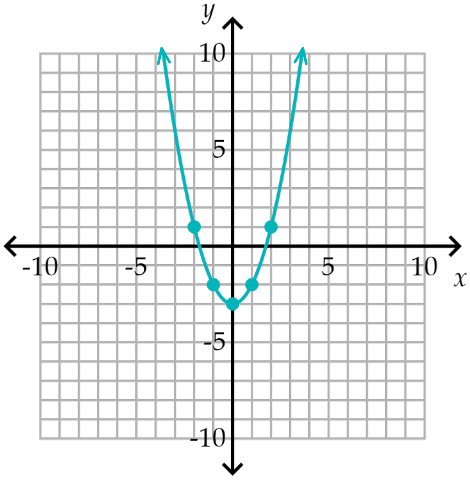


Example 1: Graph the equation $y = x^2 - 3$ to determine if it is linear or nonlinear.

Create a table with input values and find the corresponding output values.

x	y
–2	1
–1	–2
0	–3
1	–2
2	1

Plot and connect the points. The graph is not a straight line, so the equation is nonlinear.



It can be determined if an equation represents a linear or nonlinear function without graphing. A ***linear equation*** is an equation that contains only variables to the first power and constants. It is of the form $y = ax + b$, where a and b are numbers. When determining if an equation is linear or nonlinear without graphing, look at the terms. An exponent of 1 is not typically written, so any other exponent on x means the equation is nonlinear.

The table on the next page shows some linear and nonlinear functions. Notice that every linear function (except horizontal lines) only has x^1 in the equation. Nonlinear functions have x to other powers.

Linear Equations	Nonlinear Equations
$y=-x+6$	$y=\frac{4}{x}$ Note: The power of x in this equation is x^{-1}.
$y=2x-1$	$y-3=x^4+9$
$y=x$ Note: This is of the form $y=ax+b$ where a is 1 and b is 0.	$y=4x^2+x-12$ Note: There is an x^1 in this equation, but the x^2 means the equation is nonlinear.
$y-3=2(x-5)$ Note: This can be simplified to $y=ax+b$ by distributing the 2 and adding 3 to both sides.	$y=6x^3$
$y=5x$	$y=x^2$

Note on Horizontal and Vertical Lines

- Equations of the form $y = b$ are horizontal lines. Even though there is no x in the equation, horizontal lines are linear functions.
- Equations of the form $x = a$ are vertical lines and have x to the first power. However, vertical lines are not functions because they do not pass the vertical line test. There are multiple outputs for the input value.

Example 2: Determine if the equation $y=\frac{1}{2}(x-2)$ is linear or nonlinear without graphing.

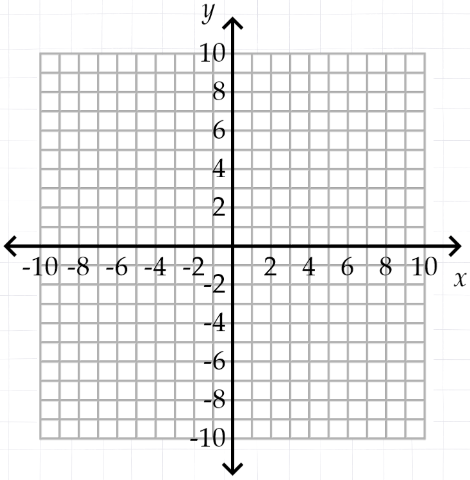
The power of x is 1 because no exponent is shown. This is a linear equation.

PRACTICE

Fill in the tables and graph each equation. Then determine if the equation represents a linear function.

1. $y = 3x - 1$

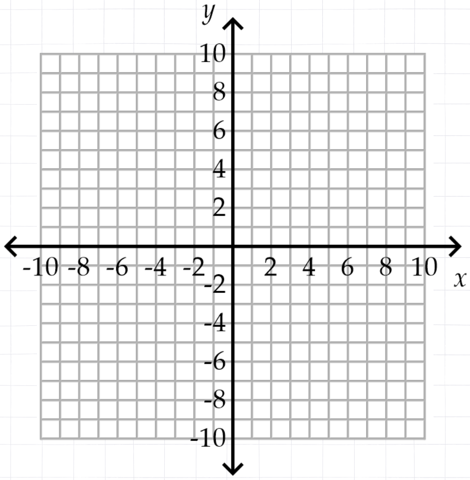
x	y
–2	
–1	
0	
1	
2	



Is it linear? _________

2. $y = \left(\frac{x}{5}\right)^2$

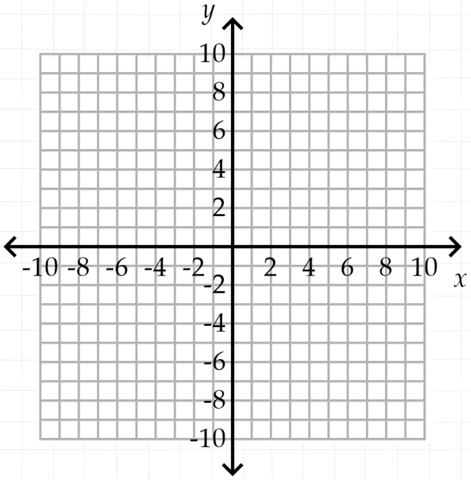
x	y
–10	
–5	
0	
5	
10	



Is it linear? _________

3. $y = 5(x + 2)$

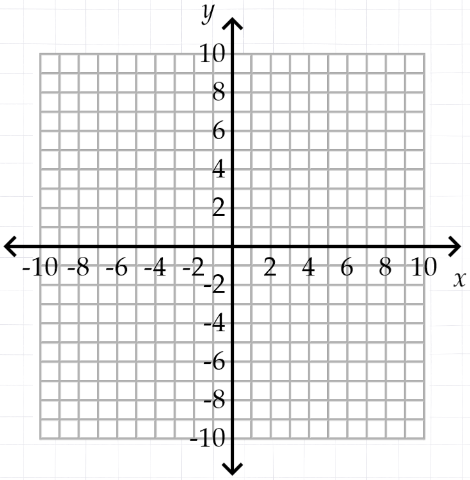
x	y
–4	
–3	
–2	
–1	
0	



Is it linear? _________

4. $y = 5$

x	y
–2	
–1	
0	
1	
2	

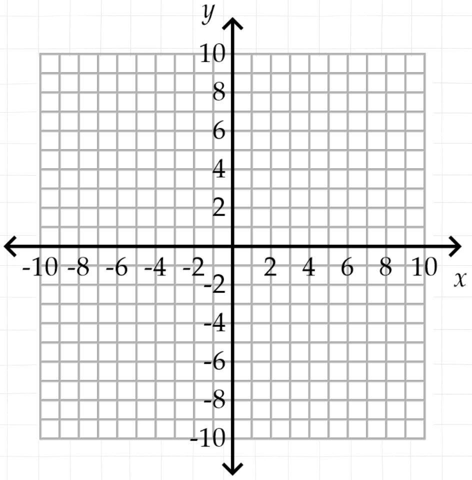


Is it linear? _________

REVIEW

1. a. Complete the table and graph the relation $y = x^2 - 5$. L37

x	y
–3	
–1	
0	
2	
3	



b. Is the relation a function? ______

2. Given the table, find the rule and write the equation for the relation. L36

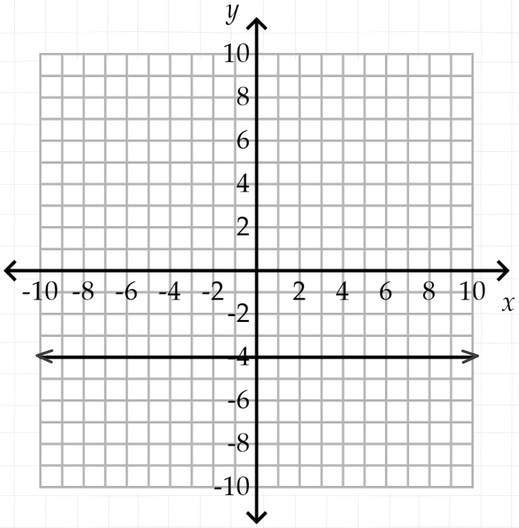
x	y
–4	–2
–2	–1
0	0
2	1
4	2

Rule: ______________________________

Equation: ___________________________

3. Write the equation of the line that is graphed below. L34

✦ Hint: All horizontal lines are of the form $y = b$.



Equation: ___________________________

4. Solve the equation. L31

$$17 - 5(x + 8) = -83$$

____

5. Mindy is making handmade bows for her family's Christmas packages. Each bow takes $1\frac{1}{3}$ feet of ribbon. How many bows can she make from a 60-foot spool of ribbon? L6

______ bows

UNIT 2 | LESSON 45

Logic Lesson 2

THE WONDER OF WINTER

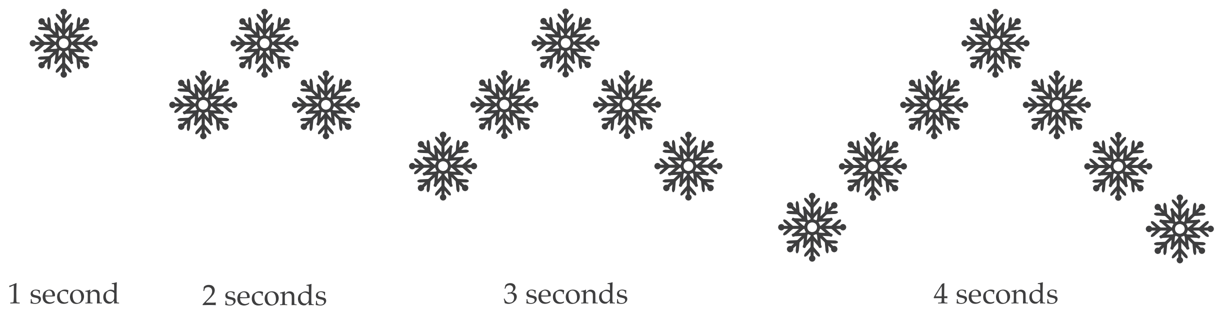
SUPPLIES: scissors

Each year, at least one septillion (1×10^{24}) snowflakes fall from the sky! Snow can be fun, beautiful, peaceful, tolerable, messy, or even annoying. Without ever naming it explicitly, Emily Dickinson shared her perspective about snow in a poem known by its first line: "It Sifts from Leaden Sieves." The logic puzzles in this lesson will refer to some of the lines from Dickinson's poem. This lesson has no video or review problems.

It sifts from Leaden Sieves -
It powders all the Wood.
It fills with Alabaster Wool
The Wrinkles of the Road -

1. Dickinson related snow falling from gray clouds and dusting the landscape in white to flour or sugar falling from a metal sieve and powdering the objects on which it lands. As snow accumulates, it covers tracks and grooves ("wrinkles") in the landscape. Suppose some snowflakes accumulated in the forest (powdering the wood) in the pattern below. Assuming the pattern continues, answer each question below.

✦ Hint: Rather than drawing more cases, think of how the number of snowflakes on the left side of each pile (including the top snowflake) corresponds to the number of seconds. Then see how the number of snowflakes in the right side of each pile (the remaining snowflakes) corresponds to the number of seconds.



a. How many snowflakes will accumulate after 5 seconds? ______ snowflakes

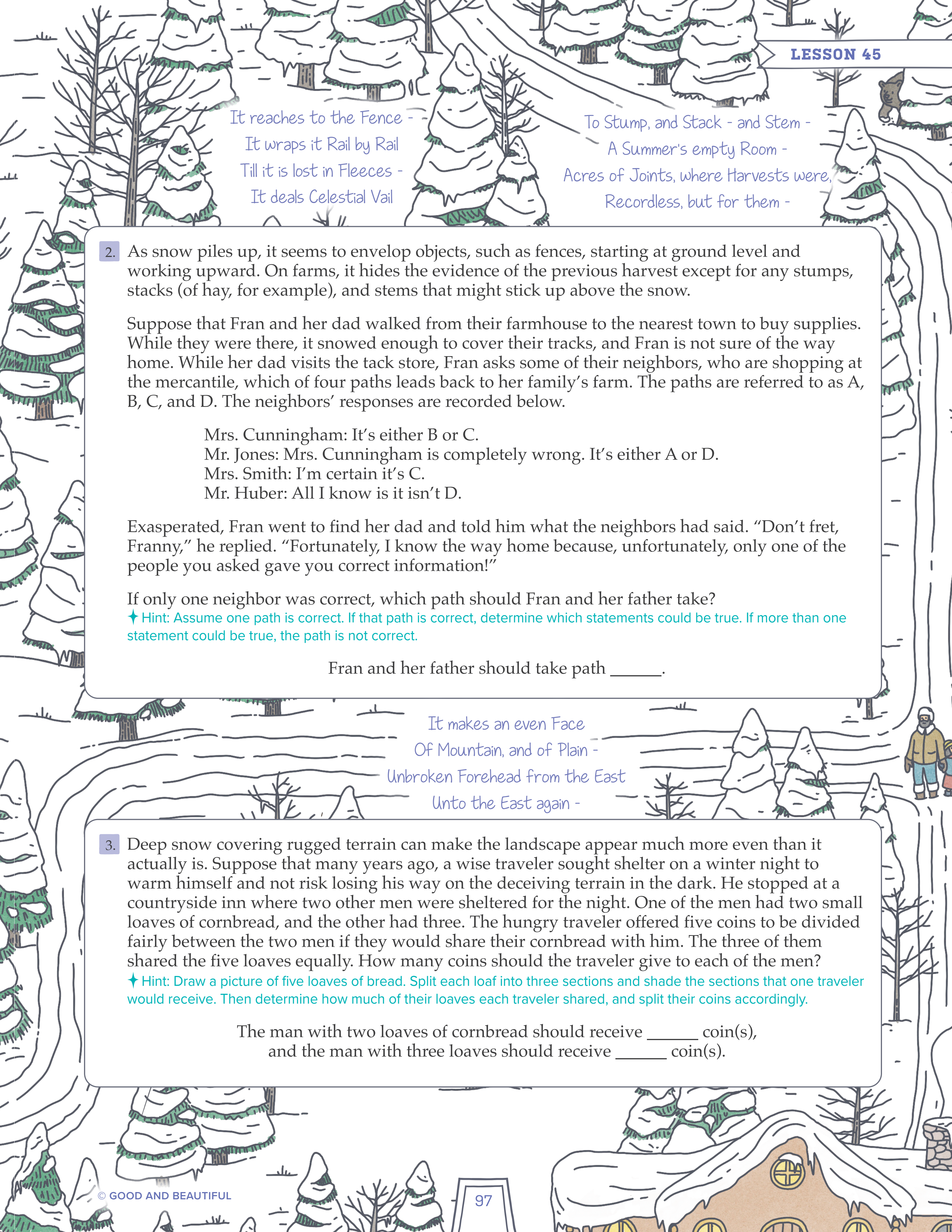
b. How many snowflakes will accumulate after 25 seconds? ______ snowflakes

c. How many snowflakes will accumulate after 100 seconds? ______ snowflakes

d. How many snowflakes will accumulate after x seconds?

✦ Hint: Write and simplify an expression that uses x.

______________ snowflakes



It reaches to the Fence -
It wraps it Rail by Rail
Till it is lost in Fleeces -
It deals Celestial Vail

To Stump, and Stack - and Stem -
A Summer's empty Room -
Acres of Joints, where Harvests were,
Recordless, but for them -

2. As snow piles up, it seems to envelop objects, such as fences, starting at ground level and working upward. On farms, it hides the evidence of the previous harvest except for any stumps, stacks (of hay, for example), and stems that might stick up above the snow.

Suppose that Fran and her dad walked from their farmhouse to the nearest town to buy supplies. While they were there, it snowed enough to cover their tracks, and Fran is not sure of the way home. While her dad visits the tack store, Fran asks some of their neighbors, who are shopping at the mercantile, which of four paths leads back to her family's farm. The paths are referred to as A, B, C, and D. The neighbors' responses are recorded below.

Mrs. Cunningham: It's either B or C.
Mr. Jones: Mrs. Cunningham is completely wrong. It's either A or D.
Mrs. Smith: I'm certain it's C.
Mr. Huber: All I know is it isn't D.

Exasperated, Fran went to find her dad and told him what the neighbors had said. "Don't fret, Franny," he replied. "Fortunately, I know the way home because, unfortunately, only one of the people you asked gave you correct information!"

If only one neighbor was correct, which path should Fran and her father take?
✦ Hint: Assume one path is correct. If that path is correct, determine which statements could be true. If more than one statement could be true, the path is not correct.

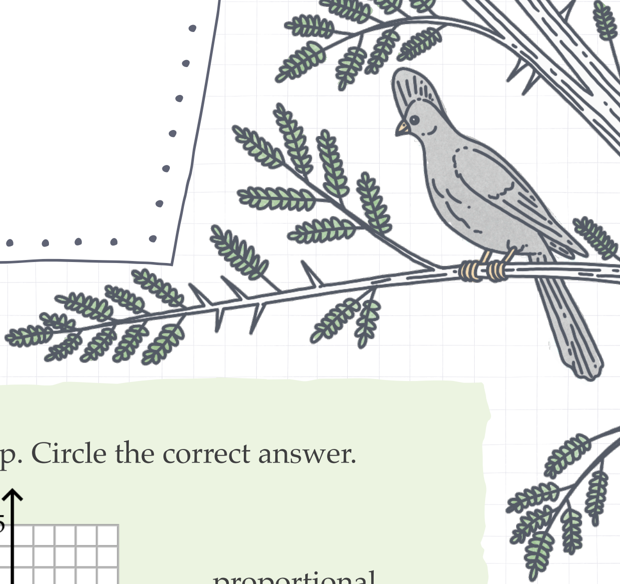
Fran and her father should take path ______.

It makes an even Face
Of Mountain, and of Plain -
Unbroken Forehead from the East
Unto the East again -

3. Deep snow covering rugged terrain can make the landscape appear much more even than it actually is. Suppose that many years ago, a wise traveler sought shelter on a winter night to warm himself and not risk losing his way on the deceiving terrain in the dark. He stopped at a countryside inn where two other men were sheltered for the night. One of the men had two small loaves of cornbread, and the other had three. The hungry traveler offered five coins to be divided fairly between the two men if they would share their cornbread with him. The three of them shared the five loaves equally. How many coins should the traveler give to each of the men?
✦ Hint: Draw a picture of five loaves of bread. Split each loaf into three sections and shade the sections that one traveler would receive. Then determine how much of their loaves each traveler shared, and split their coins accordingly.

The man with two loaves of cornbread should receive ______ coin(s),
and the man with three loaves should receive ______ coin(s).

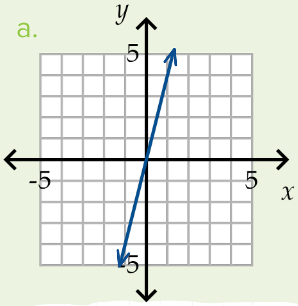
Graphing from Standard Form



WARM-UP

Determine if each graph represents a proportional relationship. Circle the correct answer.

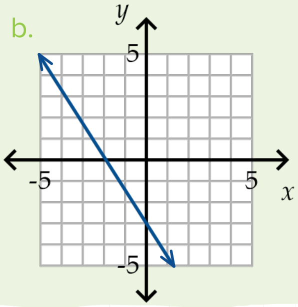
a.



proportional

not proportional

b.



proportional

not proportional

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

Recall that an x-intercept is the point on a graph where the line crosses the x-axis, and a y-intercept is the point where the line crosses the y-axis. At each intercept, the other variable is zero. For example, at the x-intercept, the value of y is zero, and at the y-intercept, the value of x is zero. Because of this, working with and using intercepts is a simple, but quite useful, process.

Two forms of linear equations that have been studied in this course are slope-intercept form and point-slope form. A third form of a linear equation is often used when working with intercepts. ***Standard form*** is a linear equation of the form $Ax + By = C$, where A, B, and C are integers.

Some equations written in standard form are shown below along with the values of A, B, and C for each equation.

$4x+5y=12$	$-3x+15y=30$	$x-y=-2$
$A=4,\ B=5,\ C=12$	$A=-3,\ B=15,\ C=30$	$A=1,\ B=-1,\ C=-2$

Linear equations written in standard form can be easily graphed by identifying the x- and y-intercepts. The coordinates of any x-intercept can be written as $(x,0)$. Substituting zero for y and solving the equation for x will yield the x-coordinate of the x-intercept. Likewise, the coordinates of any y-intercept can be written as $(0,y)$. Substituting zero for x and solving the equation for y will yield the y-coordinate of the y-intercept.

Given the equation $3x+6y=12$, the x-intercept and y-intercept are found below.

x-intercept:

Substitute zero for y and solve for x.

$$3x+6(0)=12$$
$$3x=12$$
$$\frac{3x}{3}=\frac{12}{3}$$
$$x=4$$

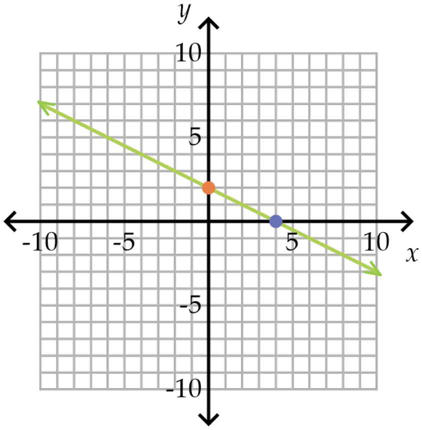
The x-intercept is $(4,0)$.

y-intercept:

Substitute zero for x and solve for y.

$$3(0)+6y=12$$
$$6y=12$$
$$\frac{6y}{6}=\frac{12}{6}$$
$$y=2$$

The y-intercept is $(0,2)$.



Because only two points are needed to draw a line, the x- and y-intercepts can be plotted on a graph to create the line for the linear equation. To the left is the graph of the line $3x+6y=12$ using the intercepts $(4,0)$ and $(0,2)$.

Once the line is graphed, the slope can be found using rise over run. The slope of this line is –2 (down 2) over 4 (right 4), which is $-\frac{1}{2}$.

KEY INFORMATION

To find an intercept, substitute zero for the other variable.

Example 1: Use the x- and y-intercepts to graph the equation $2x+3y=-18$. Then find the slope of the line.

Find the x-intercept:

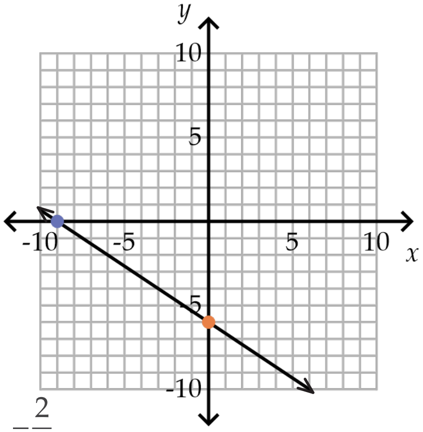
$$2x+3(0)=-18$$
$$2x=-18$$
$$\frac{2x}{2}=\frac{-18}{2}$$
$$x=-9$$

x-intercept: $(-9,0)$

Find the y-intercept:

$$2(0)+3y=-18$$
$$3y=-18$$
$$\frac{3y}{3}=\frac{-18}{3}$$
$$y=-6$$

y-intercept: $(0,-6)$



The slope is –6 (down 6) over 9 (right 9), which is $-\frac{2}{3}$.

Example 2: Use the x- and y-intercepts to graph the equation $5x-10y=20$. Then find the slope of the line.

Find the x-intercept:

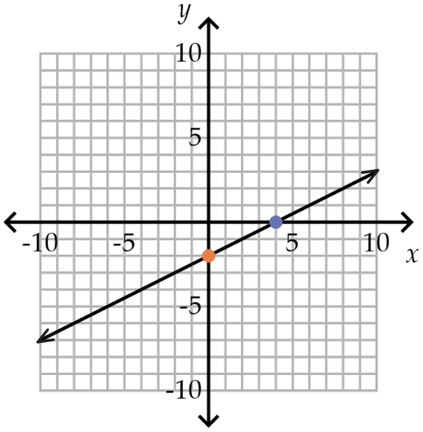
$$5x-10(0)=20$$
$$5x=20$$
$$\frac{5x}{5}=\frac{20}{5}$$
$$x=4$$

x-intercept: $(4,0)$

Find the y-intercept:

$$5(0)-10y=20$$
$$-10y=20$$
$$\frac{-10y}{-10}=\frac{20}{-10}$$
$$y=-2$$

y-intercept: $(0,-2)$



The slope is 2 (up 2) over 4 (right 4), which is $\frac{1}{2}$.

Example 3: Use the x- and y-intercepts to graph the equation $x-y=9$. Then find the slope of the line.

Find the x-intercept:

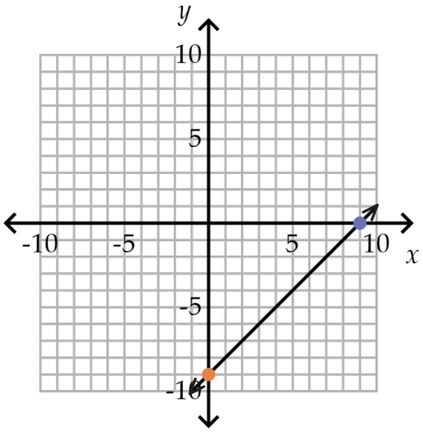
$$x-0=9$$
$$x=9$$

x-intercept: $(9,0)$

Find the y-intercept:

$$0-y=9$$
$$-y=9$$
$$\frac{-y}{-1}=\frac{9}{-1}$$
$$y=-9$$

y-intercept: $(0,-9)$



The slope is 9 (up 9) over 9 (right 9), which is 1.

PRACTICE

Complete each problem below. Find the answer in the table on the next page and cross off the phrase next to it. Once all problems have been completed, write the remaining phrases, from top to bottom, at the bottom of the page to discover a neat fact about God's creation!

1. Find the x-intercept for each equation.

a. $5x - 4y = 60$ ________

b. $-x + 2y = 6$ ________

c. $3x + 5y = -120$ ________

d. $-7x - 5y = 105$ ________

2. Find the y-intercept for each equation.

a. $5x - 4y = 60$ ________

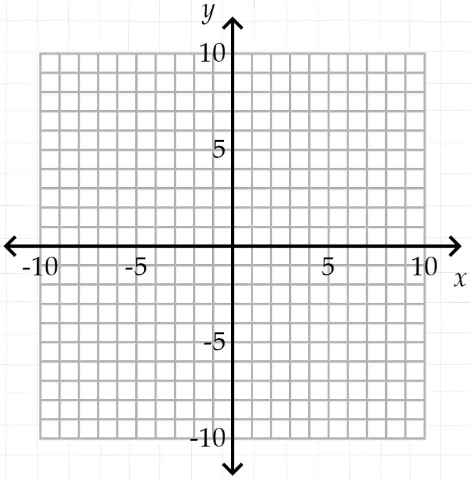
b. $-x + 2y = 6$ ________

c. $3x + 5y = -120$ ________

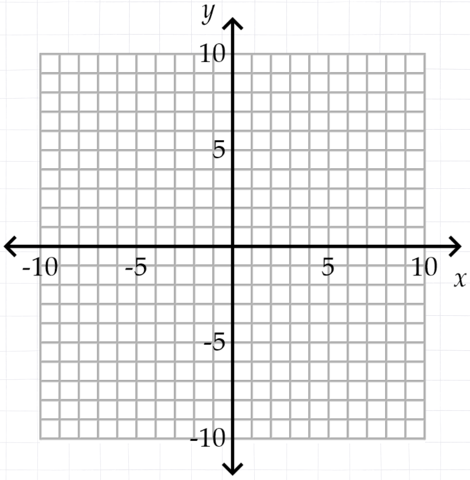
d. $-7x - 5y = 105$ ________

3. Use the x- and y-intercepts to graph each equation. Then use the graph to find the slope of each line. The slope is the answer to cross off in the table on the next page.

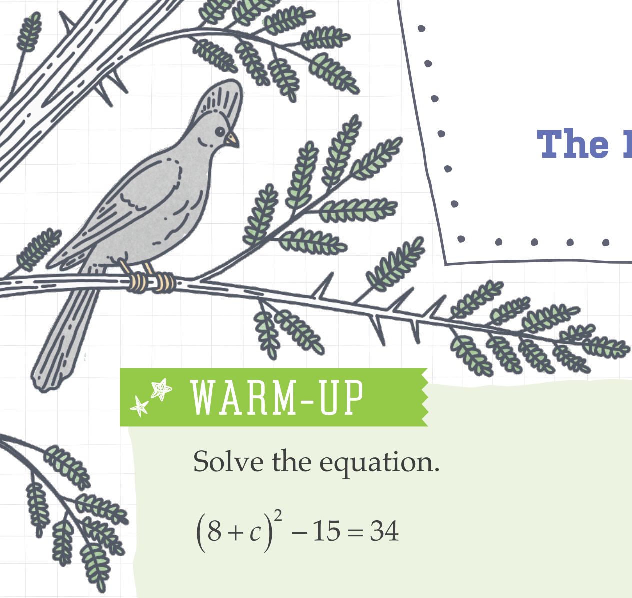
a. $-3x - 4y = 24$ Slope: ________

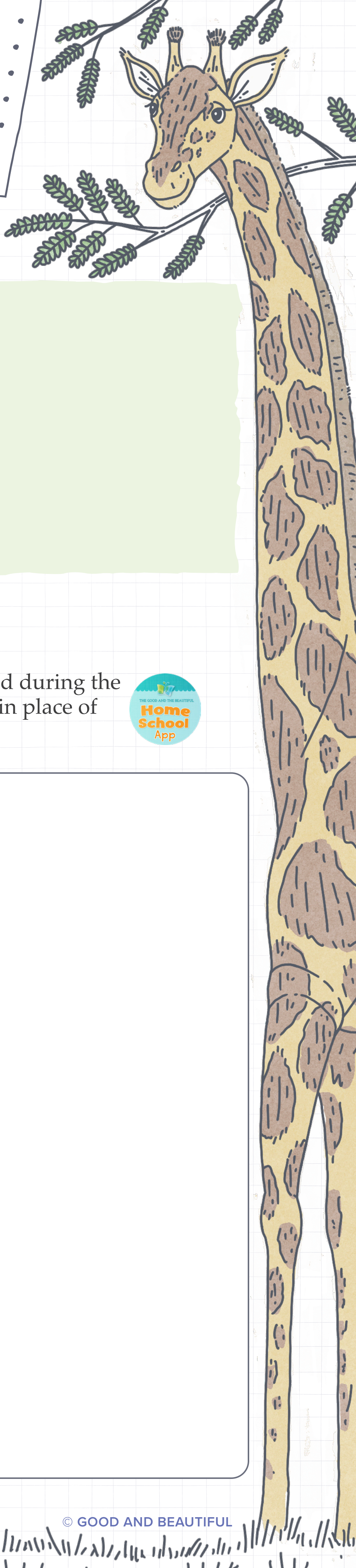


b. $6x - 8y = 24$ Slope: ________



The Pythagorean Theorem





☆ **SUPPLIES:** scissors

WARM-UP

Solve the equation.

$(8+c)^2 - 15 = 34$

______

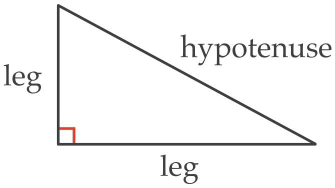
LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

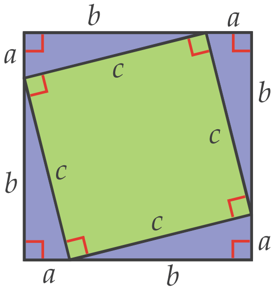
LESSON OVERVIEW

The Pythagorean theorem is named for the ancient Greek philosopher Pythagoras. A theorem is a proven statement in math. While records of reference to the Pythagorean theorem go back thousands of years, this important formula is widely used today in construction and distance calculations.

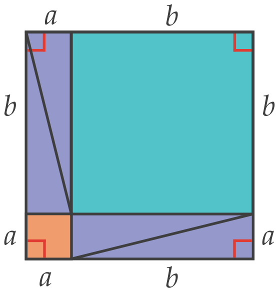


In a right triangle, one angle is a right angle as indicated by the red box. The ***Pythagorean theorem*** states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two side lengths. The ***hypotenuse*** is the longest side of a right triangle. It is the side opposite, or across from, the right angle. The ***legs*** are the two sides of a right triangle that are adjacent to the right angle. Below is the formula for the Pythagorean theorem, where a and b represent the legs and c represents the hypotenuse.

Pythagorean theorem: $a^2 + b^2 = c^2$



The Pythagorean theorem can be proven by considering a square with side lengths of $a + b$ as shown in the first diagram at the right. Notice that the inside of the square is made up of four right triangles (in purple), each with side lengths a, b, and c, and the center is a square (in green) whose area is c^2 because all side lengths are c.



If the purple triangles are rearranged, two smaller squares (in orange and blue) are formed as shown in the second diagram at the right. Notice that one smaller square has side lengths a and the other has side lengths b. The area of the smaller squares is a^2 and b^2, respectively.

Since the total area must be the same in each diagram, the sum of areas of the orange square and blue square must equal the area of the green square. Therefore, $a^2 + b^2 = c^2$.

PYTHAGOREAN TRIPLES

Pythagorean triples are groups of three integers that satisfy the Pythagorean theorem.

The integers 3, 4, and 5 are a Pythagorean triple because the sum of 3^2 and 4^2 is equal to 5^2.

$$3^2 + 4^2 = 9 + 16 = 25 \qquad 5^2 = 25$$

The numbers 6, 8, and 10 are also a Pythagorean triple because $6^2 + 8^2 = 10^2$. To determine if three numbers are a Pythagorean triple, add the squares of the two smaller numbers and see if it equals the square of the largest number.

Example 1: Do the numbers 4, 8, and 9 form a Pythagorean triple?

Add the squares of the two smaller numbers. $4^2 + 8^2 = 16 + 64 = 80$
Find the square of the largest number. $9^2 = 81$

Since $4^2 + 8^2$ does not equal 9^2, the numbers 4, 8, and 9 do not form a Pythagorean triple.

PRACTICE

1. a. Fill in the blanks to verify that 5, 12, and 13 form a Pythagorean triple.

$5^2 + 12^2 =$ ______ + ______ = ______

$13^2 =$ ______

b. Multiply 5, 12, and 13 each by 3.

$5 \bullet 3 =$ ______ $12 \bullet 3 =$ ______ $13 \bullet 3 =$ ______

c. Check if the values found in Part B are also Pythagorean triples. Circle "yes" or "no."

______^{2} + ______^{2} = ______ + ______ = ______

______^{2} = ______

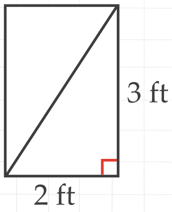
Pythagorean triple? yes / no

2. Determine if a triangle with the given side lengths is a right triangle. Write "yes" or "no" on the line.

a. 4 km, 7 km, 8 km ______

b. 20 in, 21 in, 29 in ______

3. Sarah wants to reinforce the side of a crate that is 3 ft by 2 ft by bracing it with a diagonal beam as shown below. She finds a piece of wood that is 4 ft long. Will that piece work for the brace without her having to cut it?



______

4. Follow the steps to create a proof of the Pythagorean theorem.

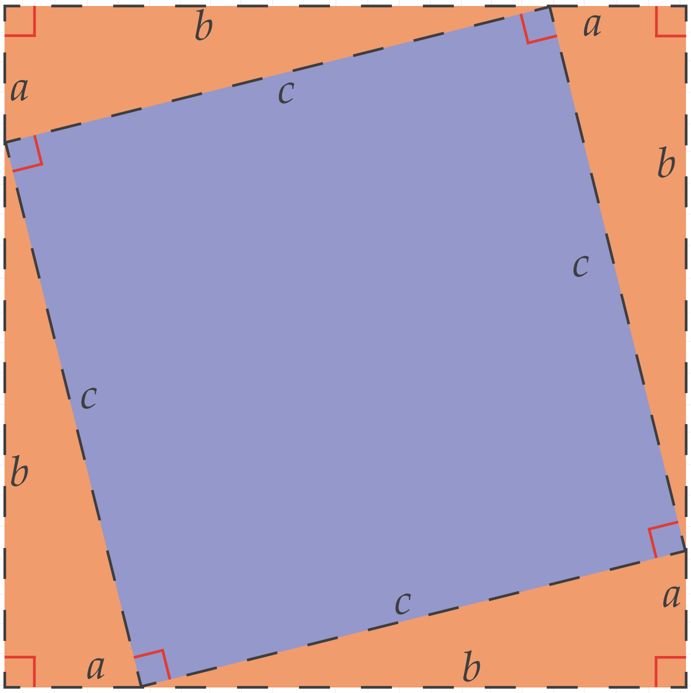
a. Consider the square on the bottom of this page. The smaller square inside has side length *c*. What is the area of the purple square?

✦ Hint: Do not measure. Write the area using a variable.

______

Note that this is the area of the large square that is NOT covered by the four orange triangles.

b. Cut on all dashed lines to cut out the four orange triangles and the purple square below.



UNIT 2 | LESSON 56

Fractions, Decimals, and Percents

WARM-UP

Complete the problem and state the meaning of the answer.

45 is what fraction of 75? ________

Meaning: ____

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

Fractions, decimals, and percents are all ways of expressing part of a whole. Knowing how to convert between each representation builds number sense and aids in understanding when encountering numbers in different forms.

CONVERTING BETWEEN FRACTIONS AND DECIMALS

Fraction → Decimal

Convert a fraction to a decimal by dividing the numerator by the denominator.

To convert $\frac{17}{20}$ to a decimal, divide 17 by 20.

$$\begin{array}{r} 0.85 \\ 20\overline{)17.00} \\ \underline{-\,160} \\ 100 \\ \underline{-\,100} \\ 0 \end{array}$$

$$\frac{17}{20} = 0.85$$

To convert $4\frac{5}{8}$ to a decimal, divide 5 by 8 and write the whole number, 4, before the decimal point.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{-\,4\,8} \\ 20 \\ \underline{-\,16} \\ 40 \\ \underline{-\,40} \\ 0 \end{array}$$

$$4\frac{5}{8} = 4.625$$

Decimal → Fraction

Convert a decimal to a fraction by writing the decimal digits in the numerator and the place value of the last decimal digit in the denominator. Simplify the fraction.

To convert 0.35 to a fraction, write the decimal digits, 35, over the place value of the 5, which is the hundredths place. Simplify.

$$0.35 = \frac{35}{100} = \frac{7}{20}$$

To convert 8.024 to a fraction, write the decimal digits, 24, over the place value of the 4, which is the thousandths place. Keep the whole number. Simplify the fraction.

$$8.024 = 8\frac{24}{1000} = 8\frac{3}{125}$$

CONVERTING BETWEEN DECIMALS AND PERCENTS

Percent → Decimal

Percent means “per hundred.” Convert a percent to a decimal by dividing by 100. Dividing by 100 moves the decimal point two places to the left.

To convert 5% to a decimal, move the decimal point two places to the left.

5.

5% = 0.05

To convert 200% to a decimal, move the decimal point two places to the left.

200.

200% = 2

Decimal → Percent

Convert a decimal to a percent by multiplying by 100. Multiplying by 100 moves the decimal point two places to the right.

To convert 0.85 to a percent, move the decimal point two places to the right.

0.85

0.85 = 85%

To convert 3.25 to a percent, move the decimal point two places to the right.

3.25

3.25 = 325%

Note: 1 is 100%. A number greater than 1 is more than 100%.

CONVERTING BETWEEN PERCENTS AND FRACTIONS

Percent → Fraction

Convert a percent to a fraction by writing the percent over 100 and simplifying.

To convert 55% to a fraction, write 55 over 100 and simplify.

$$55\% = \frac{55}{100} = \frac{11}{20}$$

To convert 120% to a fraction, write 120 over 100 and simplify.

$$120\% = \frac{120}{100} = \frac{6}{5} = 1\frac{1}{5}$$

Note: A percent greater than 100% can be written as an improper fraction or a mixed number in fraction form.

Fraction → Percent

Convert a fraction to a percent by doing one of the following:

- Make an equivalent fraction with a denominator of 100. The numerator is the percent.
- If the original denominator is not a factor of 100, convert the fraction to a decimal by dividing. Then convert the decimal to a percent by moving the decimal point two places to the right.

To convert $\frac{3}{25}$ to a percent, write an equivalent fraction with 100 in the denominator. The numerator, 12, is the percent.

$$\frac{3}{25} = \frac{12}{100} = 12\%$$

To convert $\frac{1}{8}$ to a percent, divide 1 by 8 because 8 is not a factor of 100. Then move the decimal point two places to the right.

$$\begin{array}{r} 0.125 \\ 8\overline{)1.000} \\ \underline{-\ 8} \\ 20 \\ \underline{-\ 16} \\ 40 \\ \underline{-\ 40} \\ 0 \end{array}$$

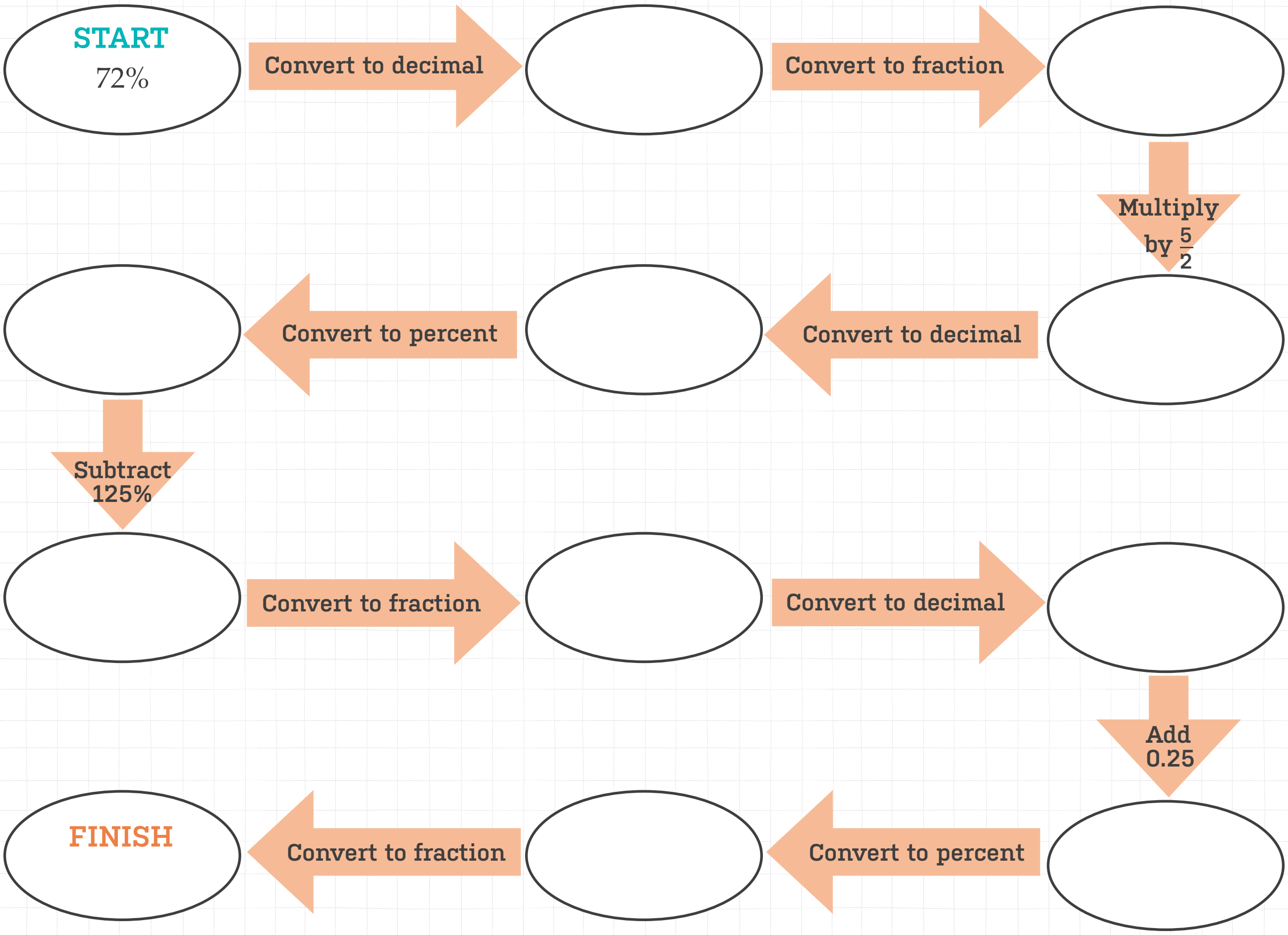
$$\frac{1}{8} = 0.125 = 12.5\%$$

To convert $3\frac{4}{5}$ to a percent, write an equivalent fraction with 100 in the denominator. The whole number, 3, is 300%.

$$3\frac{4}{5} = 3\frac{80}{100} = 300\% + 80\% = 380\%$$

PRACTICE

1. Begin at START. Follow the instructions to reach FINISH.



2. There are 50 plates at a party, 18 of which are red. What percent of the plates are red?

____

3. Of the people at the party, 74% are children. What fraction of the people at the party are children?

____

4. A banner at the party is 0.8 meters high and is printed on paper that is 1 meter high. What fraction of the paper height is not being used for the banner?

____

5. There were 40 people who said they were coming to the party, but 50 people actually attended. What percent of the people who said they were coming actually attended? In other words, 50 is what percent of 40?

____

WINTER CAMP

UNIT 2 | LESSON 58

Unit 2 Review

Complete this Unit Review to prepare for the Unit Assessment. There is no video, lesson, or practice. Because Unit Reviews include practice for an entire unit, they may take longer than regular lessons, and students may decide to take two days to finish.

Hannah is away at Winter Camp with some of her friends from church. Complete the problems related to Hannah's trip as you review material from Unit 2.

LESSON 32

1. On the first day of Winter Camp, Hannah and her friends go for a hike. They start at an elevation of 2700 ft above sea level. Their elevation increases 726 ft every hour. They finish at an elevation that is twice their initial elevation. Write an equation to model the situation, using t for the time they spent hiking, in hours.

____

LESSON 31

2. Solve the following equations.

a. $3x+1=9-x$

b. $\frac{(7+19x)}{5}=3+4x$

$x=$ ____

$x=$ ____

LESSON 33

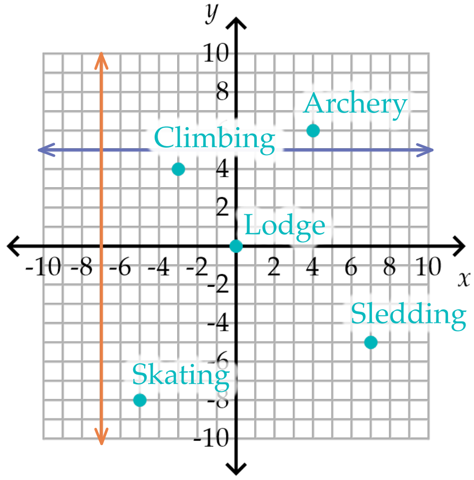
3. Hannah and her friends build a snow fort that has a perimeter given by the equation below, where A is the area of the fort. Solve the equation for A.

$$P=\frac{3}{2}A+5$$

____

LESSON 34

4. Hannah's group is given the following map of the campground.



a. In which quadrant is each activity located?

Archery: ____

Climbing: ____

Sledding: ____

Skating: ____

b. Determine the coordinates of the indicated area.

Skating: ________

Lodge: ________

c. The purple and orange lines represent trails through the grounds. Find the equation of each line.

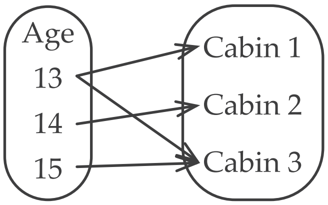
Purple: ________

Orange: ________

d. The camp director said the map is several years old and that bathrooms have been added at the point $(-9,8)$. Plot the bathrooms on the coordinate plane above.

LESSONS 35, 36

5. The following mapping diagram shows how Hannah and her three friends are assigned to cabins, relative to their ages.



a. Determine the domain and range of this relation in set notation.

Domain: ______________________________

Range: ________________________________

b. Is the relation a function? ________

Why or why not? ___________________

____

c. Identify the independent and dependent variables.

Independent variable: _______________

Dependent variable: _________________

6. The lodge makes hot cocoa based on how many people are staying there each day. The table below shows how many packages of cocoa the lodge makes relative to the number of campers. Find the rule and write the equation for the relation.

Number of Campers	Number of Cocoa Packages
10	4
20	8
30	12
40	16

Rule: ________________________________

Equation: ____________________________

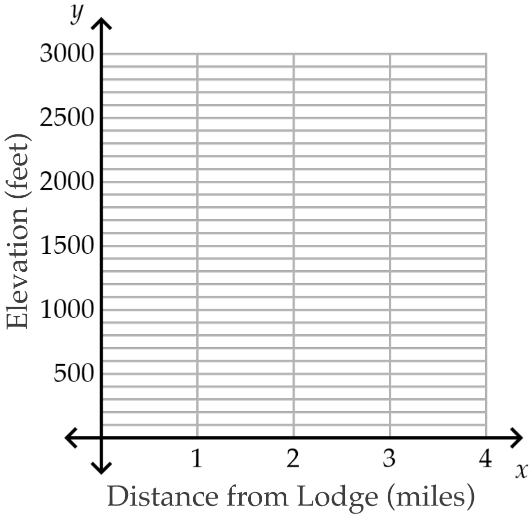
LESSON 37

7. The elevation down the side of the mountain can be modeled by the equation $y = -500x + 2700$, where x is the distance from the lodge (in miles) and y is the elevation (in feet) above sea level.

a. Use the equation to fill in the table.

x	y
0	
1	
2	
3	
4	

b. Use the table in Part A to graph the relation.



Unit 2 Assessment

○ This assessment covers concepts taught in Unit 2. Problems are designed to assess multiple skills. Read the instructions carefully and do not rush through the problems.

○ You may use the Reference Chart for the assessment. Calculators should only be used when noted. Lesson numbers are given by each problem so you can review lessons for any answers that are incorrect.

1. Solve the equations. L31

a. $3x + 18 - 2x = 9x - 30$

______

b. $5(a - 12) = 3a + 25$

______

c. $\frac{36 - p}{4} = p - 1$

______

2. Before a family trip, Logan and his sister earned money to buy souvenirs by doing extra chores. They each earned m dollars for every extra chore they did. Logan did two extra chores. He spent his extra chore money, plus $10 he had saved, on souvenirs. His sister did eight extra chores and spent $2 less than the amount she earned. L32

a. Write an expression that shows how much money Logan spent.

______

b. Write an expression that shows how much money his sister spent.

______

c. Suppose that Logan and his sister spent the same amount of money on souvenirs. Write and solve an equation to find how much they earned for each extra chore.

______ per extra chore

3. The formula for the area of a rectangle is $A = lw$.

a. Solve the formula for w.

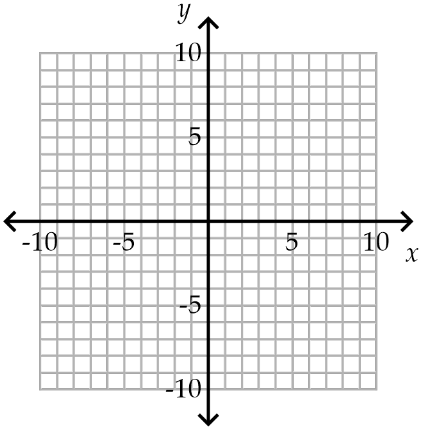
______

b. Find the width of a rectangle with an area of 27 ft^2 and a length of 9 ft. L33

______

10. For the equation $y = -3x + 2$, find the slope and y-intercept and graph the line. L41

Slope: ______ y-intercept: ________



11. Write the equation of the line in point-slope form that has the given slope and passes through the given point. Then convert the equation to slope-intercept form. L42

Slope: $\frac{3}{2}$ Point: $(4,-1)$

Point-slope form: _________________

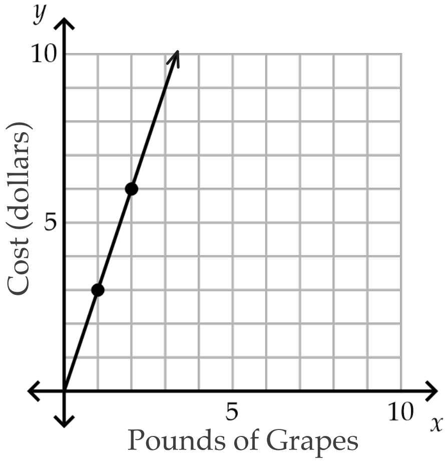
Slope-intercept form: ____________

12. Write the equation of the line that passes through the points below. L43

$(-4,-7)$ and $(0,-4)$

____

13. Use the graph to find the cost per pound of grapes. Write the equation of the line. L44



Cost: __________

Equation: _____________

14. Find the x- and y-intercepts for each equation. L46

a. $4x + 3y = 24$

x-intercept: ______ y-intercept: ______

b. $-x + y = -36$

x-intercept: ______ y-intercept: ______

UNIT 2 | LESSON 60

Enrichment: Collatz Conjecture

This is an enrichment lesson. Mastery is not expected at this level. There is no video, practice, or review. A calculator may be used for this entire lesson.

Consider a number game in which you start with any positive integer and follow just one of two rules at each step of the game. The rules are below.

- ✦ If the number is even, divide it by two.
- ✦ If the number is odd, multiply it by three and add one.

The steps are then repeated continually. Observe what happens when starting with the integer 5.

Starting integer: 5

5 is odd.	$5 \bullet 3 + 1 = 16$
16 is even.	$16 \div 2 = 8$
8 is even.	$8 \div 2 = 4$
4 is even.	$4 \div 2 = 2$
2 is even.	$2 \div 2 = 1$
1 is odd.	$1 \bullet 3 + 1 = 4$ ← This is a repeat of a previous answer.

Once a repeat answer is found, the process will just loop. In this example, the answers will cycle as shown below.

$$\begin{tikzcd}4 \arrow[r] & 2 \arrow[r] & 1 \arrow[ll, bend left]\end{tikzcd}$$

Try it!

Begin with the given integer and continue until you reach a repeat answer. Write the first repeat answer on the line.

1. Starting integer: 6

2. Starting integer: 7

3. Starting integer: 8

Repeat answer: _____

Repeat answer: _____

Repeat answer: _____

4. What do you notice about the first repeated answer in the examples above?

____

____

In Problem 8, the cycle of answers becomes this:

$$\begin{tikzcd} -5 \arrow[r] & -14 \arrow[r] & -7 \arrow[r] & -20 \arrow[r] & -10 \arrow[llll, bend left] \end{tikzcd}$$

In contrast, Problem 9 has a smaller loop:

$$\begin{tikzcd} -2 \arrow[r] & -1 \arrow[l, bend left] \end{tikzcd}$$

Problem 10 has a much larger loop:

$$\begin{tikzcd} -17 \arrow[r] & -50 \arrow[r] & -25 \arrow[r] & -74 \arrow[r] & -37 \arrow[r] & -110 \arrow[r] & -55 \arrow[r] & -164 \arrow[r] & -82 \arrow[dllllllll] \\ -41 \arrow[r] & -122 \arrow[r] & -61 \arrow[r] & -182 \arrow[r] & -91 \arrow[r] & -272 \arrow[r] & -136 \arrow[r] & -68 \arrow[r] & -34 \arrow[ullllllll, bend left] \end{tikzcd}$$

When observing negative integers, the Collatz conjecture is clearly not true because at least three different cycles are obtained as shown above. Mathematicians do not yet know if these three negative loops are the only loops that can be obtained when starting with a negative integer or whether there are others. It is fascinating that math problems exist that are simple to understand but that no one actually knows the answers to yet!

Fun fact: Mathematical loops are widely used in coding and computer programming!

LESSONS
61–90

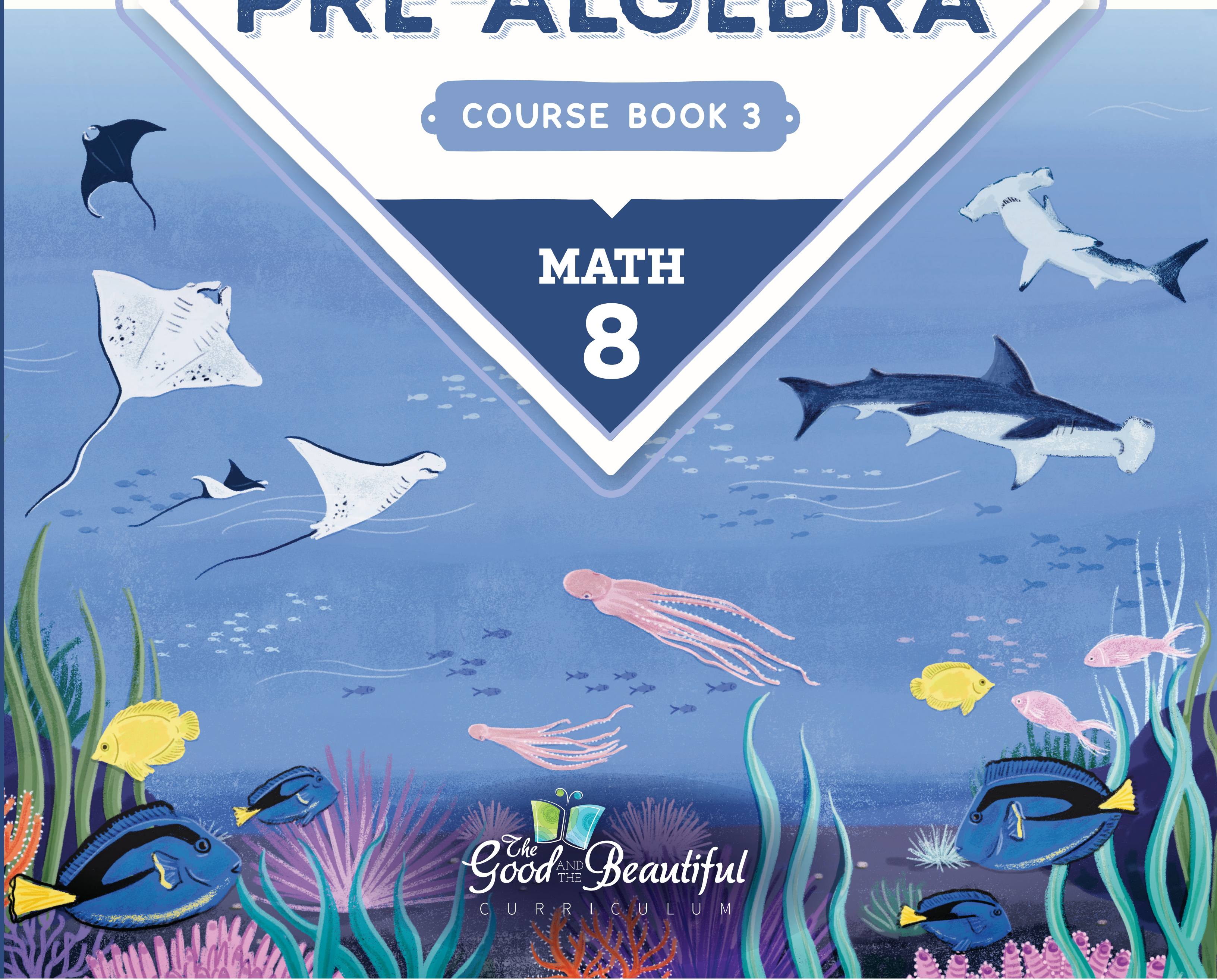
UNIT
3

Simply Good and Beautiful

PRE-ALGEBRA

COURSE BOOK 3

MATH
8



The Good and the Beautiful
CURRICULUM

COURSE BOOK 3

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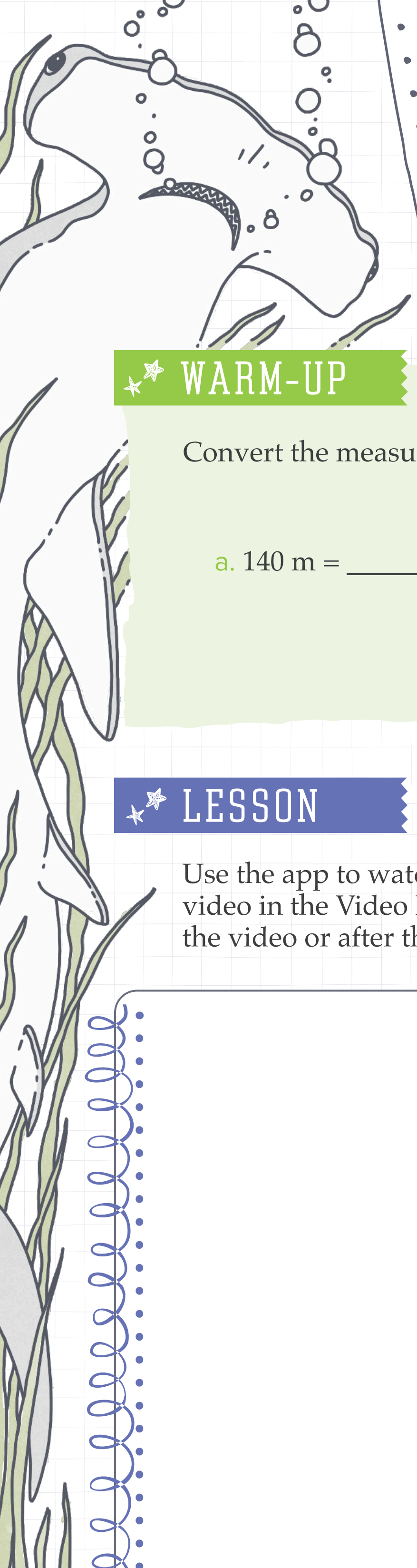
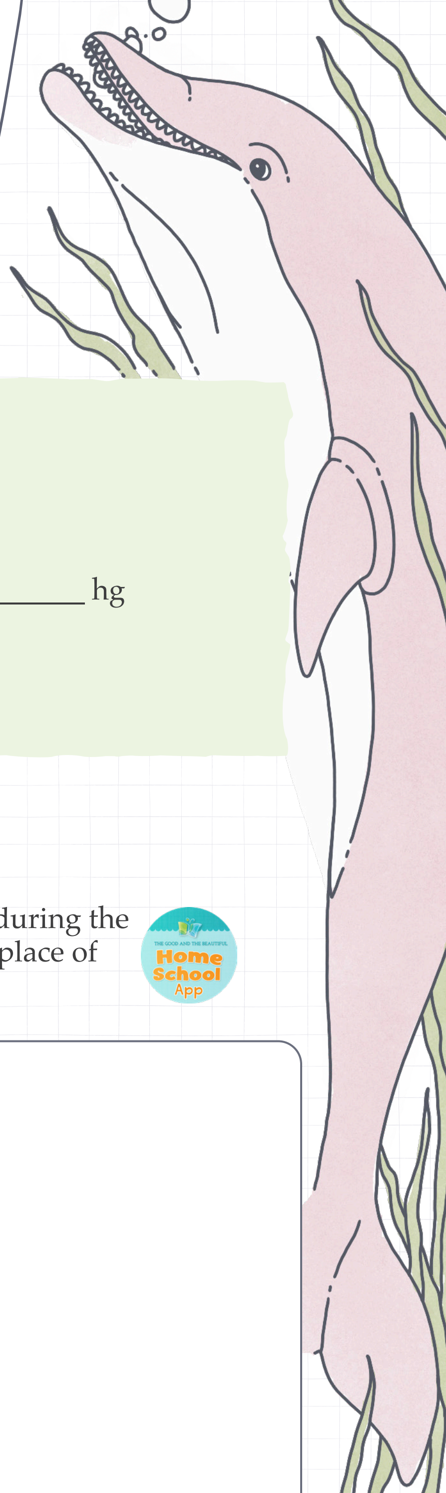
UNIT 3 | LESSON 68

Unit Conversions and Unit Multipliers





WARM-UP

Convert the measurements in the metric system.

a. 140 m = ________ km

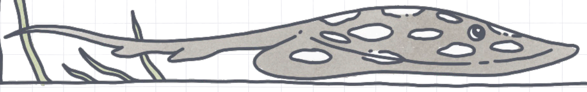
b. 5000 daL = ________dL

c. 0.008 cg = ________ hg

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE



LESSON OVERVIEW

Unit multipliers are used to convert between different units of measure while maintaining equality. Converting within a system of measurement (example: feet to yards) can be done quickly by multiplying or dividing by the conversion factor, as shown in the previous lesson. However, unit multipliers can also be used. Unit multipliers are particularly helpful when converting *between* systems of measurement, such as converting from miles to kilometers.

UNIT MULTIPLIERS

A ***unit multiplier*** is a fraction that has units and is equal to 1. Some examples are below.

$$\frac{12\text{ in}}{1\text{ ft}} \qquad \frac{4\text{ qt}}{1\text{ gal}} \qquad \frac{5280\text{ ft}}{1\text{ mi}} \qquad \frac{16\text{ oz}}{1\text{ lb}} \qquad \frac{2000\text{ lb}}{1\text{ t}}$$

Each fraction above is equal to 1 because the quantity in the numerator and the quantity in the denominator represent the same amount. The reciprocal of a unit multiplier is also a unit multiplier.

To convert units using unit multipliers, multiply the original measurement by a unit multiplier that has the current unit in the denominator. Remember, when a number or value is in the numerator and denominator of a fraction, it will cancel. For example, 345 feet can be converted to yards using a conversion factor that has both feet and yards in it: 3 feet = 1 yard. Writing the unit multiplier with 3 feet in the denominator will allow the unit of feet to cancel. The resulting unit is yards.

$$345\ \cancel{\text{ft}} \bullet \frac{1\text{ yd}}{3\ \cancel{\text{ft}}} = \frac{345\text{ yd}}{3} = 115\text{ yd}$$

REMEMBER: Any whole number can be considered to be in the numerator of a fraction. Multiply fractions straight across. Multiplying by 1 does not have to be shown.

This process works because multiplying by a unit multiplier does not change the value of the original measurement. A unit multiplier is equal to 1, and multiplying by 1 does not change the value of something. Note: The above conversion could have been done by dividing 345 by 3 because there are 3 feet in 1 yard, but it is helpful to learn how unit multipliers work through simple examples.

Often, more than one unit multiplier is needed for a conversion. If conversion factors between the original unit and ending unit are not known, use known conversion factors as stepping stones. Write the unit multiplier so the original unit is in the denominator. Then write the next unit multiplier with the new unit in the denominator. In this way, units will continue to cancel until the desired unit is left.

Example 1: Convert 3.5 miles to inches.

Note: The conversion factor from miles to inches is not commonly known, but the conversion factor from miles to feet is. A unit multiplier can be used to convert from miles to feet and then from feet to inches.

$$3.5\ \cancel{\text{mi}} \bullet \frac{5280\ \cancel{\text{ft}}}{1\ \cancel{\text{mi}}} \bullet \frac{12\text{ in}}{1\ \cancel{\text{ft}}} = \frac{3.5 \bullet 5280 \bullet 12\text{ in}}{1} = 221{,}760\text{ in}$$

Example 2: Convert 2 tons to ounces.

$$2\ \cancel{t} \bullet \frac{2000\ \cancel{lb}}{1\ \cancel{t}} \bullet \frac{16\text{ oz}}{1\ \cancel{lb}} = \frac{2 \bullet 2000 \bullet 16\text{ oz}}{1} = 64{,}000\text{ oz}$$

If a conversion factor, such as the conversion from cups to gallons, is commonly used but cannot be remembered, other unit multipliers can be used. In the example below, the conversions 1 pint = 2 cups, 1 quart = 2 pints, and 1 gallon = 4 quarts are used in place of the conversion from cups to gallons.

Example 3: A fish tank can hold 40 cups of water. What is the capacity of the fish tank in gallons?

$$40\ \cancel{c} \bullet \frac{1\ \cancel{pt}}{2\ \cancel{c}} \bullet \frac{1\ \cancel{qt}}{2\ \cancel{pt}} \bullet \frac{1\text{ gal}}{4\ \cancel{qt}} = \frac{40\text{ gal}}{2 \bullet 2 \bullet 4} = 2.5\text{ gal}$$

The fish tank can hold 2.5 gallons.

Note: When entering this in the calculator, parentheses must be used around the denominator because 40 is divided by the product of 2, 2, and 4.

CONVERTING BETWEEN SYSTEMS

Unit multipliers can be used to convert between systems of measurement, such as between the metric system and the US customary system. A chart of conversion factors between systems is shown below.

Length	Weight	Volume
1 in = 2.54 cm	1 oz ≈ 28.35 g	1 qt ≈ 0.95 L
1 mi ≈ 1.6 km	1 kg ≈ 2.2 lb	1 gal ≈ 3.8 L
1 m ≈ 1.1 yd	1 t ≈ 907 kg	Some of these conversions are rounded approximations, but they may be used as unit multipliers.

The following examples use unit multipliers from the table above to convert between the US customary system and the metric system.

Example 4: Carly is running a 10 km race for a charity event. How many miles will she run?

$$10\ \cancel{km} \bullet \frac{1\text{ mi}}{1.6\ \cancel{km}} = \frac{10\text{ mi}}{1.6} = 6.25\text{ mi}$$

Carly will run 6.25 miles.

Example 5: Aaron lives in the US, and his cousin Bryan lives in the UK. They are discussing their weight-lifting goals. Bryan is using a 7.5-kg weight, and Aaron is using a 15-lb weight. Who is using a heavier weight?

$$7.5\ \cancel{kg} \bullet \frac{2.2\text{ lb}}{1\ \cancel{kg}} = \frac{7.5 \bullet 2.2\text{ lb}}{1} = 16.5\text{ lb}$$

Bryan is using a heavier weight.

Example 6: Find the number of liters in one pint of cream.

$$1\ \cancel{\text{pt}} \bullet \frac{1\ \cancel{\text{qt}}}{2\ \cancel{\text{pt}}} \bullet \frac{0.95\ \text{L}}{1\ \cancel{\text{qt}}} = \frac{0.95\ \text{L}}{2} = 0.475\ \text{L}$$

There are 0.475 liters in one pint.

CONVERTING AREA

Area is given in square units, such as square centimeters or square feet. When converting squared units, use two of the same unit multiplier. This will allow the squared units to cancel and will result in a new squared unit. It can help to write out squared units like ft^2 as ft • ft so the cancellation can be seen easily.

Example 7: Sierra is sewing aprons as presents for her cousins. She has a total of 18 yd^2 of fabric. How many square feet of fabric does Sierra have?

Multiply by the unit multiplier twice to cancel both units of yards.

$$18\ \cancel{\text{yd}} \bullet \cancel{\text{yd}} \bullet \frac{3\ \text{ft}}{1\ \cancel{\text{yd}}} \bullet \frac{3\ \text{ft}}{1\ \cancel{\text{yd}}} = \frac{18 \bullet 3 \bullet 3\ \text{ft} \bullet \text{ft}}{1} = 162\ \text{ft}^2$$

Sierra has 162 square feet of fabric.

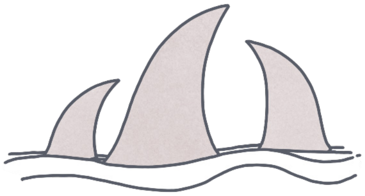
Example 8: Beth is designing a banner for her soccer team. The banner she wants to purchase is 20 ft^2. She has a piece of felt that is 8100 cm^2. Will Beth's piece of felt fit on the banner she wants to purchase?

To determine if the felt will fit, convert 20 ft^2 to cm^2.

$$20\ \cancel{\text{ft}} \bullet \cancel{\text{ft}} \bullet \frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}} \bullet \frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}} \bullet \frac{2.54\ \text{cm}}{1\ \cancel{\text{in}}} \bullet \frac{2.54\ \text{cm}}{1\ \cancel{\text{in}}}$$

$$= \frac{20 \bullet 12 \bullet 12 \bullet 2.54 \bullet 2.54\ \text{cm} \bullet \text{cm}}{1} = 18{,}580.608\ \text{cm}^2$$

The area of the banner is larger than the area of the felt, so the felt will fit on the banner.



PRACTICE

A calculator may be used for this entire practice section.

1. Write two unit multipliers for each conversion factor.

a. 4 qt = 1 gal

__________ __________

b. 1.6 km = 1 mi

__________ __________

c. 1 oz = 28.35 g

__________ __________

d. 1 mi = 1760 yd

__________ __________

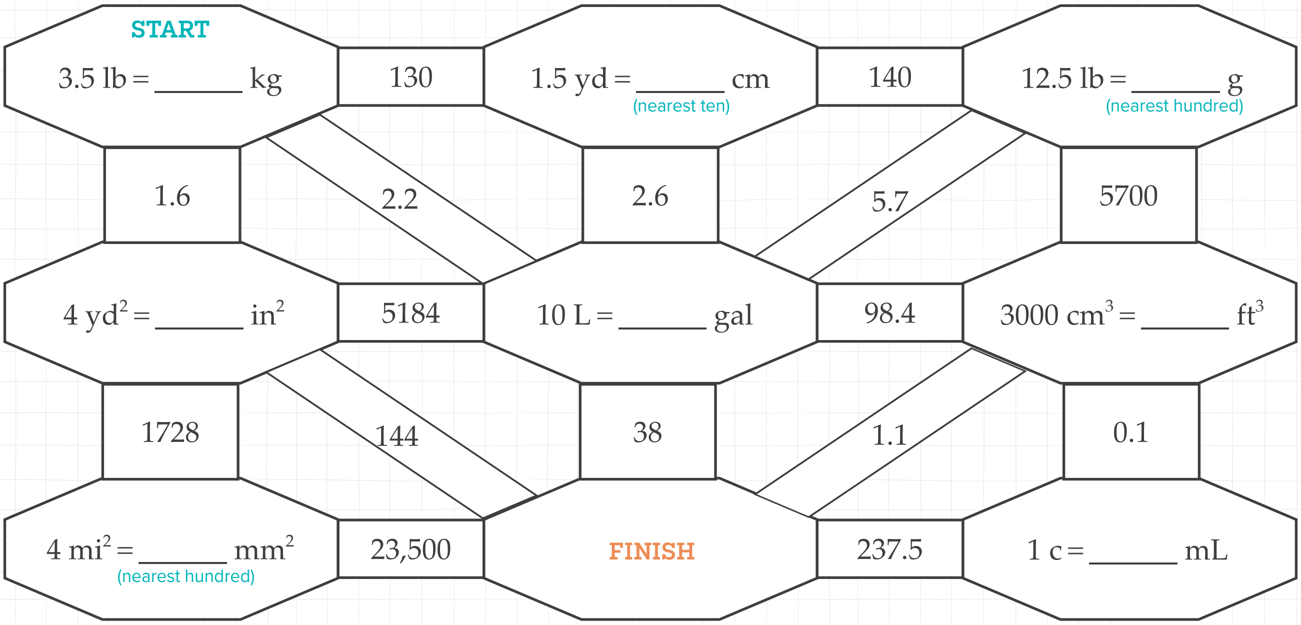
2. Beatrice has a recipe that calls for 1.5 quarts of milk, but she lives in a country where the milk is measured in liters. How many liters of milk will she need for the recipe?

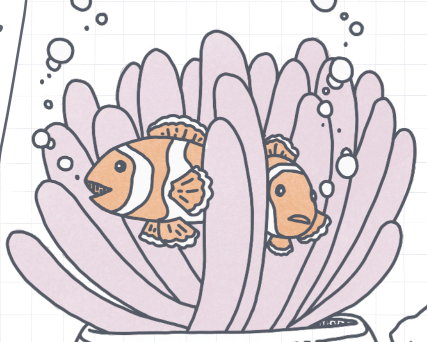
____

3. A soccer field is 7140 m^2 in area, and an American football field is 57,600 ft^2 in area. Determine which field is larger by converting 7140 m^2 to ft^2.

The __________ field is larger.

4. Begin at START. Complete each conversion to reach FINISH. Round to the nearest tenth unless otherwise indicated.





UNIT 3 | LESSON 73

Polygons and Interior Angles

WARM-UP

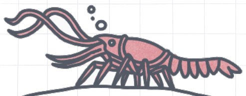
Two angles of a triangle have measures of 48° and 71°. Find the third angle measure.

____

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

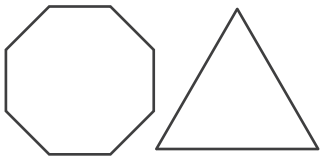


LESSON OVERVIEW

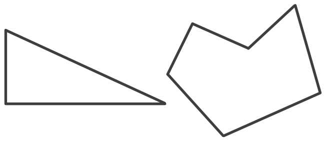
Shapes like triangles, squares, and pentagons are called polygons. Polygons can be found everywhere. Some naturally occur in nature, like hexagons in beehives, while others can be found in modern art or architecture.

Polygons are two-dimensional, closed shapes with straight sides. A ***regular polygon*** is a polygon that has all sides of equal length and all angles of equal measure. Some regular polygons have special names. An equilateral triangle is a regular triangle. A square is a regular quadrilateral.

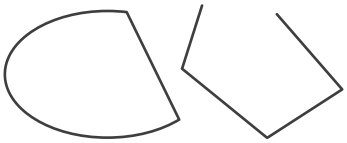
Regular Polygons:



Irregular Polygons:



Not Polygons:



*curved sides

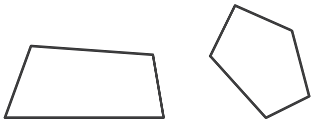
*not closed

Polygons, whether regular or irregular, are named for their number of sides. Any polygon with more than 12 sides is referred to as an *n*-gon, where *n* is the number of sides.

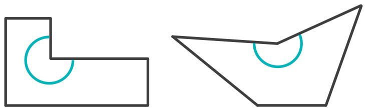
3 sides	4 sides	5 sides	6 sides	7 sides
triangle	quadrilateral	pentagon	hexagon	heptagon
8 sides	9 sides	10 sides	11 sides	12 sides
octagon	nonagon	decagon	hendecagon	dodecagon

Polygons can be convex or concave. In a ***convex polygon***, all interior angles measure less than 180°. In a ***concave polygon***, one or more interior angles measure more than 180°.

Convex Polygons:



Concave Polygons:

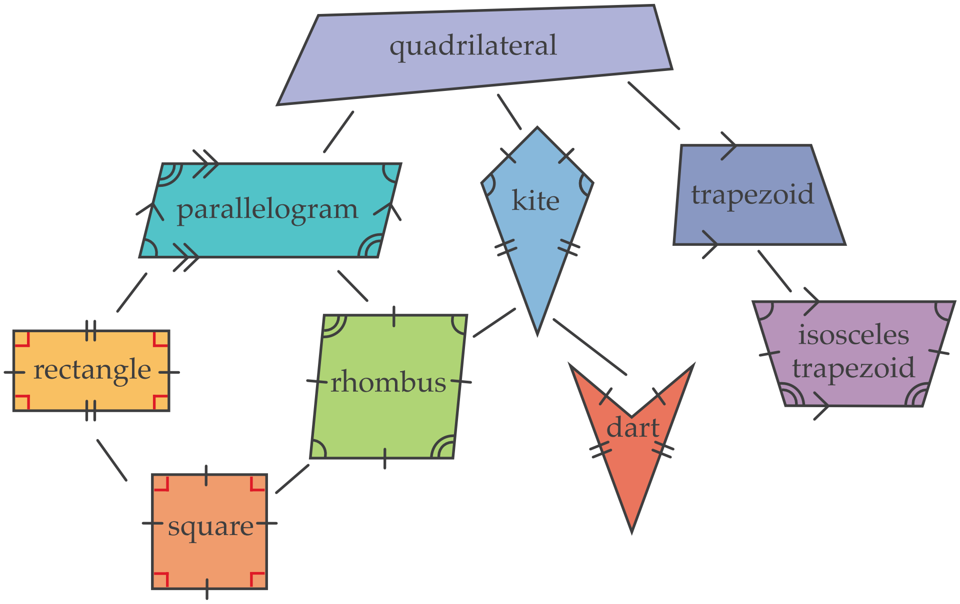


Note: The interior angle
marked is greater than 180°.

Notice that convex polygons have all angles pointing outward from the middle of the shape. Concave polygons have one or more angles pointing inward toward the middle of the shape.

CLASSIFYING QUADRILATERALS

Quadrilaterals are polygons with exactly four sides. Below is a diagram showing the classifications of quadrilaterals in relation to one another. Shapes below other shapes are part of that classification. For example, a rectangle is a type of parallelogram, which is a type of quadrilateral. The definition of each shape is below the diagram. Note: The same number of arrows indicates parallel lines, and the same number of tick marks or arcs indicates congruent sides or angles.



Shape	Definition
parallelogram	a quadrilateral with two pairs of parallel sides
rectangle	a quadrilateral with four right angles
rhombus	a quadrilateral with four sides of equal length
kite	a quadrilateral with two sets of adjacent, congruent sides
square	a quadrilateral with four right angles and four sides of equal length
dart	a kite that has a concave angle
trapezoid	a quadrilateral with exactly one pair of parallel sides
isosceles trapezoid	a trapezoid in which the two non-parallel sides are the same length

Example 1: Classify a square in as many ways as possible and give a reason for each classification.

A square is a type of rectangle because it has four right angles.

A square is a type of rhombus because it has four sides of equal length.

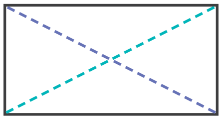
A square is a type of parallelogram because it has two pairs of parallel sides.

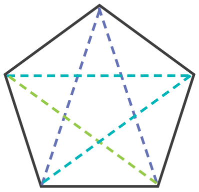
A square is a type of kite because it has two sets of adjacent, congruent sides.

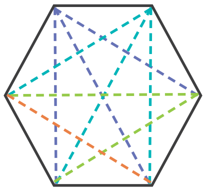
A square is a type of quadrilateral because it is a polygon with four sides.

POLYGON ANGLES AND DIAGONALS

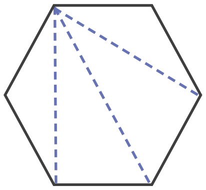
A ***diagonal*** is a straight line within a polygon that connects two of the polygon's nonadjacent vertices. The more sides and vertices a polygon has, the more diagonals it has. Note: To make it easier to see the number of diagonals in the polygons below, diagonals coming from the same vertex are the same color, starting at the top and going clockwise.


2 diagonals


5 diagonals


9 diagonals

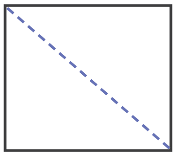
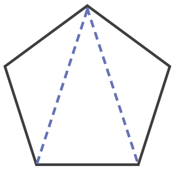
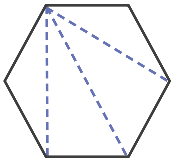
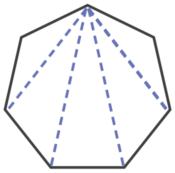
Some of the diagonals in a polygon can be used to divide the shape into triangles. When divided into the fewest number of triangles, the sum of the interior angles of the polygon can be found by using the fact that the sum of the interior angles of a triangle is 180°. Convex polygons can be divided into the fewest number of triangles by drawing all diagonals from one vertex as shown below.



The fewest number of triangles a hexagon can be divided into is four. The sum of the interior angles of a hexagon can be found by multiplying the number of triangles by 180°.

$4 \bullet 180^\circ = 720^\circ$ The interior angle sum of a hexagon is 720°.

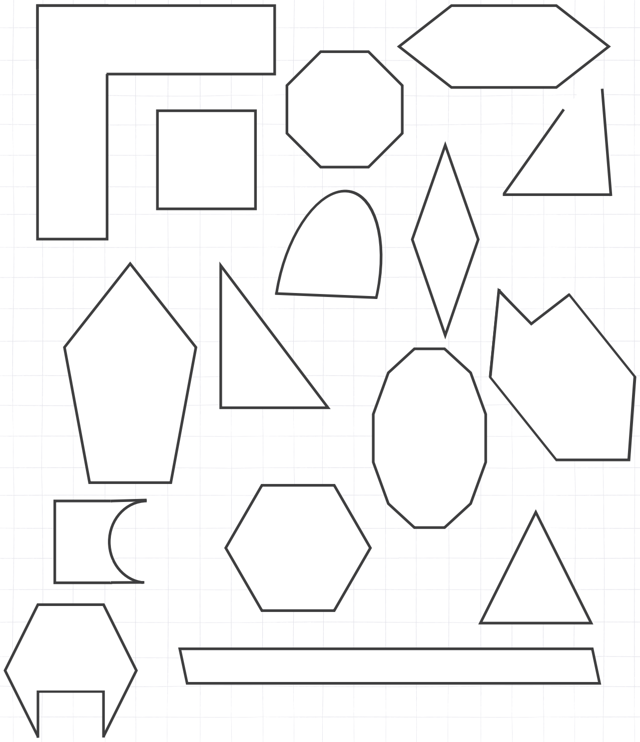
The fewest number of triangles a polygon can be divided into is two less than the number of sides in the polygon. The table below shows the interior angle sum of polygons with n sides.

	Quadrilateral	Pentagon	Hexagon	Heptagon
Number of sides (n)	4	5	6	7
Number of triangles ($n - 2$)	2 	3 	4 	5 
Interior angle sum $(n-2) \bullet 180^\circ$	$2 \bullet 180^\circ = 360^\circ$	$3 \bullet 180^\circ = 540^\circ$	$4 \bullet 180^\circ = 720^\circ$	$5 \bullet 180^\circ = 900^\circ$

The fewest number of triangles a polygon can be divided into is $n - 2$. The interior angle sum of a polygon is $(n-2) \bullet 180^\circ$, where n is the number of sides of the polygon. To find a missing angle measure in a polygon, first calculate the interior angle sum. Then write and solve an equation with the sum of all angle measures equal to the interior angle sum.

PRACTICE

1. Use the shapes below for Parts A–D.



a. Cross off any shape that is not a polygon.

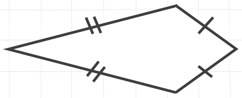
b. On each polygon, write the name according to the number of sides. For example, write "quadrilateral" instead of "rectangle" or "square."

c. Circle any regular polygons.

d. Color in any concave polygons.

2. Classify the polygons in as many ways as possible and give a reason for each classification.

a.



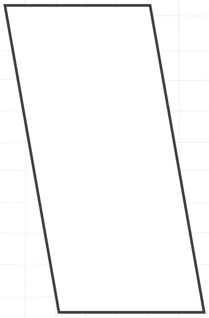
Shape	Reason

b.

Shape	Reason

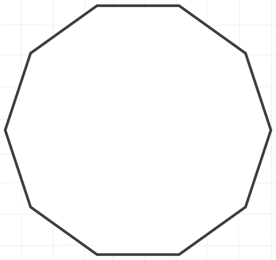
3. Draw diagonals to divide each shape into the fewest number of triangles. Then determine the sum of the interior angles of the polygon.

a.



Interior angle sum: ________

b.

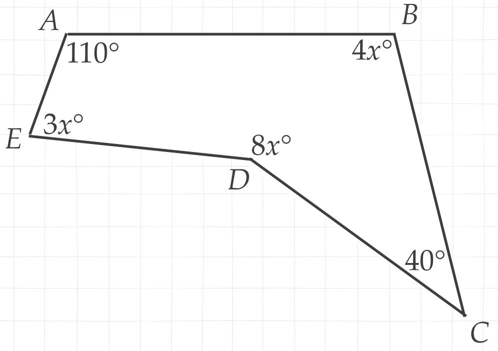


Interior angle sum: ________

4. Find the indicated angle measures in each polygon.

✦ Hint: First use the interior angle sum formula. Then write an equation setting the sum of all angles equal to the interior angle sum.

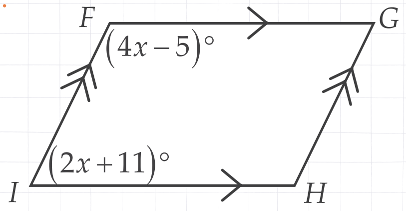
a.



$m\angle B =$ ________

$m\angle D =$ ________

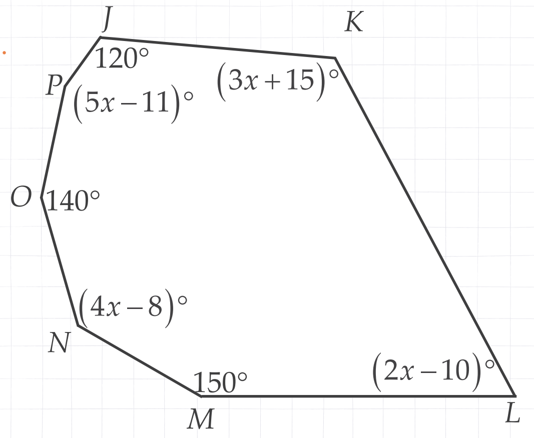
b.



$m\angle G =$ ________

$m\angle H =$ ________

c.

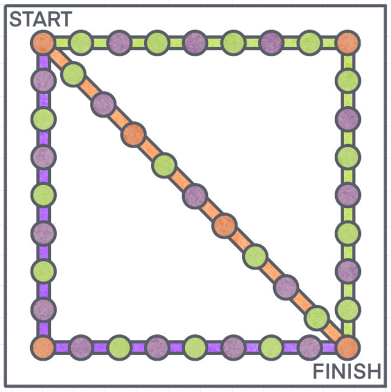


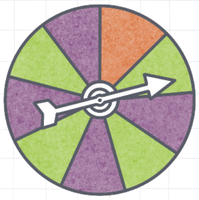
$m\angle K =$ ________

$m\angle N =$ ________

REVIEW

1. Noah is creating a board game. The board on which the game is played is a 15-inch by 15-inch square. A diagram of the game board and game spinner are shown below. Use the given information to answer the questions.





a. At the beginning of the game, each player will spin the spinner to determine the color of the path he or she will follow from START to FINISH. Write the probability of spinning each path as a fraction. L63

orange: ______ green: ______ purple: ______

b. What is the length of the orange path (diagonal distance of the game board)? Round the answer to the nearest tenth of an inch. L53

✦ Hint: Use the given game board dimensions and the Pythagorean theorem.

____

c. What is the probability of spinning orange and then green? Round to the nearest hundredth of a percent. L64

____

d. Classify the triangle formed by the green path and the orange path by angles and by sides. L72

________________ and ________________

e. Noah is designing a triangular image to be printed on the board. He is considering making the sides of the triangle 2 inches, 1.5 inches, and 4 inches. Determine if a triangle can be formed by the lengths Noah is considering. Write "yes" or "no" on the line. L72

____

f. Several families from Noah's church have agreed to test his game. If there are 35 people who will test the game, and the ratio of children to adults is 3 : 2, how many adults will test the game? L66

✦ Hint: Find the ratio of adults to total people, and then create a proportion.

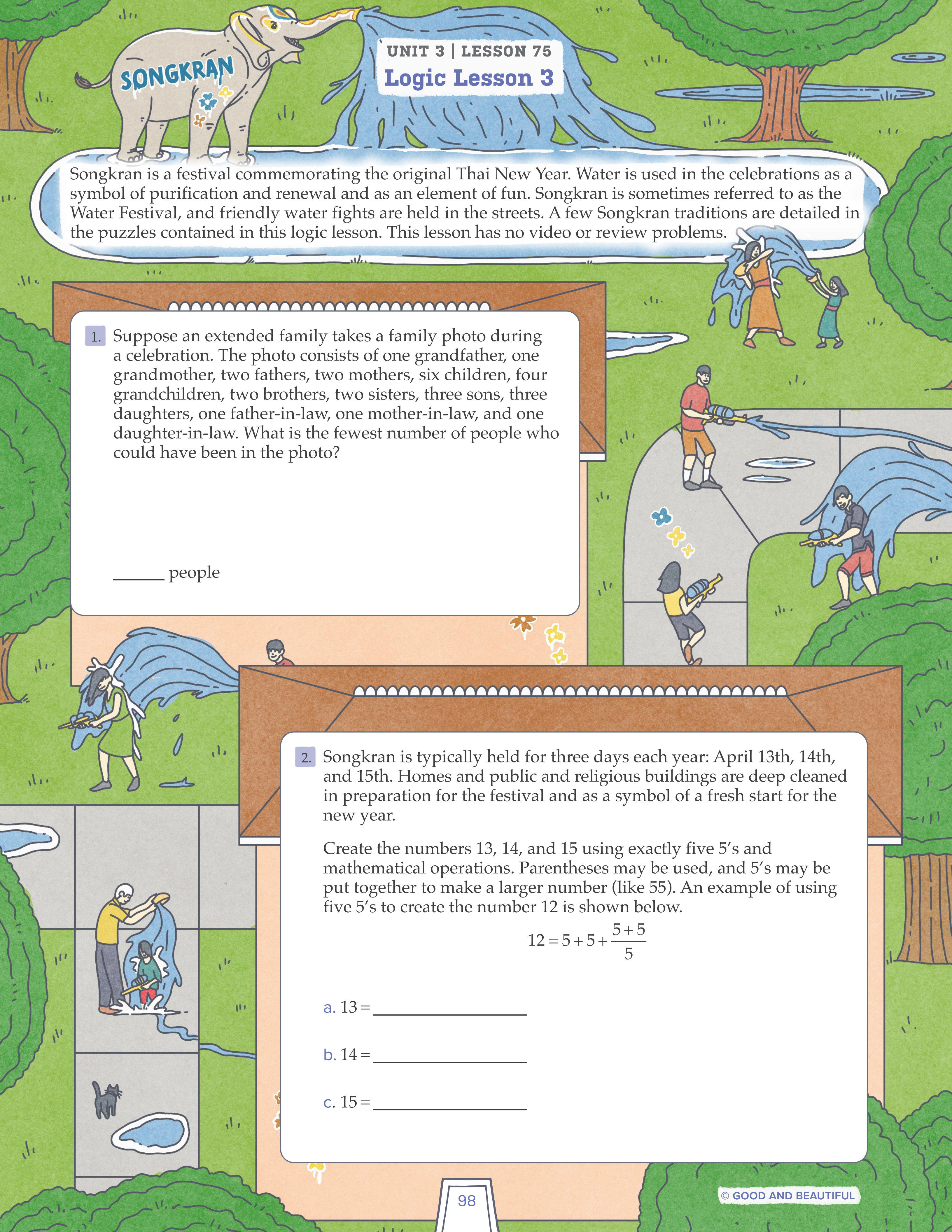
__________ adults

g. Once the game is produced, Noah estimates that 32% of total game sales (revenue) will be profit. If Noah sets a goal to make $5000 in profit in the first six months, how much does he need in sales revenue? L57

____

h. If each game is sold for the same unit price, the sales revenue can be modeled by the equation $y = kx$, where y is revenue, k is the price of each game, and x is the number of games sold. If each game is sold for $25, use the equation and the answer to Part G to determine the number of games Noah needs to sell in the first six months to meet his goal. L44

__________ games



UNIT 3 | LESSON 75

Logic Lesson 3

Songkran is a festival commemorating the original Thai New Year. Water is used in the celebrations as a symbol of purification and renewal and as an element of fun. Songkran is sometimes referred to as the Water Festival, and friendly water fights are held in the streets. A few Songkran traditions are detailed in the puzzles contained in this logic lesson. This lesson has no video or review problems.

1. Suppose an extended family takes a family photo during a celebration. The photo consists of one grandfather, one grandmother, two fathers, two mothers, six children, four grandchildren, two brothers, two sisters, three sons, three daughters, one father-in-law, one mother-in-law, and one daughter-in-law. What is the fewest number of people who could have been in the photo?

   ______ people

2. Songkran is typically held for three days each year: April 13th, 14th, and 15th. Homes and public and religious buildings are deep cleaned in preparation for the festival and as a symbol of a fresh start for the new year.

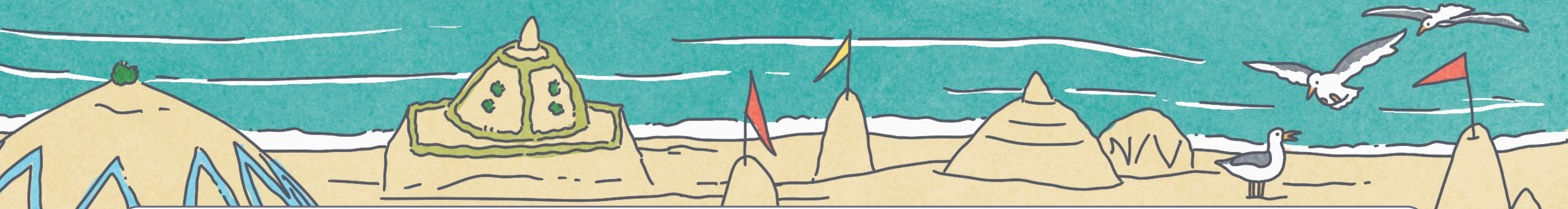
   Create the numbers 13, 14, and 15 using exactly five 5's and mathematical operations. Parentheses may be used, and 5's may be put together to make a larger number (like 55). An example of using five 5's to create the number 12 is shown below.

$$12 = 5 + 5 + \frac{5+5}{5}$$

a. 13 = ________________

b. 14 = ________________

c. 15 = ________________



5. Some additional Songkran traditions include building sand pagodas outside temples, dressing in traditional Thai clothing to perform traditional dances, releasing fish or birds into the wild, making traditional food to share with family and/or religious leaders, making floral garlands to give to elder relatives, and playing games. Suppose that on the day after Songkran, four Thai children were asked their favorite day and event of the recent festival. Each child shared a different favorite event, but because the festival lasts only three days, two of the children stated that their favorite day of the festival was the second day, April 14th. Read the clues to find each child's favorite event and day.

✦ Hints: Once you know something for certain, put a ✓ in that square and fill in the rest of the row and column of that 4 x 4 box with Xs. You may need to go through the clues more than once.

Clue #1: A boy and a girl each stated that April 14th was their favorite day of the festival, but neither of them stated that the Water Fight or Dancing was their favorite event.

Clue #2: The boy who chose the Water Fight as his favorite enjoyed the final day of the festival the most.

Clue #3: The boy who chose Making Garlands enjoyed the second day of the festival the most.

Clue #4: Dara's favorite day was not the first day.

Clue #5: Phet did not choose Making Garlands.

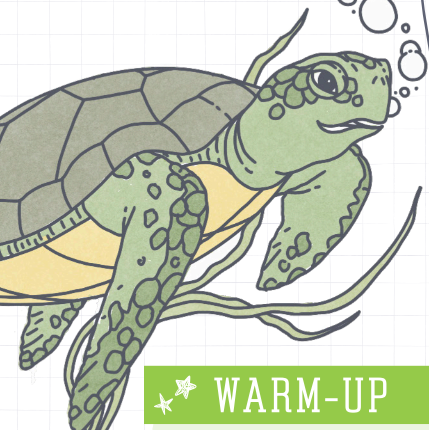
	Favorite Day				Favorite Event			
	April 13th	April 14th	April 14th	April 15th	Dancing	Making Garlands	Water Fight	Building Sand Pagodas
Dara (girl)								
Mali (girl)								
Saksit (boy)								
Phet (boy)								
Dancing								
Making Garlands								
Water Fight								
Building Sand Pagodas								

a. Dara's favorite day: ________________
Dara's favorite event: ____________________

b. Mali's favorite day: ________________
Mali's favorite event: ____________________

c. Saksit's favorite day: ________________
Saksit's favorite event: ____________________

d. Phet's favorite day: ________________
Phet's favorite event: ____________________



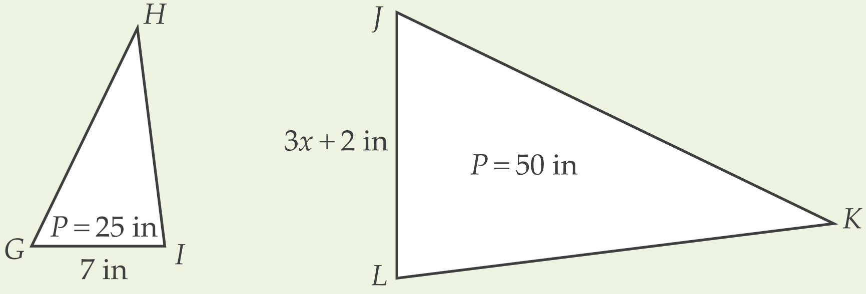
UNIT 3 | LESSON 77

Drawings and Constructions

☆ SUPPLIES: ruler, compass, and protractor

WARM-UP

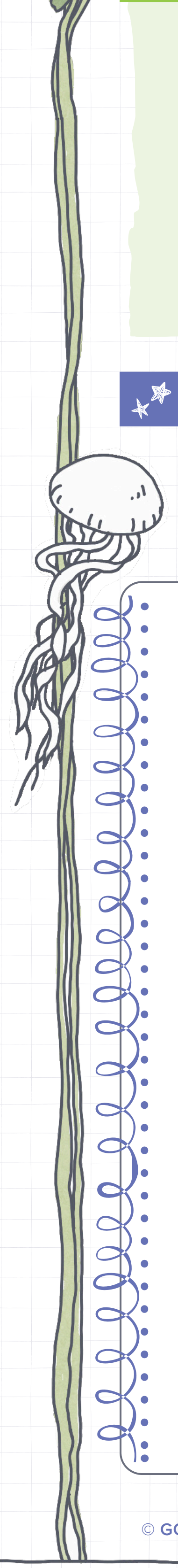
Given that $\triangle GHI \sim \triangle JKL$, use the perimeters to find the length of side JL.



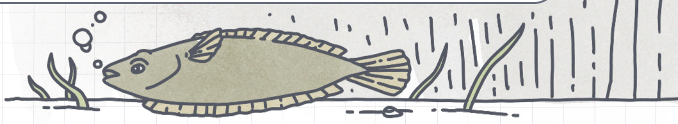
____

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.



VIDEO PRACTICE

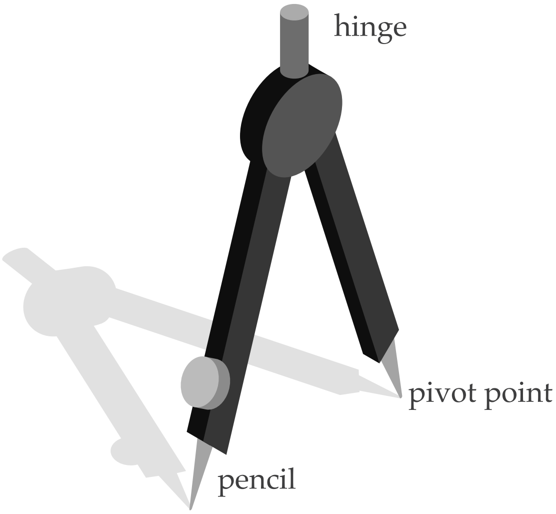


LESSON OVERVIEW

Geometric constructions are different from freehand drawings because of the precision used when completing them. Constructions only use two tools: a straightedge and a compass. A protractor is not a valid tool for creating constructions. Drawings can be made freehand or with other tools like a protractor or ruler.

USING A COMPASS

A compass is a tool used to construct circles and arcs, which can be used to construct many other figures. A compass can be held at the top as if it were a key being turned on the table. The compass contains a hinge for adjusting the distance between the pivot point and the pencil that are at the ends of the compass. The pivot point marks the center of a circle that would be created if the compass were turned a complete 360°. The distance between the pencil and pivot point is the length of the radius of that circle.



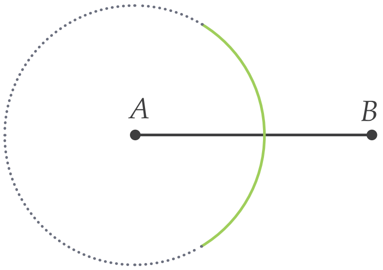
CONSTRUCTING PERPENDICULAR BISECTORS

A ***perpendicular bisector*** is a line, ray, or line segment that divides a line segment into two congruent parts and is perpendicular to the line segment.

Steps for constructing a perpendicular bisector for line segment *AB*:

1. Place the pivot point of the compass on one endpoint of the line segment. Open the compass to more than half the length of the line segment.

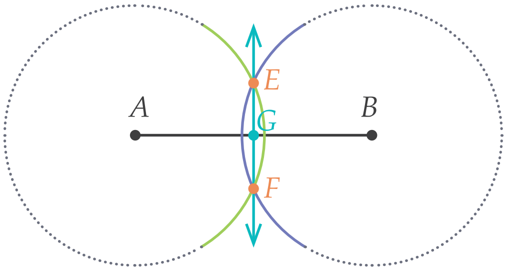
2. Draw part of a circle (an arc). This arc should extend above and below the line segment.



3. Keep the same radius on the compass and move the pivot point to the other endpoint of the line segment. Draw another arc that intersects the first arc in two places.

4. Draw points at the intersections of the arcs.

5. Use a straightedge to connect the intersection points. Line *EF* is a perpendicular bisector for line segment *AB*.

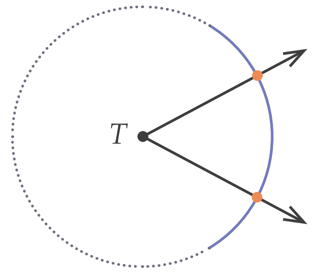


Note: The perpendicular bisector creates two congruent line segments. If point *G* is placed at the intersection of line segment *AB* and the perpendicular bisector, line segment *AG* is congruent to line segment *GB*.

CONSTRUCTING ANGLE BISECTORS

An ***angle bisector*** is a line, ray, or line segment that divides an angle into two angles of equal measure.

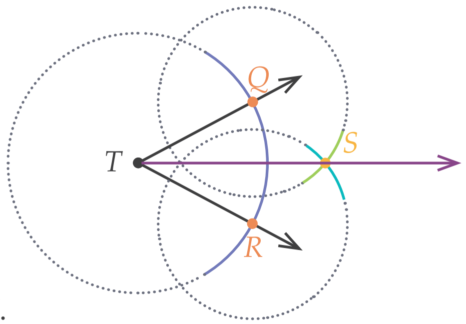
Steps for constructing an angle bisector for angle *T*:



1. Place the pivot point of the compass on the vertex.

2. Draw part of a circle (an arc) that intersects both sides of the angle.

3. Draw points at the intersections of the arc and the angle.



4. Place the pivot point of the compass on one of the intersection points. Open the compass to more than half the distance to the other intersection point. Draw an arc.

5. Keep the same radius on the compass and move the pivot point to the other point of intersection on the angle. Draw another arc that intersects the arc from Step 4.

6. Draw a point at the intersection of the arcs from Steps 4 and 5. Use a straightedge to connect this point with the vertex of the angle. Ray *TS* is the angle bisector for angle *T*.

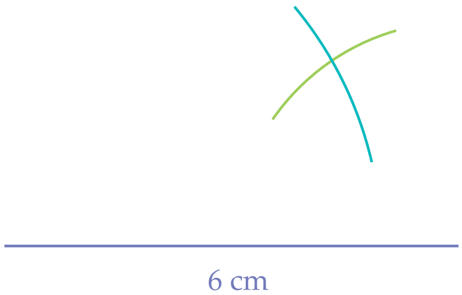
Note: The angle bisector creates two congruent angles. If the intersection points from Step 3 are named *Q* and *R* as shown in the figure, then angle *QTS* is congruent to angle *STR*.

Perpendicular bisectors and angle bisectors can be constructed using only a compass and straightedge. There are many other types of geometric constructions that are taught in higher-level geometry courses. The following instructions use a ruler and a protractor, so they are considered drawings, not constructions.

DRAWING A TRIANGLE GIVEN THREE SIDE LENGTHS

To draw a triangle given three side lengths, a compass and ruler can be used.

Steps for drawing a triangle with side lengths of 3 cm, 5 cm, and 6 cm:



1. Use the ruler to measure and draw the first side. 6 cm is shown first here, but any side can be drawn first.

2. Set the radius of the compass to the length of another side. 3 cm is used next here. Put the pivot point of the compass on one endpoint of the first side and draw an arc above that side. Any point on this arc is 3 cm from that endpoint.

Note: Some compasses have measurements on the hinge for setting the radius. A ruler can also be used to measure the distance between the pivot point and pencil.

3. Set the radius of the compass to the length of the third side, which is 5 cm in this example. Put the pivot point on the other endpoint of the first side and draw another arc that intersects the first arc. Any point on this arc is 5 cm from that endpoint.

PRACTICE

1. a. Construct a perpendicular bisector for the line segment below.

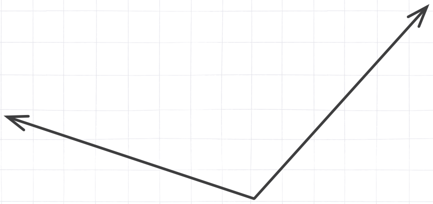
b. Verify that the construction in Part A bisects the line segment by measuring each side of the line segment and writing the measurements below.

Left side: ________ Right side: ________

c. Verify that the construction in Part A is perpendicular to the line segment by measuring the angle made with the given line segment.

____

2. a. Construct an angle bisector for the angle below.



b. Verify that the construction in Part A bisects the angle by measuring the two angles created by the bisector.

Left angle measure: ______

Right angle measure: ______

3. Draw a triangle for each set of side lengths.

a. 2 in, 2.5 in, 3 in

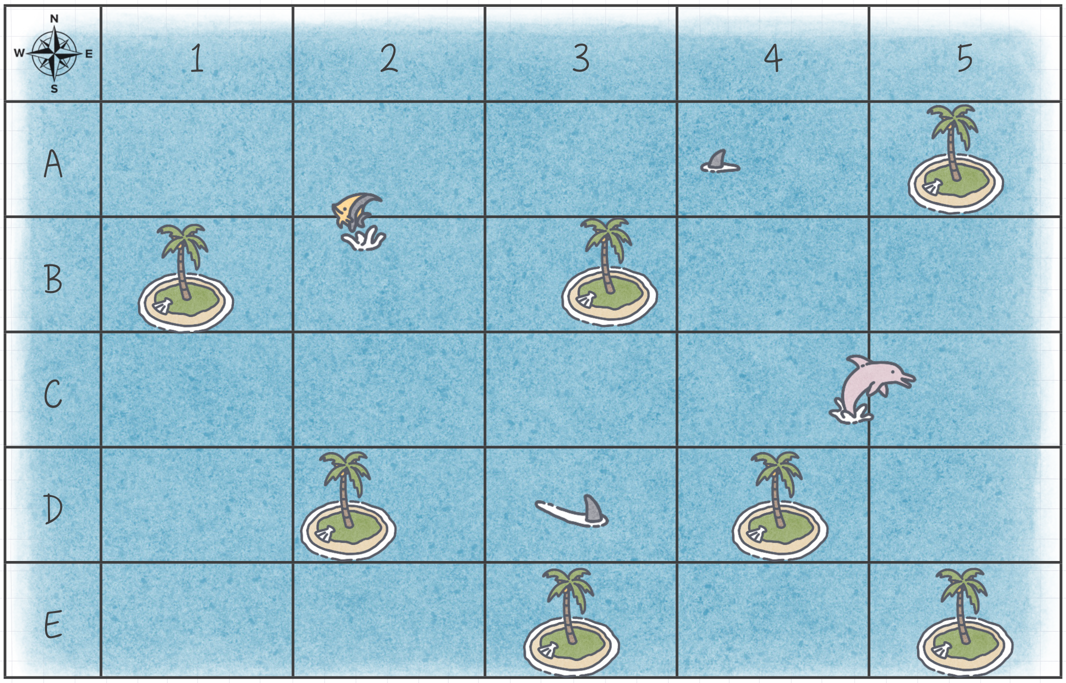
b. 4 cm, 3 cm, 4 cm

4. Construct a triangle for each set of angle measures.

a. 55°, 70°, 55°

b. 115°, 30°, 35°

5. A treasure map is shown below. Follow the clues to find the location of the treasure.



a. Draw a line segment from the base of the palm tree in D2 to the base of the palm tree in B1.
✦ Hint: The base of the palm tree is where the tree meets the island.

b. Construct a perpendicular bisector for the line segment drawn in Part A.

c. The perpendicular bisector created in Part B forms four angles where it intersects the line segment from Part A. Using only a compass and straightedge, bisect the angle that faces east.

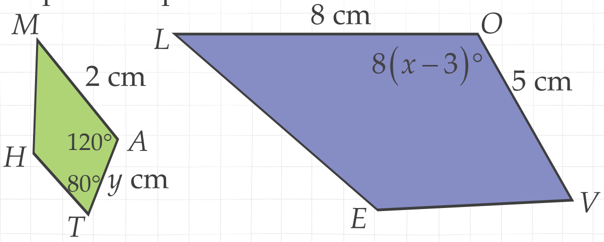
d. Extend the angle bisector from Part C until it hits an island. The treasure is on this island. In what section of the map is the treasure island located?

____

REVIEW

A calculator may be used for this entire review section.

1. Given that $LOVE \sim MATH$, use the figures to complete the problems.



a. Name the polygons above according to their number of sides. L73

____

b. Circle the correct word from each row to describe the polygons. L73

Circle one: regular / irregular

Circle one: concave / convex

c. Determine the value of x. L76

____

d. Determine the value of y. L76

____

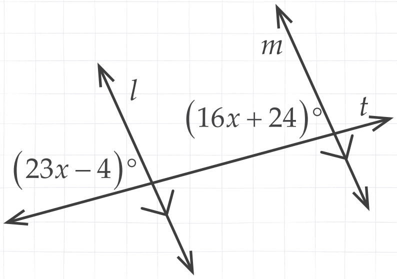
2. Convert 23,225 cm^2 to square feet. Round to the nearest square foot. L68

____

3. Oliver builds wooden scale models of farm equipment. The diameter of each tractor wheel on a model is 3 inches, and the diameter of the actual tractor wheel is 42 inches. What scale factor did Oliver use for the model tractor? L69

______

4. Use the figure below to answer the questions.



a. What is the relationship between the two angles whose measures are labeled with expressions? L71

________________ angles

b. Are angles with this relationship congruent or supplementary? Circle one. L71

congruent supplementary

c. Use the answer from Part B and the given expressions to write and solve an equation to find the value of x and the measures of the angles. L25, L31

$x =$ ______ $(23x-4)^\circ =$ ______

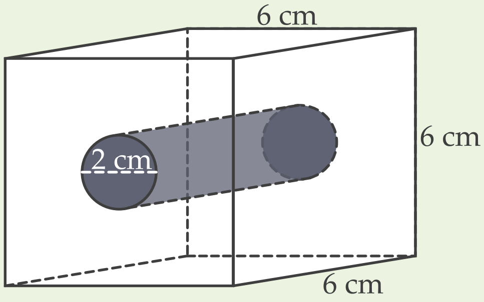
$(16x+24)^\circ =$ ______

UNIT 3 | LESSON 87

Solving One-Step and Two-Step Inequalities

WARM-UP

Find the volume of the composite solid.



LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

An ***inequality*** is a comparison of two expressions that does not use an equal sign. Inequality symbols and their meanings are shown in the chart below.

Symbol	Meaning
$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to
$\neq$	not equal to

$x < 7$ means that the value of x must be less than 7. Any number less than 7 is a possible value for x. This includes numbers like 6.98, 0, and –46. Any number that is *not* less than 7 is not a possible value for x. Some numbers that do not satisfy the inequality include 7, 45, and 1258.

ONE-STEP INEQUALITIES

An inequality is solved similarly to how an equation is solved. Always perform the same operations on both sides of the inequality to maintain the same relationship that was originally expressed.

Example 1: Solve for a.

$$a + 11 > -19$$
$$a + 11 - 11 > -19 - 11$$
$$a > -30$$

Example 2: Solve for x.

$$x - 6 < 18$$
$$x - 6 + 6 < 18 + 6$$
$$x < 24$$

To determine if a number satisfies an inequality, substitute the number into the inequality and see if a true statement results. If the statement is true, the number satisfies the inequality. If the statement is false, the number does not satisfy the inequality.

Example 3: Determine if the numbers in the set $\{0,\ 2.5,\ 11,\ -11,\ -12,\ -23\}$ are solutions to the inequality $b - 12 \geq -23$.

Check each value in the set in the original inequality.

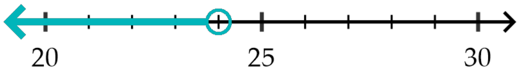
$0 - 12 = -12$	$-12 \geq -23$	0 satisfies the inequality.
$2.5 - 12 = -9.5$	$-9.5 \geq -23$	2.5 satisfies the inequality.
$11 - 12 = -1$	$-1 \geq -23$	11 satisfies the inequality.
$-11 - 12 = -23$	$-23 \geq -23$	–11 satisfies the inequality.
$-12 - 12 = -24$	$-24 < -23$	–12 does *not* satisfy the inequality.
$-23 - 12 = -35$	$-35 < -23$	–23 does *not* satisfy the inequality.

The numbers 0, 2.5, 11, and –11 are all solutions to the inequality $b - 12 \geq -23$. The numbers –12 and –23 are not solutions.

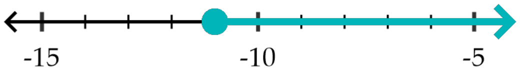
Inequalities have an infinite number of solutions. Solutions can be written with an inequality symbol, and they can also be graphed on a number line. Below are the graphs of the solutions to Examples 1, 2, and 3. Any number in the shaded part (including decimal values) is a solution to the inequality. An open circle on the number is used when the symbol is < or >. The number at the circle is not part of the solution set. A closed circle on the number is used when the symbol is ≤ or ≥. The number at the circle is part of the solution set.

$a > -30$

$x < 24$



$b \geq -11$

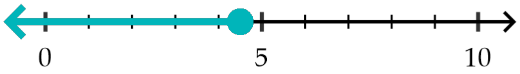


Example 4: Solve and graph the inequality.

$$5q \leq 22.5$$

$$\frac{5q}{5} \leq \frac{22.5}{5}$$

$$q \leq 4.5$$



When multiplying or dividing both sides of an inequality by a negative value, the inequality sign switches directions. The table below demonstrates how the relationship between numbers changes when both sides of an inequality are multiplied or divided by a negative number.

Original Inequality	Multiply by 2	Multiply by -2	Divide by 6	Divide by -6
$18 > 12$	$18 \bullet 2 \quad 12 \bullet 2$ $36 > 24$	$18 \bullet (-2) \quad 12 \bullet (-2)$ $-36 < -24$ sign changes	$18 \div 6 \quad 12 \div 6$ $3 > 2$	$18 \div (-6) \quad 12 \div (-6)$ $-3 < -2$ sign changes

Anytime both sides of an inequality are multiplied or divided by a negative number, remember to switch the direction of the inequality symbol.

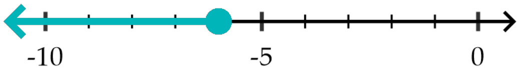
Example 5: Solve and graph the inequality.

$$-8f \geq 48$$

$$\frac{-8f}{-8} \geq \frac{48}{-8}$$

$$f \leq -6$$

Note: The sign switches.



TWO-STEP INEQUALITIES

Inequalities may require multiple steps to solve. As when solving equations, isolate the variable by performing the order of operations in reverse. First undo any addition or subtraction. Then undo any multiplication or division. Remember to switch the inequality symbol when multiplying or dividing by a negative number.

Example 6: Solve and graph the inequality.

$$12t + 15 > 39$$

Subtract 15 from both sides.

$$12t + 15 - 15 > 39 - 15$$

$$12t > 24$$

Divide both sides by 12.

$$\frac{12t}{12} > \frac{24}{12}$$

$$t > 2$$

Note: There is an open circle on 2 because 2 is not part of the solution set.

Example 7: Solve and graph the inequality.

$$-\frac{3}{4}p + 21 \geq 24$$

Subtract 21 from both sides.

$$-\frac{3}{4}p + 21 - 21 \geq 24 - 21$$

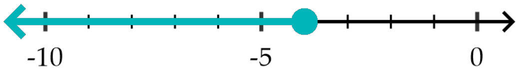
$$-\frac{3}{4}p \geq 3$$

Multiply both sides by the reciprocal of $-\frac{3}{4}$.

$$-\frac{4}{3} \bullet \left(-\frac{3}{4}p\right) \geq 3 \bullet \left(-\frac{4}{3}\right)$$

Multiplying by a negative switches the inequality symbol.

$$p \leq -4$$



Example 8: Solve and graph the inequality.

$$-3x - 15 \leq 6$$

Add 15 to both sides.

$$-3x - 15 + 15 \leq 6 + 15$$

$$-3x \leq 21$$

Divide both sides by –3.

$$\frac{-3x}{-3} \leq \frac{21}{-3}$$

Dividing by a negative switches the inequality symbol.

$$x \geq -7$$

KEY INFORMATION

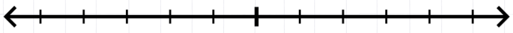
Use an open circle for < or >. Use a closed circle for ≤ or ≥.

PRACTICE

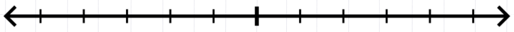
Solve and graph each inequality. Find the solution graph in the table on the next page, and cross off the word or phrase next to it. Once all problems have been completed, write the remaining words or phrases (from top to bottom) on the lines at the bottom of the page to discover the answer to the joke.

1. $a+2<7$ ____________

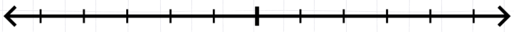
2. $b-4>3$ ____________



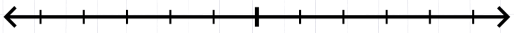
3. $3c \geq -27$ ____________



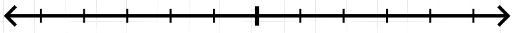
4. $-8d<20$ ____________



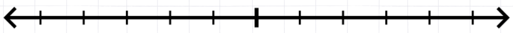
5. $2e+5 \leq 17$ ____________



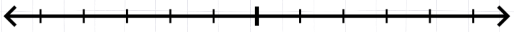
6. $4f-6>-30$ ____________

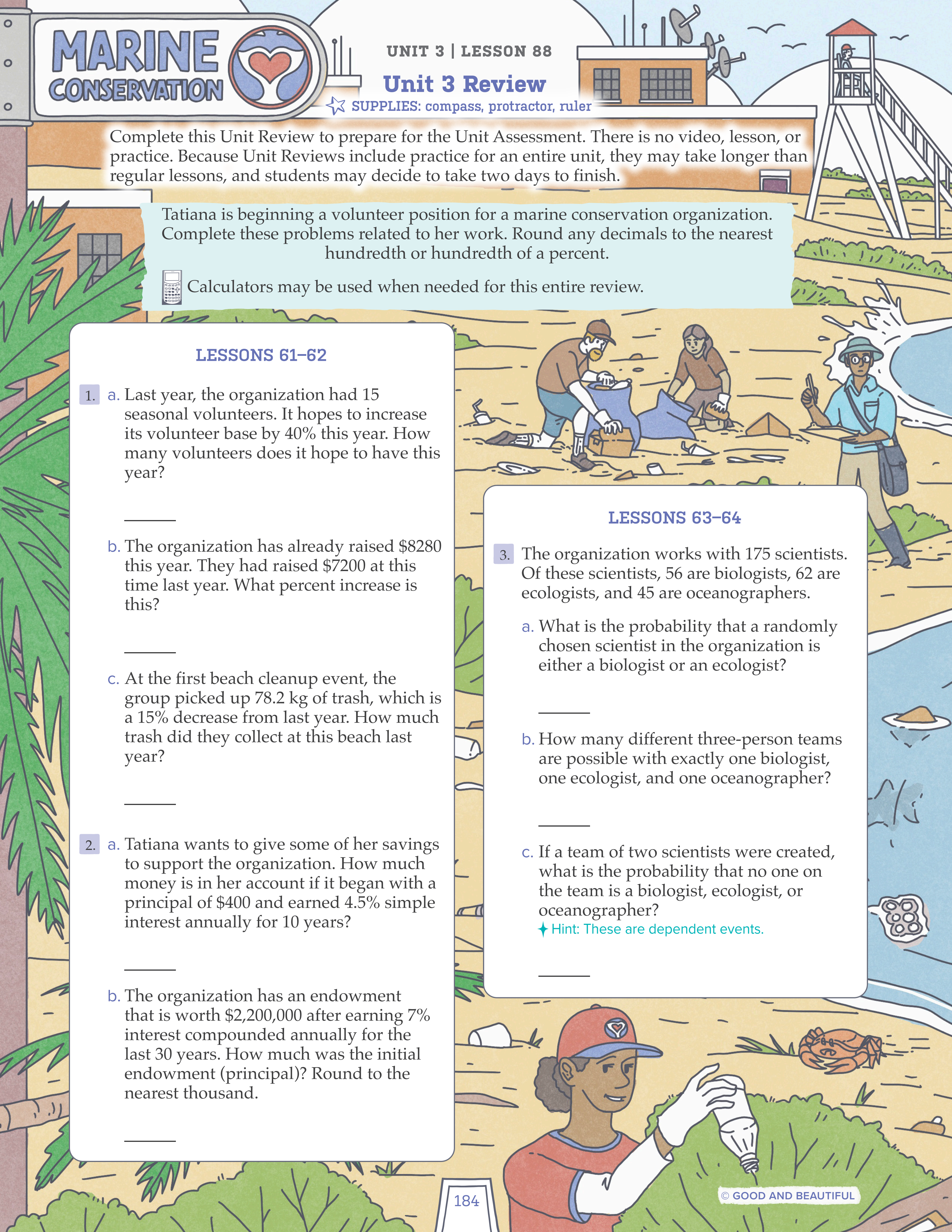


7. $-g+2 \geq 17$ ____________



8. $\frac{1}{2}h-4 \leq 7$ ____________





Unit 3 Review

☆ SUPPLIES: compass, protractor, ruler

Complete this Unit Review to prepare for the Unit Assessment. There is no video, lesson, or practice. Because Unit Reviews include practice for an entire unit, they may take longer than regular lessons, and students may decide to take two days to finish.

Tatiana is beginning a volunteer position for a marine conservation organization. Complete these problems related to her work. Round any decimals to the nearest hundredth or hundredth of a percent.

Calculators may be used when needed for this entire review.

LESSONS 61–62

1. a. Last year, the organization had 15 seasonal volunteers. It hopes to increase its volunteer base by 40% this year. How many volunteers does it hope to have this year?

______

b. The organization has already raised $8280 this year. They had raised $7200 at this time last year. What percent increase is this?

______

c. At the first beach cleanup event, the group picked up 78.2 kg of trash, which is a 15% decrease from last year. How much trash did they collect at this beach last year?

______

2. a. Tatiana wants to give some of her savings to support the organization. How much money is in her account if it began with a principal of $400 and earned 4.5% simple interest annually for 10 years?

______

b. The organization has an endowment that is worth $2,200,000 after earning 7% interest compounded annually for the last 30 years. How much was the initial endowment (principal)? Round to the nearest thousand.

______

LESSONS 63–64

3. The organization works with 175 scientists. Of these scientists, 56 are biologists, 62 are ecologists, and 45 are oceanographers.

a. What is the probability that a randomly chosen scientist in the organization is either a biologist or an ecologist?

______

b. How many different three-person teams are possible with exactly one biologist, one ecologist, and one oceanographer?

______

c. If a team of two scientists were created, what is the probability that no one on the team is a biologist, ecologist, or oceanographer?

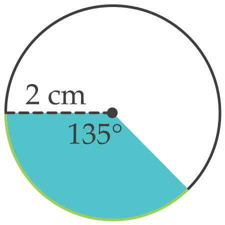
✦ Hint: These are dependent events.

______

LESSONS 79–80

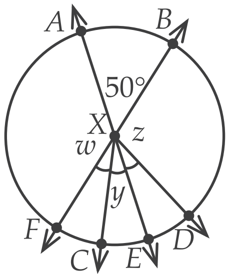
17. Tatiana is in charge of designing buttons for an awareness campaign and is considering different designs.

a. If the radius of the button is 2 cm, determine the green arc length and the area of the blue sector below.



Green arc length: ______ Blue sector area: ______

b. Find the missing angle measures.



$w =$ ______

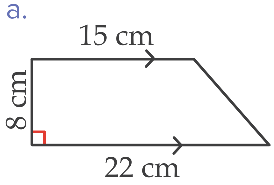
$y =$ ______

$z =$ ______

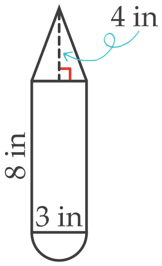
LESSONS 80–81

18. The conservatory is planning on repainting some of its walls and displays. Determine the area of the following figures.

a.



b.

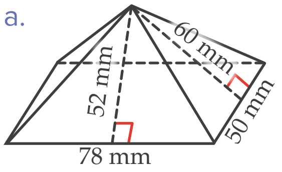


________ ________

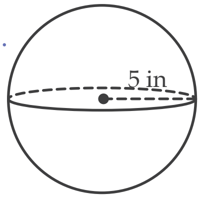
LESSONS 82–83

19. Determine the surface area of the following solids.

a.



b.

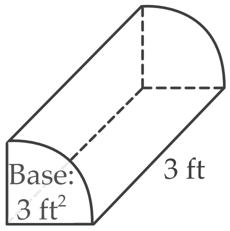


________ ________

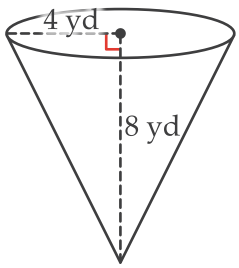
LESSONS 84–86

20. The marine conservation organization also works with aquariums. Find the volume of the following tanks at a nearby aquarium.

a.



b.



________ ________

Unit 3 Assessment

○ This assessment covers concepts taught in Unit 3. Problems are designed to assess multiple skills. Read the instructions carefully and do not rush through the problems.

○ You may use the Reference Chart for the assessment. Lesson numbers are given by each problem so you can review lessons for any answers that are incorrect.

Calculators may be used throughout this assessment.

1. The value of Cynthia's house has increased from $350,000 to $420,000. Find the percent increase on the value of her home. L61

____

2. A savings account starts with $450. Find the new balance in the account after 12 years of 3.5% interest compounded annually. L62

____

3. A jar contains 120 pink beads, 80 yellow beads, 100 blue beads, 150 green beads, and 50 white beads. L63

a. Find the probability of randomly picking a bead of each color from the jar on one pick.

Pink: ________ Green: ________

Yellow: ________ White: ________

Blue: ________

b. Find the expected number of beads of each color that would likely be in a scoop of 50 beads.

Pink: ________ Yellow: ________

Blue: ________ Green: ________

White: ________

4. Jessie was given six pairs of earrings for her birthday. If there are two pairs of studs, one pair of hoops, and three dangly pairs, find the probability that she will take out a dangly pair first and a pair of hoops second. L64

____

5. A package of 46 cookies costs $3.78. Find the unit rate of cost per cookie to the nearest cent. L65

____

6. Rob's pasta recipe uses 500 grams of uncooked pasta and half of a yellow onion. If he wants to use 0.75 onions, how much pasta should he use? L66

____

7. Perform the operations. L67

a. 4 lb 6 oz • 3 = ________

b. 2 ft 5 in + 4 ft 8 in = ________

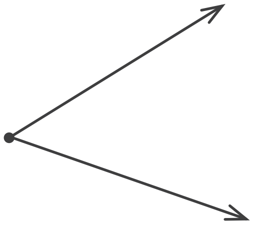
8. Siera is flying from California to Heathrow, London. Her luggage must be below 40 lbs on her departure flight. When she flies home, she must check her luggage on a scale that measures in kg. Use the conversion 1 kg ≈ 2.2 lb to determine the maximum number of kg her luggage can weigh. Round to the nearest hundredth. L68

____

9. A redwood tree is 320 feet tall. In a picture, the tree measures 4 inches tall. What is the scale from the real tree to the picture? L69

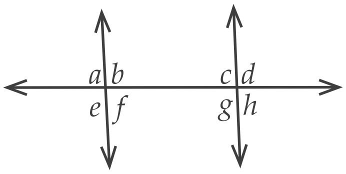
____

10. Classify the angle by measure. L70



Circle one: acute obtuse right

11. List the vertical angle pairs from the diagram. L71



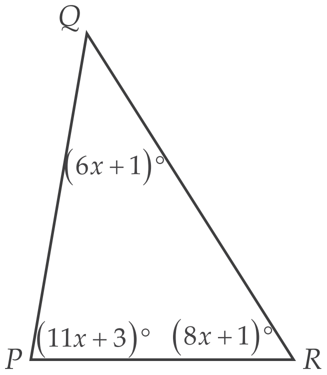
________ and ________

________ and ________

________ and ________

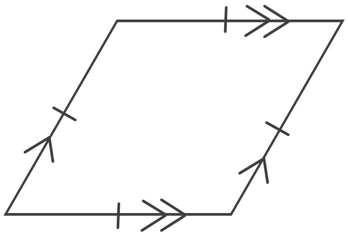
________ and ________

12. Find the measure of angle P. L72



____

13. Classify the polygon in as many ways as possible and give a reason for each classification. L73



Shape	Reason

LESSONS
91–120

UNIT
4

Simply Good and Beautiful

PRE-ALGEBRA

COURSE BOOK 4

MATH
8

The Good and the Beautiful
CURRICULUM

COURSE BOOK 4

Table of Contents

Unit 4 Overview

LESSONS 91–120

CONCEPTS COVERED

- Adding and subtracting polynomials
- Algebraic rules for reflections
- Algebraic rules for translations
- Analyzing data containing outliers
- Analyzing methods of solving systems
- Bias in survey questions
- Bias in survey samples
- Box plots
- Calculating equations for lines of best fit
- Clusters and outliers in data
- Comparing transformations
- Dilations on a coordinate plane
- Dividing binomials by monomials
- Dividing monomials
- Factoring the GCF from binomials and trinomials
- Finding missing data values using mean
- First, second, and third quartiles in data
- Frequency tables
- Graphing linear inequalities
- Greatest common factor of monomials
- Histograms
- Identifying solutions to systems
- Interpreting lines of best fit
- Interquartile ranges
- Joint and marginal frequencies
- Least common multiple of monomials
- Line plots
- Linear equations with infinite solutions
- Linear equations with no solutions
- Linear equations with one solution
- Lines of symmetry
- Measures of central tendency
- Multiplying binomials
- Multiplying monomials
- Multiplying monomials and binomials
- Order of rotational symmetry
- Performing multiple transformations
- Qualitative and quantitative data
- Ranges in data
- Reflectional symmetry
- Reflections on a coordinate plane
- Relative frequencies
- Rotations on the coordinate plane
- Scale factor of dilation
- Scatter plots
- Simplifying polynomials
- Skewed data
- Solving and graphing multi-step inequalities
- Solving systems by elimination
- Solving systems by graphing
- Solving systems by substitution
- Stem-and-leaf plots
- Surveys, samples, and sample size
- Systems of linear equations
- Translations on a coordinate plane
- Two-way tables
- Types of correlation
- Types of random samples (simple, stratified, systematic)
- Using solutions in other equations and expressions
- Writing inequalities from word problems

UNIT 4 | LESSON 92

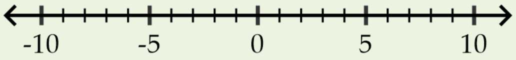
Graphing Linear Inequalities

SUPPLIES: colored pencils

WARM-UP

Solve and graph the inequality.

$4t - 8 \geq 9t - 23$



______

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

Equations and inequalities have similar properties. Recall that a linear function forms a straight line when graphed. A linear *inequality* is a linear function written with a <, >, ≤, or ≥. Just as with linear equations, linear inequalities contain only variables to the first power and constants. Some examples of linear inequalities in different forms are shown below.

Slope-Intercept Form:	$y < 3x - 12$	$y \le -\frac{2}{3}x + 8$	$y > x$
Standard Form:	$3x - 5y > 15$	$2x + 18y \le -48$	$x + y < 22$

Graphing a linear inequality is similar to graphing a linear equation. Once the inequality is in slope-intercept form, plot the y-intercept and use the slope to plot additional points. However, a linear inequality is not always drawn with a solid line.

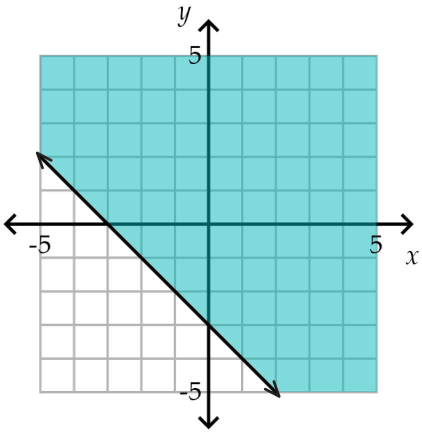
- Use a dashed line for inequalities with < or > symbols. ←- - - -→
  A dashed line shows that points on the line itself are *not* part of the solution set.
- Use a solid line for inequalities with ≤ or ≥ symbols. ←——→
  A solid line shows that points on the line itself *are* part of the solution set.

A linear inequality has infinite solutions. Recall that when graphing an inequality on a number line, the number line is shaded to show all possible solutions. Similarly, when graphing a *linear* inequality, the coordinate plane is shaded to show all possible solutions. The coordinate plane is shaded on one side of the line depending on where the solution set lies. In general, when inequalities are in slope-intercept form, a < or ≤ symbol indicates that the solution set is "below" the line, and a > or ≥ symbol indicates that the solution set is "above" the line.

Two examples of linear inequalities and their graphs are shown below.

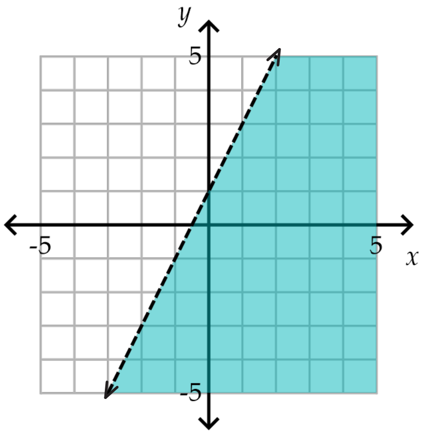
$y \ge -x - 3$

- solid line
- shaded "above" the line



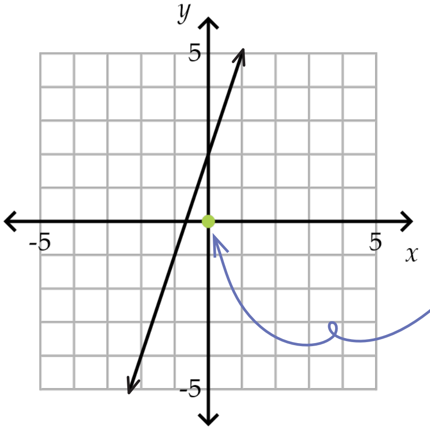
$y < 2x + 1$

- dashed line
- shaded "below" the line



To determine which side of the line to shade on, substitute an ordered pair into the inequality and see if a true statement results. If the ordered pair satisfies the inequality, shade on the side of the line where the ordered pair is located. If the ordered pair does not satisfy the inequality, shade on the other side of the line. Any ordered pair other than one located on the line itself may be used to determine where to shade.

The line for the inequality $y \le 3x + 2$ is shown below. To determine where to shade, the ordered pair $(0,0)$ is substituted into the inequality to see if a true statement results.



Test $(0,0)$:

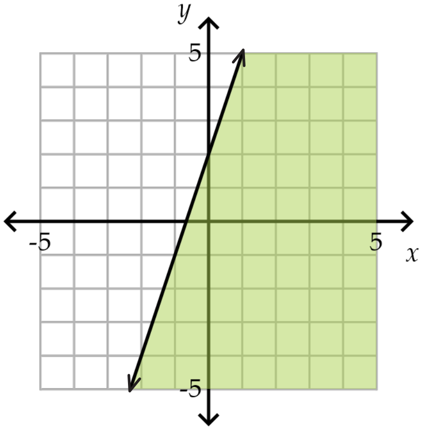
$y \le 3x + 2$

$0 \overset{?}{\le} 3(0) + 2$

$0 \overset{?}{\le} 0 + 2$

$0 \overset{?}{\le} 2$ ✓

Because a true statement results, shade on this side of the line. $(0,0)$ is part of the solution.



Example 1: Graph the inequality $y < 2x + 5$.

Slope: 2 y-intercept: $(0,5)$ Use a dashed line.

Test $(0,0)$ to determine shading:

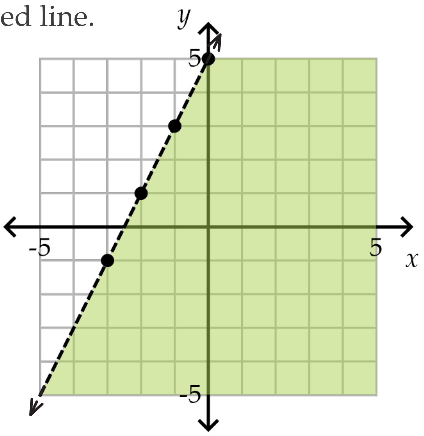
$y < 2x + 5$

$0 \overset{?}{<} 2(0) + 5$

$0 \overset{?}{<} 0 + 5$

$0 \overset{?}{<} 5$ ✓

Shade on the side of the line where $(0,0)$ is located.



Example 2: Graph the inequality $y \ge -\frac{2}{3}x$.

Slope: $-\frac{2}{3}$ y-intercept: $(0,0)$ Use a solid line.

Because $(0,0)$ is on the line, it cannot be used as a test point. Use a point with numbers that are easy to work with.

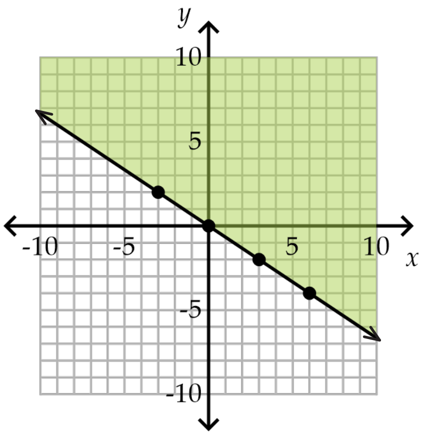
Test $(3,0)$:

$y \ge -\frac{2}{3}x$

$0 \overset{?}{\ge} -\frac{2}{3}(3)$

$0 \overset{?}{\ge} -2$ ✓

Shade on the side of the line where $(3,0)$ is located.

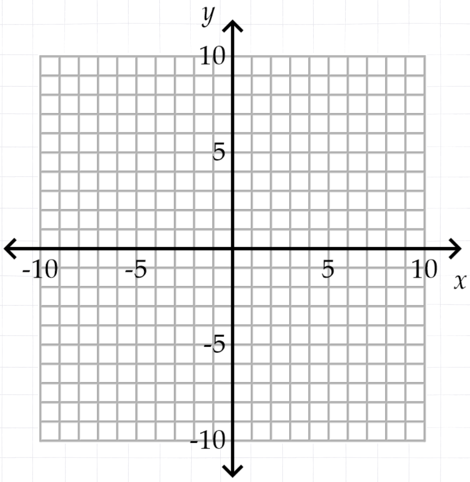


PRACTICE

1. Graph the inequalities. Use the indicated colors to shade.

Red: $y > 2x - 6$

Yellow: $y < \frac{3}{4}x$



2. Fill in the table to see if each ordered pair satisfies each inequality. Write "yes" or "no" in each box. Use the graph from Problem 1 to check your answers.

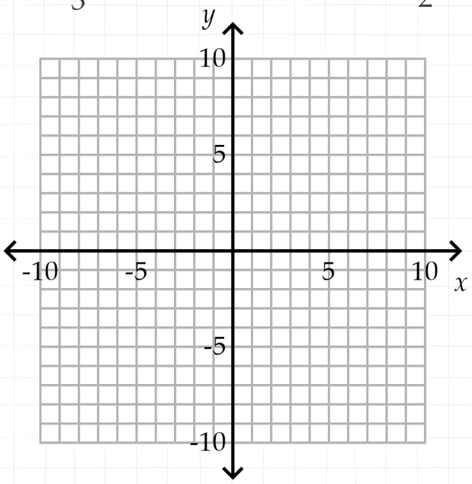
✦ Hint: If a point satisfies both inequalities, it will be shaded orange.

Ordered pair	$y > 2x - 6$	$y < \frac{3}{4}x$
$(0,4)$		
$(0,-4)$		
$(4,0)$		
$(-4,0)$		

3. Graph the inequalities. Use the indicated colors to shade.

Blue: $y \le -\frac{5}{3}x + 3$

Yellow: $y > \frac{1}{2}x - 1$



4. Fill in the table to see if each ordered pair satisfies each inequality. Write "yes" or "no" in each box. Use the graph from Problem 3 to check your answers.

✦ Hint: If a point satisfies both inequalities, it will be shaded green.

Ordered pair	$y \le -\frac{5}{3}x + 3$	$y > \frac{1}{2}x - 1$
$(0,4)$		
$(0,-4)$		
$(4,0)$		
$(-4,0)$		

REVIEW

1. Solve the inequality in Part A and Part B.

a. $-15 - 2x \leq 24$ L87

______

b. $\frac{5}{3}x + 18 > 2x + 22$ L91

______

c. Color the circle next to each value blue if it satisfies the inequality in Part A, red if it satisfies the inequality in Part B, and purple if it satisfies both inequalities. L87

○ $x = 0$

○ $x = -10$

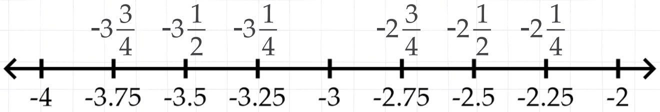
○ $x = -30$

○ $x = -16$

2. 252 is $\frac{21}{4}$ of what number? L55

______

3. Complete each problem mentally. Use the number line model for help. Write each answer as an integer.



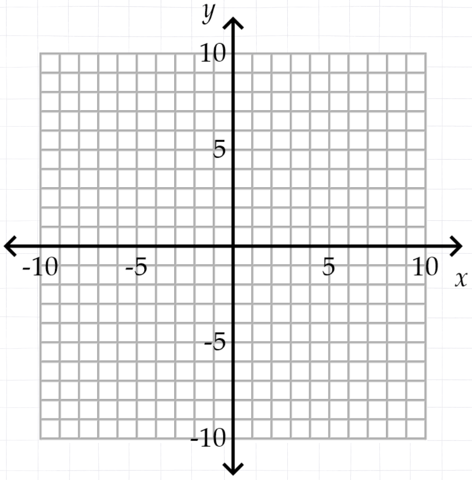
a. $-2.25 + 0.25$ ______

b. $-2\frac{1}{4} - \frac{3}{4}$ ______

c. $-2\frac{1}{2} - 1\frac{1}{2}$ ______

4. Use the x- and y-intercepts to graph the equation. Then use the graph to find the slope of the line. L46

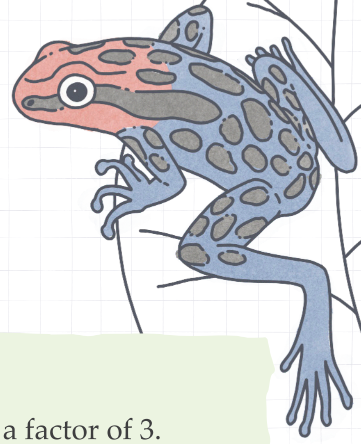
$9x - 6y = -18$



Slope: ______

UNIT 4 | LESSON 103

Polynomials

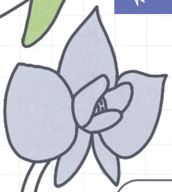


WARM-UP

Find the coordinates of the image if the preimage is dilated from the origin by a factor of 3.

Preimage	$(3,-2)$	$(7,-2)$	$(7,-4)$	$(3,-4)$
Image				

LESSON



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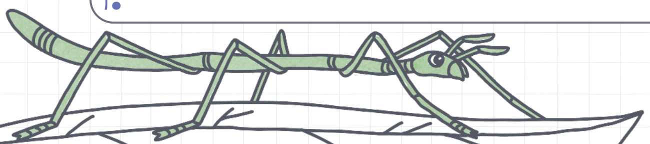
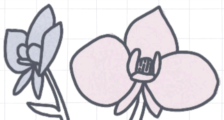
VIDEO PRACTICE





LESSON OVERVIEW

Operations can be performed on expressions with multiple terms and variables as well as on individual numbers. Recall that an ***expression*** is a number, variable, or combination of numbers and variables joined by operations. A ***term*** is one part of an expression, which may be a number, a variable, or a product of numbers and variables. The type of expression discussed in today's lesson is a polynomial. A ***polynomial*** is an expression with variables, coefficients, and constants in which terms are combined by addition, subtraction, or multiplication. Below are some examples of polynomials:

$-3x + 5$ $23a - 15b^5 + ab$ $y^2 + 12xy - 3x^2$ $fg^3h^5 + 11g^5h$

Polynomials can be classified by the number of terms they have. The table below shows the names of polynomials with one, two, and three terms.

Number of terms	Type of polynomial	Examples
1	monomial	$3x^4$ $-ab^2$ $23g$
2	binomial	$x^2 + y^5$ $-18ac + 6c^5$ $14 + v^5$
3	trinomial	$x^2 + xy - y^2$ $2a^5 - 4a^4b + a^3b^2$

Note: Polynomials with more than three terms are just called polynomials.

Like terms are terms with the same variables raised to the same power. Below are some examples of like terms.

x^2 and $-3x^2$
Both terms contain x^2.

ab^2c^5 and $10c^5b^2a$
Both terms contain ab^2c^5.

$24gh$ and $-gh$
Both terms contain gh.

A polynomial can be simplified by combining like terms. To combine like terms, add or subtract the coefficients. The variables and exponents on each variable remain the same. When all like terms have been combined, the polynomial is simplified.

Example 1: Simplify the polynomial.

$34x + 2x^2 - 15x - 5x^2$
$= 34x + 2x^2 - 15x - 5x^2$
$= 34x - 15x + 2x^2 - 5x^2$
$= 19x - 3x^2$

The orange terms can be combined by adding 34 and –15. The blue terms can be combined by adding 2 and –5.

Note: These are not like terms. The polynomial is simplified.

Example 2: Simplify the polynomial.

$125ac^2 + 55ac^2 - 64ab + 24ab$
$= 125ac^2 + 55ac^2 - 64ab + 24ab$
$= 180ac^2 - 40ab$

The orange terms can be combined by adding 125 and 55. The blue terms can be combined by adding –64 and 24.

ADDING AND SUBTRACTING POLYNOMIALS

To add two polynomials together, combine all like terms. In the example below, the polynomials $34x^2 + 5y - 10$ and $10x^2 - 8x + 3y$ are added together. Terms are first rearranged so like terms are next to each other.

$34x^2 + 5y - 10 + 10x^2 - 8x + 3y$

$= 34x^2 + 10x^2 + 5y + 3y - 10 - 8x$

$= 44x^2 + 8y - 10 - 8x$ This is the simplified answer.

Example 3: Add the polynomials $-11b^4 + 8b^3 - 3bc$ and $10 + 15b^3 + 2b^4$.

Write the polynomials with a plus sign between them.

$-11b^4 + 8b^3 - 3bc + 10 + 15b^3 + 2b^4$ Rearrange the terms.

$= -11b^4 + 2b^4 + 8b^3 + 15b^3 - 3bc + 10$ Combine like terms.

$= -9b^4 + 23b^3 - 3bc + 10$

Example 4: Add the polynomials $3jk - jk^2 + 15j$ and $23j + 5jk + 9j^2k$.

Write the polynomials with a plus sign between them.

$3jk - jk^2 + 15j + 23j + 5jk + 9j^2k$ Rearrange the terms.

$= 3jk + 5jk - jk^2 + 15j + 23j + 9j^2k$ Combine like terms.

$= 8jk - jk^2 + 38j + 9j^2k$

To subtract two polynomials, parentheses are written around the polynomial being subtracted. The minus sign can be thought of as a coefficient of –1. The –1 is distributed to each term in the second polynomial. The coefficient of –1 does not need to be written, but it is shown in the example below for clarity. Terms are then rearranged so like terms are next to each other.

$45a^2 + 15ab - 30b^2 - \left(22a^2 - 3ab + 18b^2\right)$

$= 45a^2 + 15ab - 30b^2 - 1\left(22a^2 - 3ab + 18b^2\right)$

$= 45a^2 + 15ab - 30b^2 - 22a^2 + 3ab - 18b^2$

$= 45a^2 - 22a^2 + 15ab + 3ab - 30b^2 - 18b^2$

$= 23a^2 + 18ab - 48b^2$ This is the simplified answer.

Example 5: Subtract $24c^2 + 52cd + 25d^2$ from $3cd - 15d^2 + 5c^2$.

Write the polynomials with a minus sign between them and parentheses around the polynomial being subtracted.

$3cd - 15d^2 + 5c^2 - \left(24c^2 + 52cd + 25d^2\right)$ Distribute the minus sign.

$= 3cd - 15d^2 + 5c^2 - 24c^2 - 52cd - 25d^2$ Rearrange the terms.

$= 3cd - 52cd - 15d^2 - 25d^2 + 5c^2 - 24c^2$ Combine like terms.

$= -49cd - 40d^2 - 19c^2$

Example 6: Subtract $-4h^3 + 8h^2 - 12h$ from $2h^3 + 6h - 9h^2$.

Write the polynomials with a minus sign between them and parentheses around the polynomial being subtracted.

$2h^3 + 6h - 9h^2 - (-4h^3 + 8h^2 - 12h)$ Distribute the minus sign.

$= 2h^3 + 6h - 9h^2 + 4h^3 - 8h^2 + 12h$ Rearrange the terms.

$= 2h^3 + 4h^3 + 6h + 12h - 9h^2 - 8h^2$ Combine like terms.

$= 6h^3 + 18h - 17h^2$

PRACTICE

Five in a Row

Add or subtract the given polynomials. Problems do not need to be completed in order. Cross out the answer in the table on the next page. Continue completing problems until five in a row (vertically, horizontally, or diagonally) are crossed off. For extra practice, complete all remaining problems.

Polynomials	***A* + *B*** Add the polynomials.	***A* – *B*** Subtract polynomial ***B*** from polynomial ***A***.
A: $2a + 3a^2 + 1$ ***B***: $-a^2 - a$		
A: $5ba - b^2$ ***B***: $b + a^2 + b^2$		
A: $-4a^2 - ab + b^2a$ ***B***: $a^2b - 3a^2 + ba$		
A: $-a^3 + a^2 - a + 1$ ***B***: $a^3 - a^2 + a - 1$		
A: $10b + 4b^2 - 5$ ***B***: $-7b^2 - 2 + 3b$		
A: $-2a^3 + ab - 3ab^2$ ***B***: $2b^2 + 4a^3 + 5ab^2$		
A: $b^2a + ab - b^2 + 3a$ ***B***: $-2a - 2ab^2 + 3ab$		
A: $-a^2 - 4b + 3ab - 5$ ***B***: $2ab - 3a^2 - a^3 + 1$		

Polynomials	***A* + *B*** Add the polynomials.	***A* – *B*** Subtract polynomial ***B*** from polynomial ***A***.
A: $4a^2 + 1 - 3a$ ***B***: $-a + a^2 - 4$		
A: $-2b^2a + b^2$ ***B***: $2b^2 + 2a^2 + ab^2$		
A: $3a^2 + 3ab - 5ab^2$ ***B***: $-a^2b + 2a^2 + ba$		
A: $2b^3 - 6b^2 + 4a + 2$ ***B***: $-b^2 + 3b^3 - 2a - 6$		

Note: Answers in the chart below may have terms in a different order.

$a + 2a^2 + 1$	$3ab^2 - 2ab - b^2 + 5a$	$5a^2 - 3 - 4a$	$7b + 11b^2 - 3$	$-b^3 - 5b^2 + 6a + 8$
$5ba + b + a^2$	$-3ab^2 - b^2 - 2a^2$	$-a^2 - 2ab + b^2a - a^2b$	$5a^2 + 4ab - 5ab^2 - a^2b$	$2a^3 + ab + 2ab^2 + 2b^2$
$a^2 + 2ab - 5ab^2 + a^2b$	$-7a^2 + b^2a + a^2b$	Free Space	$2a^2 - 4b + ab - 6 + a^3$	$-6a^3 + ab - 8ab^2 - 2b^2$
$5b^3 - 7b^2 + 2a - 4$	$5ba - 2b^2 - b - a^2$	$-ab^2 + 4ab - b^2 + a$	$3a + 4a^2 + 1$	$-4a^2 - 4b + 5ab - 4 - a^3$
$-2a^3 + 2a^2 - 2a + 2$	$13b - 3b^2 - 7$	$3a^2 + 5 - 2a$	$-ab^2 + 3b^2 + 2a^2$	0

Logic Lesson 4

SWIM-BIKE-RUN-MATH

A triathlon is a race that includes three events: swimming, cycling, and running. Men's and women's triathlons have been part of the summer Olympic Games since the year 2000. Some of the distances in competitive triathlons are called the sprint, standard / Olympic, long course, and ultra. All of these logic puzzles are related to triathlons. This lesson has no video or review problems.

A calculator may be used for this entire lesson.

ONE HUNDRED FIVE

One type of triathlon includes a 2.4-mile swim, a 112-mile bike ride, and a 26.2-mile run (a full marathon). In 2023, Zimbabwean Sean Conway set a world record when he completed 105 of these triathlons in 105 consecutive days!

Find three consecutive numbers that add to 105.

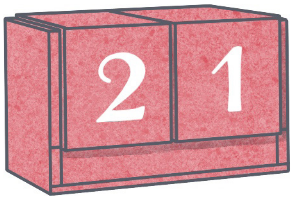
______, ______, and ______

FAMILY FEAST

After finishing a sprint triathlon, two fathers and two sons eat breakfast at a diner. The bill totals exactly $33 including tax and a tip, and they split the cost evenly. Each person pays a whole-number amount of money (no cents). Explain how this could be true.

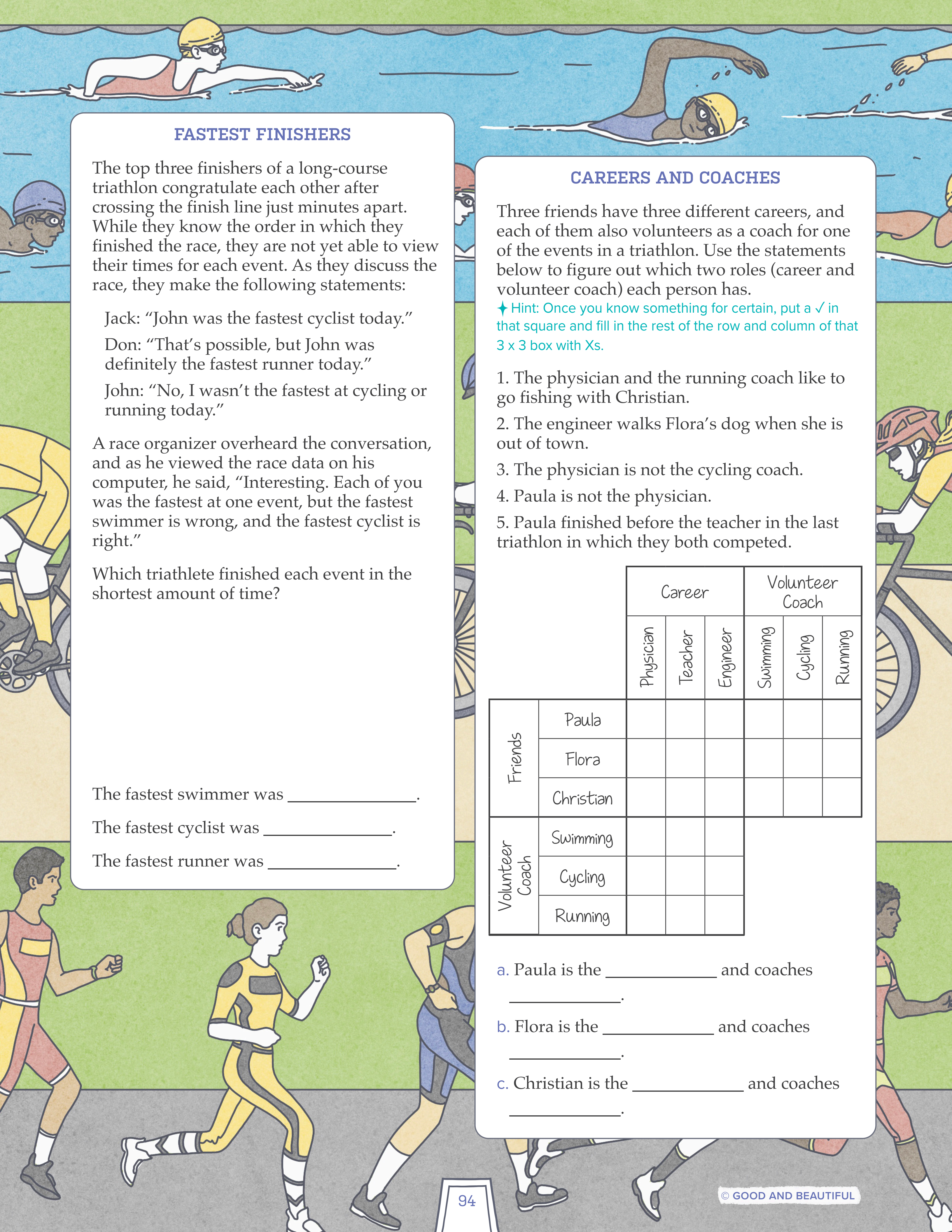
BIKE-SHOP BLOCKS

A triathlete takes her bike to a local shop for a tune-up before her next triathlon. While waiting, she notices a calendar made from wooden blocks like the one shown below. What numbers must be painted on each cube in order to show all the possible dates in a month? Assume 9 can be made by turning 6 upside down and that the blocks can switch places.



Numbers on one block: ________________

Numbers on the other block: ________________



FASTEST FINISHERS

The top three finishers of a long-course triathlon congratulate each other after crossing the finish line just minutes apart. While they know the order in which they finished the race, they are not yet able to view their times for each event. As they discuss the race, they make the following statements:

Jack: "John was the fastest cyclist today."

Don: "That's possible, but John was definitely the fastest runner today."

John: "No, I wasn't the fastest at cycling or running today."

A race organizer overheard the conversation, and as he viewed the race data on his computer, he said, "Interesting. Each of you was the fastest at one event, but the fastest swimmer is wrong, and the fastest cyclist is right."

Which triathlete finished each event in the shortest amount of time?

The fastest swimmer was _______________.

The fastest cyclist was _______________.

The fastest runner was _______________.

CAREERS AND COACHES

Three friends have three different careers, and each of them also volunteers as a coach for one of the events in a triathlon. Use the statements below to figure out which two roles (career and volunteer coach) each person has.

✦ Hint: Once you know something for certain, put a ✓ in that square and fill in the rest of the row and column of that 3 x 3 box with Xs.

1. The physician and the running coach like to go fishing with Christian.

2. The engineer walks Flora's dog when she is out of town.

3. The physician is not the cycling coach.

4. Paula is not the physician.

5. Paula finished before the teacher in the last triathlon in which they both competed.

		Career			Volunteer Coach		
		Physician	Teacher	Engineer	Swimming	Cycling	Running
Friends	Paula						
	Flora						
	Christian						
Volunteer Coach	Swimming						
	Cycling						
	Running						

a. Paula is the _____________ and coaches _____________.

b. Flora is the _____________ and coaches _____________.

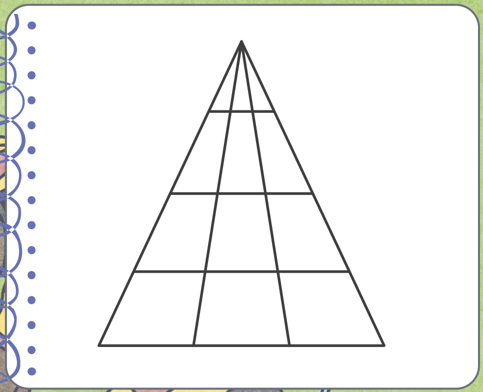
c. Christian is the _____________ and coaches _____________.

TRIATHLONS, TRIANGLES, AND TIMERS

Sisters Tallie and Tandee are training together to compete in a triathlon, but today is a rest day in their training program. Finding it difficult to talk about something other than their upcoming race, Tallie has an idea. “Tandee, let’s not forget that we both love puzzles and riddles as much as we love swimming, biking, and running. To distract ourselves from thinking about the triathlon, let’s have a competition with triangles!” Tandee agrees, and Tallie sketches the image at the right. “The winner is whoever finds the most triangles in 15 minutes,” Tallie says.

“Wait!” replies Tandee. “Don’t start a timer on your phone. We have a 7-minute and an 11-minute sand hourglass timer in the kitchen. Before we start working on the triangles problem, let’s figure out how to keep track of 15 minutes using the sand timers.”

First figure out how to use a 7-minute and an 11-minute sand hourglass timer to measure 15 minutes. Write an explanation below.



How many triangles are in the sketch? For a challenge, set a timer for 15 minutes to see if you can find all the triangles in that time.

✦ Hint: Sketch the figure multiple times in the space above or on a separate sheet of paper. Use colored pencils to color the triangles in an organized way.

______ triangles

Factoring

WARM-UP

Divide $12x^8 + 15x^3$ by $3x^2$.

____

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

LEAST COMMON MULTIPLE OF MONOMIALS

Recall that the least common multiple (LCM) of two numbers is the smallest positive number that is a multiple of both numbers. The least common multiple of 3 and 4 is 12. 12 is the smallest number that is a multiple of 3 and a multiple of 4. The least common multiple of monomials can be found as well. To find the LCM of monomials, it can help to write the factorization of each monomial. Then find the product of all factors from each monomial, but do not repeat factors that have already been included from another factorization.

To find the LCM of $9f^2g^3$ and $12fg^2$, first write the factorization of each monomial. Note: The prime factorization is written for coefficients.

$$9f^2g^3 = 3 \bullet 3 \bullet f \bullet f \bullet g \bullet g \bullet g \qquad 12fg^2 = 2 \bullet 2 \bullet 3 \bullet f \bullet g \bullet g$$

The LCM must include all factors from both monomials but should not include any extras. The purple factors below are the factors that both monomials have in common. They are only written once.

$$\text{LCM: } 2 \bullet 2 \bullet 3 \bullet 3 \bullet f \bullet f \bullet g \bullet g \bullet g = 36f^2g^3$$

The LCM of $9f^2g^3$ and $12fg^2$ is $36f^2g^3$. Notice that f^2 and g^3 are the variables with the highest exponents in the original monomials. Because every factor needs to be part of the LCM, the highest power of each variable will be the factor to use in the LCM. Instead of writing the factorization for each monomial, find the LCM of the coefficients and multiply by the highest power of each variable.

Example 1: Find the LCM of m^2n^5 and m^3n.

Highest power of each variable in the original monomials: m^3, n^5

Multiply the highest power of each variable.

LCM: m^3n^5

Example 2: Find the LCM of $15x^3y^2$ and $9x^5y^4$.

Prime factors of the coefficients: $15 = 3 \bullet 5$ $\quad 9 = 3 \bullet 3$

LCM of the coefficients: $3 \bullet 3 \bullet 5 = 45$

Highest power of each variable in the original monomials: x^5, y^4

Multiply the LCM of the coefficients by the highest power of each variable.

LCM: $45x^5y^4$

Example 3: Find the LCM of $5a^5b^3c$, $20b^7c^4$, and $30c^2d$.

Prime factors of the coefficients: $5 = 5$ $\quad 20 = 2 \bullet 2 \bullet 5$ $\quad 30 = 2 \bullet 3 \bullet 5$

LCM of the coefficients: $2 \bullet 2 \bullet 3 \bullet 5 = 60$

Highest power of each variable in the original monomials: a^5, b^7, c^4, d

Multiply the LCM of the coefficients by the highest power of each variable.

LCM: $60a^5b^7c^4d$

PRACTICE

1. Complete Parts A–D to find the least common multiple and greatest common factor of the two monomials.

   a. Write the factorization of each monomial.

   $36a^2bc^5 =$ ______________________________

   $15a^5b^4c^2 =$ ______________________________

   b. Find the LCM of $36a^2bc^5$ and $15a^5b^4c^2$.

   ____

   c. In the factorizations from Part A, circle common factors.

   d. Find the GCF of $36a^2bc^5$ and $15a^5b^4c^2$.

   ____

2. Complete Parts A–D to find the least common multiple and greatest common factor of the two monomials.

   a. Write the factorization of each monomial.

   $12t^3uv^5 =$ ______________________________

   $5tu^4v^3 =$ ______________________________

   b. Find the LCM of $12t^3uv^5$ and $5tu^4v^3$.

   ____

   c. In the factorizations from Part A, circle common factors.

   d. Find the GCF of $12t^3uv^5$ and $5tu^4v^3$.

   ____

3. Use the highest or lowest power of the exponents to find the least common multiple (LCM) and greatest common factor (GCF) of the monomials.

Monomials	LCM	GCF
$16h^3i^4j$ $22h^5j^5$		
$8m^5n^3$ $50m^7n^2$		
$14p^4qr^7$ $52p^3q^{15}r^8$		

4. Use the GCF to factor each polynomial.

a. $3a^3b^5 + 6a^5b^2$

____

b. $12cd^5 - 20c^2d^3$

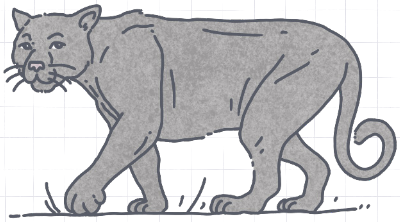
____

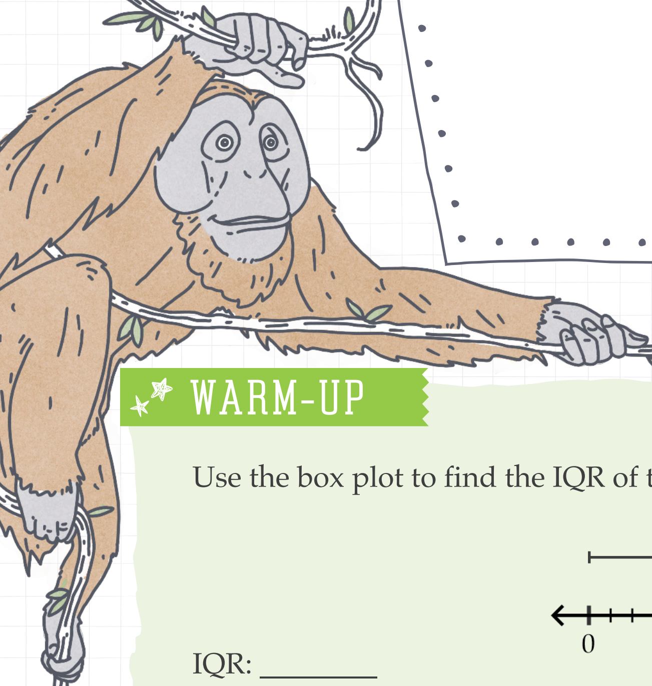
c. $45e^4f^5g^2 + 63e^4f^2g^2$

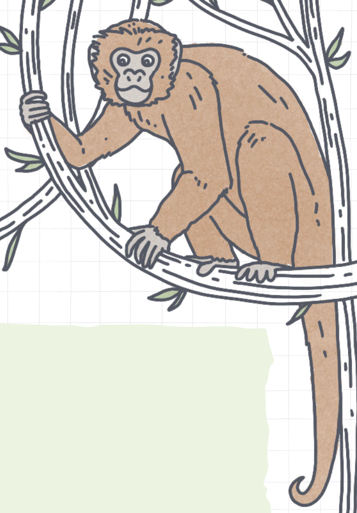
____

d. $91h^2i^3j^4 - 13hi^2j^3$

____





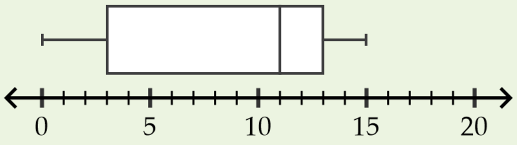


UNIT 4 | LESSON 112

Scatter Plots

WARM-UP

Use the box plot to find the IQR of the data.



IQR: ________

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

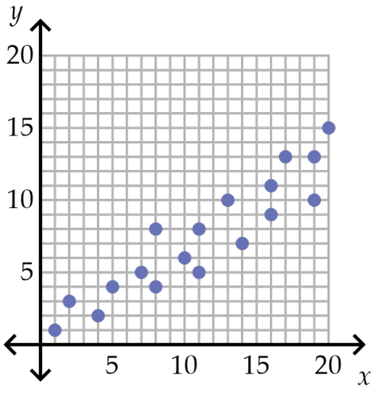
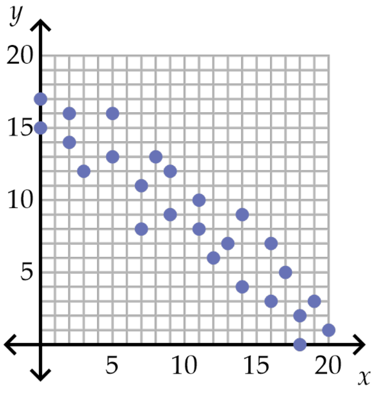
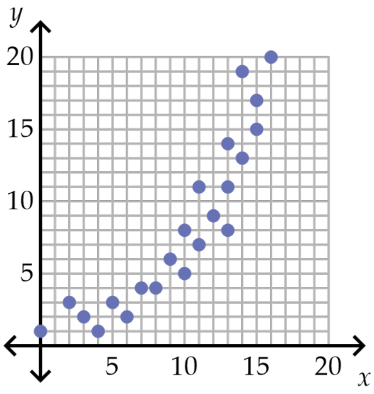
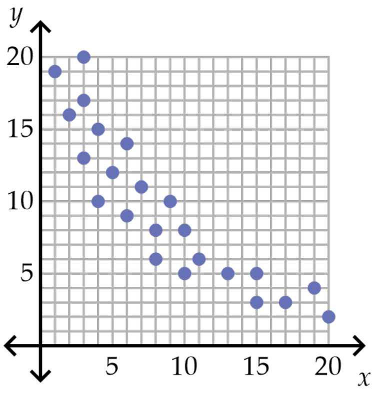
VIDEO PRACTICE

LESSON OVERVIEW

A ***scatter plot*** is a graph on the coordinate plane with one point for each data value. There may be more than one output (y-value) for a given input (x-value). Scatter plots are a visual way to organize data to see trends, patterns, and relationships, as well as to make predictions and inferences about the data.

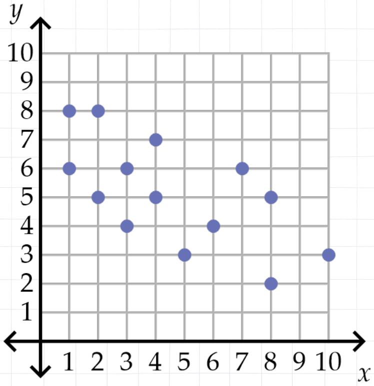
TYPES OF CORRELATION

A ***correlation*** is a relationship between two variables. Correlation can be seen on a scatter plot and can be described in a number of ways. The table below shows four scatter plots with different types of correlation. Each graph shows a different type of relationship between the x-values and the y-values.

Types of Correlation	Positive: Data points generally increase from left to right. As x increases, y increases.	Negative: Data points generally decrease from left to right. As x increases, y decreases.
Linear: Data points generally form a line.	positive linear correlation 	negative linear correlation 
Nonlinear: Data points generally form a curve	positive nonlinear correlation 	negative nonlinear correlation 

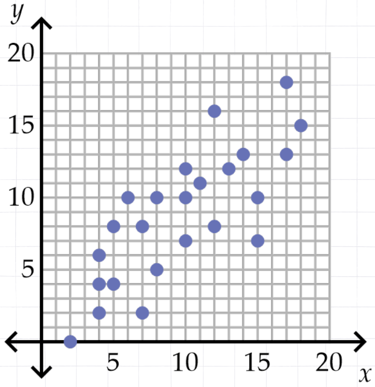
PRACTICE

Seven scatter plots are shown on this page. Each scatter plot represents an answer in Section I and in Section II. Read each problem and find the graph that is the correct answer. Write the letter shown below the graph for Section I or Section II on the line. For example, if the answer to Problem 1 in Section I is the graph at the right, write a V on the line. If the answer to Problem 1 in Section II is this graph, write an L on the line. When finished, fill in the letters on the lines at the end of the practice to answer the riddle.

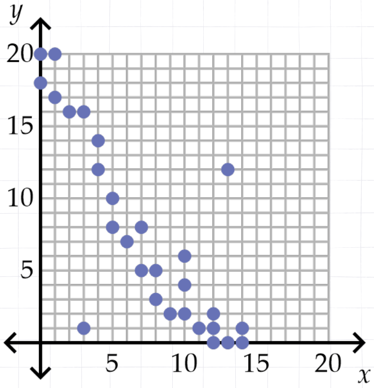


Section I: V Section II: L

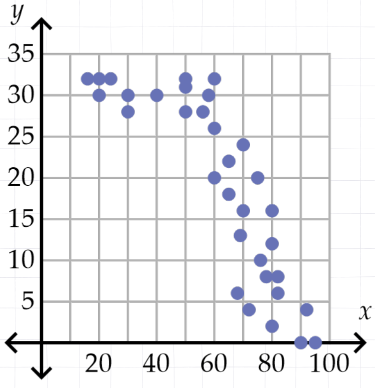
Riddle: Why should you never tell a scatter plot a secret?



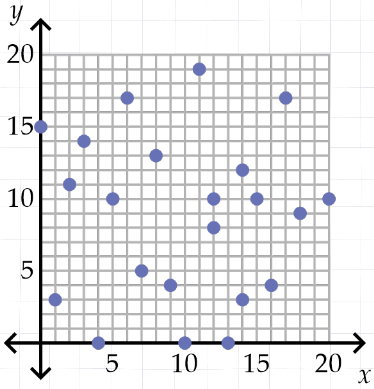
Section I: S Section II: R



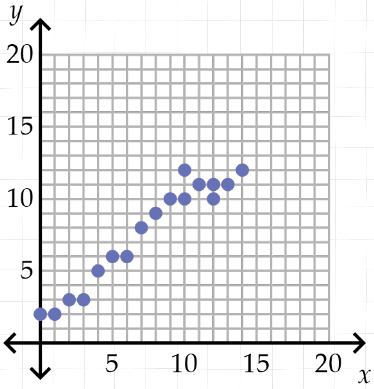
Section I: D Section II: P



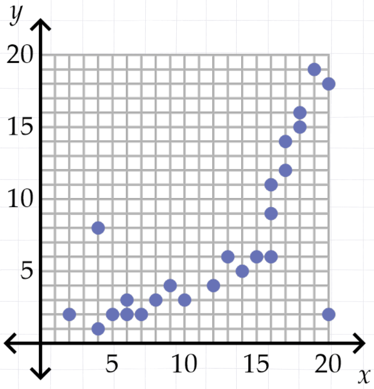
Section I: H Section II: A



Section I: W Section II: T



Section I: C Section II: E



Section I: O Section II: I

Section I: Match the description of correlation and outliers to the correct scatter plot.

1. strong, positive, linear ______

2. strong, negative, nonlinear ______

3. weak, negative, linear ______

4. positive, nonlinear, some outliers ______

5. strong, negative, some outliers ______

6. weak, positive, no outliers ______

7. no noticeable correlation ______

Section II: A research company compiled data on dental healthcare. Match each scenario with the scatter plot that most likely represents the situation and answer the questions.

8. a. The number of cavities a person has (y) has a weak correlation with how much candy they eat (x).

______

b. What does the correlation in the scatter plot indicate?

____

____

9. a. The number of cavities a person has (y) is strongly correlated with how often they brush their teeth each week (x), but there are notable outliers.

______

b. What do the outliers in the scatter plot indicate?

____

____

10. After childhood, the number of teeth a person has (y) is strongly correlated with age (x), with significant loss occurring at advanced ages.

______

11. The number of cavities children have (x) does not appear to be correlated with the number of cavities their parents have (y).

______

12. A dentist's monthly income in thousands of dollars (y) is positively correlated with the number of daily clients he or she has (x), but the correlation is not linear. There are some outliers.

______

13. a. At a dentist's office, oral hygiene is measured on a scale with higher numbers corresponding to better oral health. The oral hygiene of a person (y) is strongly correlated with frequency of flossing (x).

______

b. What does the correlation in the scatter plot indicate?

____

____

14. The amount of plaque buildup (y) is weakly correlated with the cost of the toothbrush used (x).

______

Riddle Answer:

12 11   7 12 14 14   6 9 8 13 10 5   12 11

10 14 14   4 3 13 8   11 2 13   9 14 10 1 13 !

REVIEW

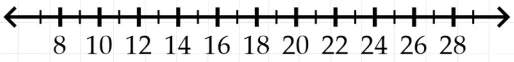
A calculator may be used for this entire review section.

1. The data set below shows the height (in thousands of feet, rounded to the nearest thousand) of the highest peak on each continent. Create a box plot for the data and find the IQR. L111

19, 19, 20, 23, 29, 16, 7

Minimum: _____ Maximum: _____

Q1: _____ Q2: _____ Q3: _____

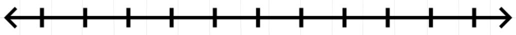


IQR: _____

2. Solve and graph the inequality. L91

$$\frac{3}{4}\left(8x-24\right)<\frac{15x-27}{2}$$

____

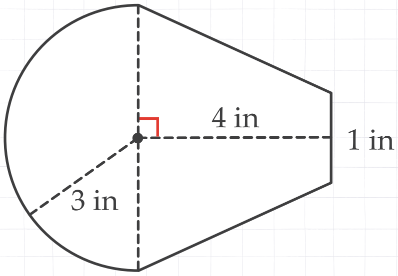


3. Alicia is using an app to learn a foreign language. She set a goal to spend an average of 20 minutes per day, six days a week, practicing on the app. The numbers of minutes she practiced each day from Monday to Friday are listed below. How many minutes does she need to practice on Saturday in order to average 20 minutes per day this week? L110

22, 15, 18, 20, 23

_____ minutes

4. Find the area of the composite figure to the nearest hundredth. L81



____

Unit 4 Review

☆ SUPPLIES: protractor, colored pencils

Complete this Unit Review to prepare for the Unit Assessment. There is no video, lesson, or practice. Because Unit Reviews include practice for an entire unit, they may take longer than regular lessons, and students may decide to take two days to finish.

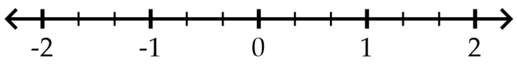
A rainforest is a forest that receives a lot of rainfall, generally over 80 inches a year. Complete the problems in this review. At the end of the review, write the green word on the line that corresponds to the solution to discover some amazing facts about rainforests.

LESSONS 91–92

1. Solve and graph the inequality.

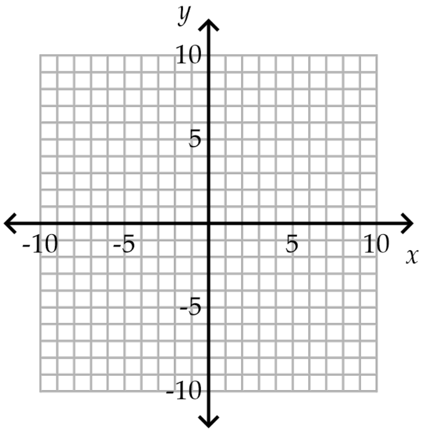
$$3x + 2 \le 4 - 3x$$

$x \le$ ________ surface



2. a. Graph the linear inequality.

$$5x + 2y > 20$$



b. One of the points below satisfies the inequality from Part A. Use the graph to determine which point it is and use that word as the clue.

$(0,0)$	$(1,6)$	$(6,1)$
evaporating	absorbing	falling

LESSONS 93–96

3. Find the value of $2(4a+b)-1$ if $\frac{2a}{3}-1=5$ and $2b=4b+12$.

______ ten

4. Solve each equation or system of equations. Use the specified method if one is indicated. Write “none” or “infinitely many” if there is not a unique solution.

a. $5+3(2u-4)=-7-(-6u)$

________________ canopy

b. $2v+3=-3(4-v)-v$

________________ medicines

c. Substitution
$x=3y+1$
$2x-4y=6$

________________ inches

d. Elimination
$6x+2y=8$
$-9x-3y=-12$

____

RAINFOREST FACTS:

1. Roughly ______________ of the earth's ______________ is ______________ .

$y=\frac{5}{4}x+\frac{1}{2}$ $\frac{1}{3}$ 20

2. A ______________ ______________ can take as long as ______________ minutes to

$(6,1)$ 28 59

fall from the upper ______________ of a ______________ all the way to the ground.

infinitely many 20

3. During years of heavy ______________ , a ______________ might receive as many as

$(x,y)\rightarrow(x+4,y+4)$ 20

400 ______________ of ______________ in a single year.

$(7,2)$ 3

4. About ______________ of modern ______________ originate from ______________ in

$(0,5)$ none 38

the ______________ . The ______________ ______________ Institute in the United

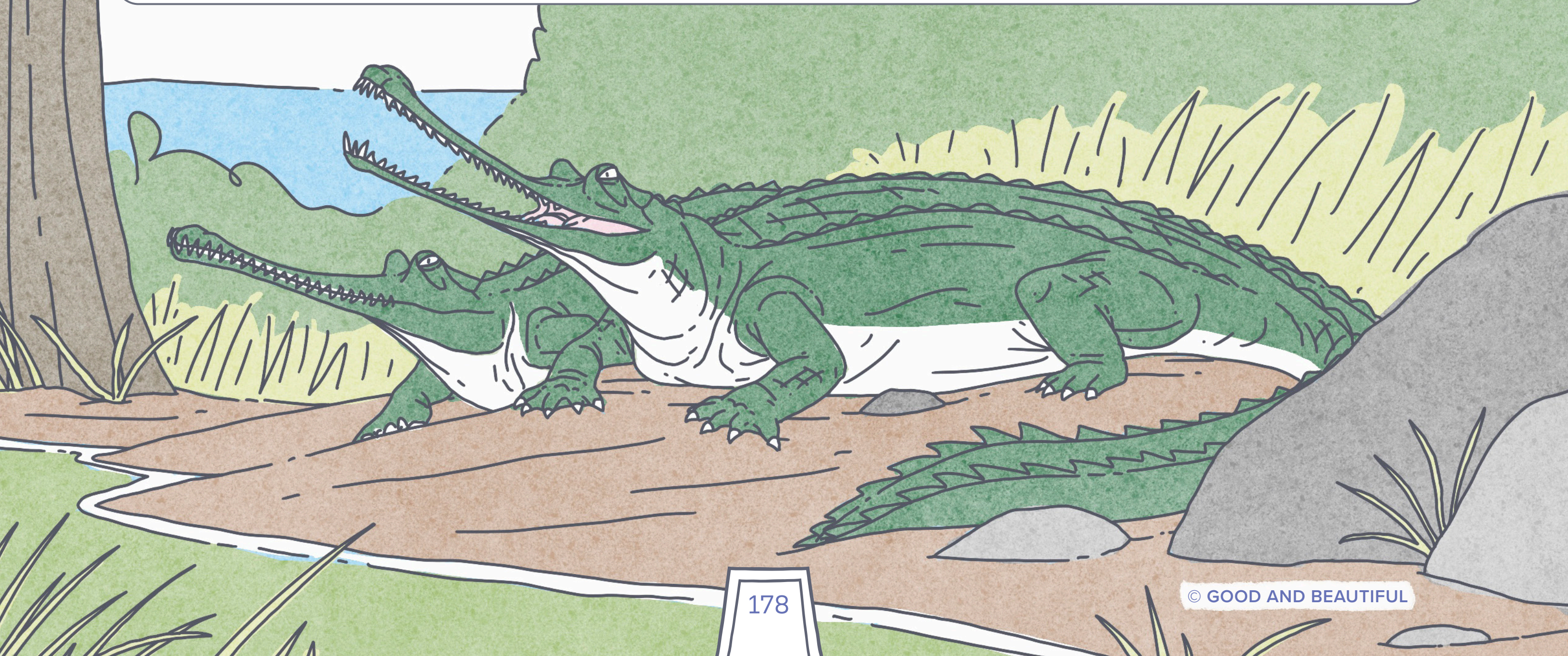
20 10.53% 2

States estimates that as many as ______________ of ______________ -fighting

$2e^2-e-3$ $2kl$

______________ originate from ______________ ______________ .

3.1 20 38



Course Assessment

SUPPLIES: protractor, colored pencils

This assessment covers concepts taught in Pre-Algebra. Problems are designed to assess multiple skills. Read the instructions carefully and do not rush through the problems. You may use the Reference Chart for the assessment. Lesson numbers are given by each problem so you can review lessons for any answers that are incorrect. Calculators may be used where needed on the entire assessment.

1. Simplify the expression. L16, 19–21

a. $\sqrt{49}$ ______

b. $\sqrt[3]{512}$ ______

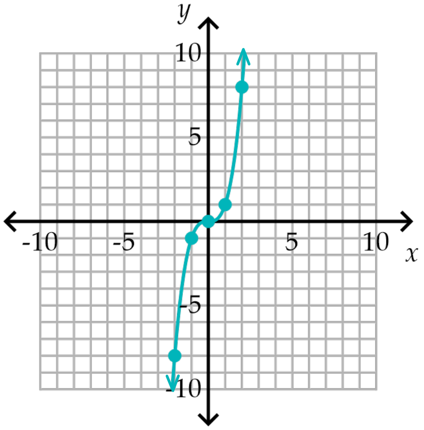
c. 2^{-3} ______

d. $\sqrt{2} \bullet \sqrt{8}$ ______

e. $5\sqrt{3} - 2\sqrt{3}$ ______

f. $\sqrt[3]{\frac{1000 - 2 \bullet 68}{8^2 - 4 \bullet 15}}$ ______

2. Fill in the table with the ordered pairs shown on the graph of the relation. Then determine the rule and equation for the relation. L37



x	y

Rule: ____________________

Equation: ____________

3. a. Fill in the table to find the change in x and y. L39

Change in x	x	y	Change in y
	–2	3	
	–1	4	
	0	5	
	1	6	
	2	7	

b. Is the rate of change constant? ______

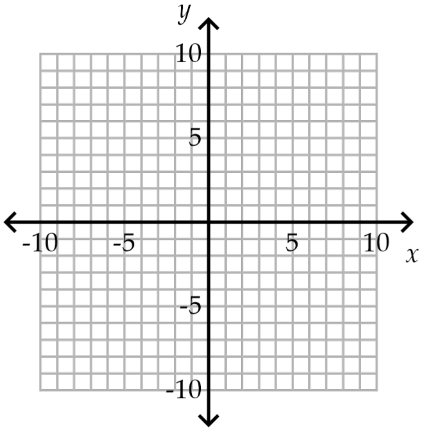
c. Does the table represent a linear function? ______

4. Three linear equations are given below. Graph each line. Then identify a pair of parallel lines and perpendicular lines. L46–47, 49

A $y = -\frac{1}{2}x + 7$

B $y - 2x = -6$

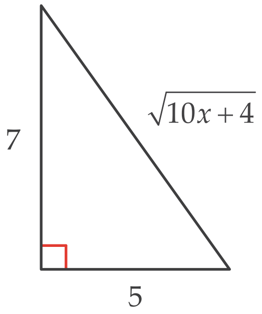
C $y = 2x + 3$



Parallel lines: _____ and _____

Perpendicular lines: ____ and ____

5. Use the Pythagorean theorem to solve for x. L50, 52–53



____

6. Chase bought a car five years ago for $42,250. Now Chase's car is worth $32,110. What is the percent decrease in the value of Chase's car after five years? L61

____

7. A bag contains the following tiles: 8 circles, 6 squares, 7 triangles, and 4 stars. L63–64

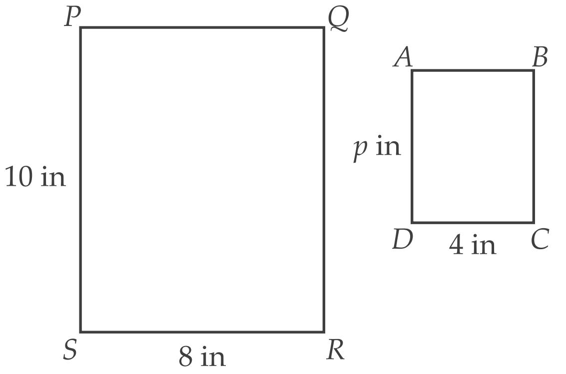
a. Find the probability (as a percent) of drawing a triangle.

______

b. Find the probability (as a percent) of drawing a square, not replacing it, and then drawing a star.

______

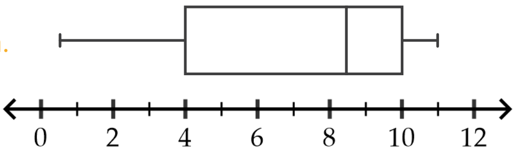
8. Given that $PQRS \sim ABCD$, find the value of p. L66, 74, 76



____

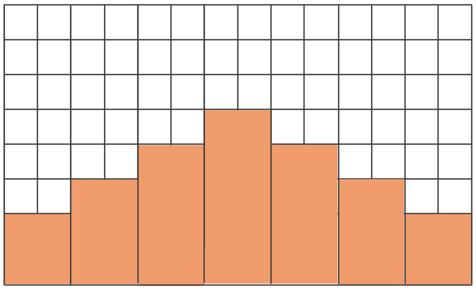
24. Determine whether each situation represents a data set that is symmetric, right-skewed, or left-skewed. L114

a.



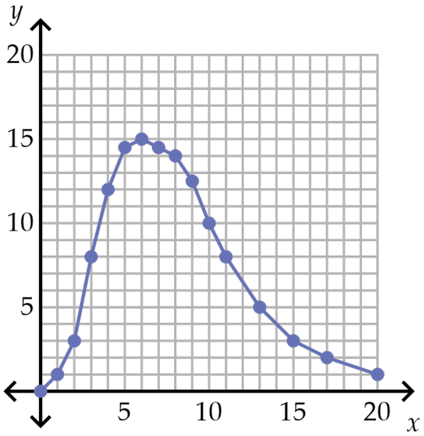
____

b.



____

c.



____

25. Given the two-way table, determine what percent of the total sample are females who picked tennis. L115

	Male	Female	Total
Tennis	140	95	235
Pickleball	110	155	265
Total	250	250	500

____

26. Determine if the survey questions are biased or unbiased. L116

a. Do you prefer a truck or an SUV?

____

b. Do you prefer to read a book or watch a movie?

____

c. Do you prefer to watch the challenging sport of soccer or boring football?

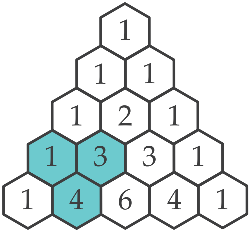
____

Enrichment: Pascal's Triangle

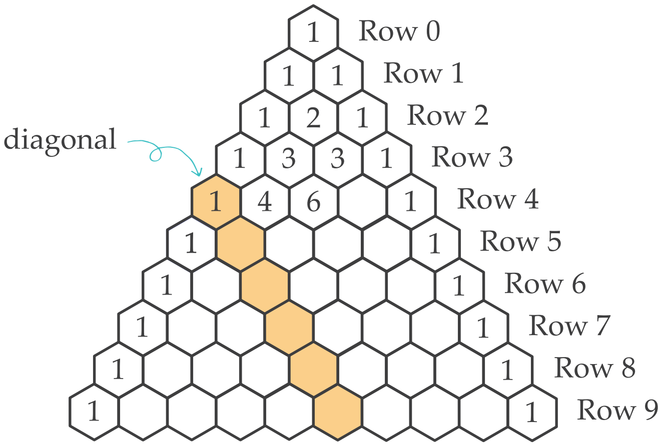
This is an enrichment lesson. Mastery is not expected at this level. There is no video, practice, or review. A calculator may be used for this entire lesson.

Blaise Pascal was a famous French philosopher, writer, inventor, physicist, and mathematician who lived in the 1600s. He made significant contributions to the fields of probability and geometry.

One of Pascal's most famous works is called Pascal's triangle. Pascal's triangle is made up of numbers that form a triangular shape with the digit 1 on the left and right edge of each row. The other values in each row are found by adding the two values directly above it. For example, in part of Pascal's triangle at the right, the number 4 is found by adding the 1 and 3 above it.



Additional rows can be added at the bottom of Pascal's triangle to make the triangle larger and larger. The top row is considered Row 0. A diagonal is highlighted in yellow below. Pascal's triangle has many patterns that can be seen in the rows and diagonals.



Try it!

1. Fill in the missing values of Pascal's triangle above.

2. Spend five minutes studying the numbers in this triangle and their arrangement. Write any observations you notice about patterns in the triangle.

Another interesting characteristic of Pascal's triangle can be found by looking at the following example. Suppose there are three different hamburger toppings to choose from: cheese, lettuce, and tomato. If any combination of toppings can be chosen, how many different burgers can be made? Note that choosing cheese and tomato is the same as choosing tomato and cheese because the order of the toppings does not matter.

0 toppings: 1 burger can be made with 0 toppings.
Options: no toppings

1 topping: 3 different burgers can be made with 1 topping.
Options: cheese or lettuce or tomato

2 toppings: 3 different burgers can be made with 2 toppings.
Options: cheese and lettuce, cheese and tomato, lettuce and tomato

3 toppings: 1 burger can be made with 3 toppings.
Options: cheese, lettuce, and tomato

Look at the number of possible burgers with 0, 1, 2, and 3 toppings: 1, 3, 3, 1. This is the third row of Pascal's triangle. The row sum is the number of possible combinations: 8.

Try it!

7. Suppose there are four different hamburger toppings to choose from: cheese, lettuce, tomato, and pickles. If any combination of toppings can be chosen, how many different burgers can be made? Remember that the order of the toppings does not matter.

a. 0 toppings: _____

Options: ____________________________

b. 1 topping: _____

Options: ____________________________

____

c. 2 toppings: _____

Options: ____________________________

____

____

d. 3 toppings: _____

Options: ____________________________

____

____

e. 4 toppings: _____

Options: ____________________________

____

f. Number of possible combinations: ______

The answers to Parts A–E should be the numbers in the fourth row of Pascal's triangle, and the answer to Part F is the sum of the numbers in that row.

The rows of Pascal's triangle give information about *combinations*. In math, combinations refer to the number of ways to choose a certain number of items. In the example with three toppings, there were three ways to choose two of the three toppings. In the example with four toppings, there were six ways to choose two of the four toppings. Pascal's triangle has so many more interesting properties!

UNITS

1–4

Simply Good and Beautiful

PRE-ALGEBRA

ANSWERS & SOLUTIONS

MATH

8

The Good and the Beautiful

CURRICULUM

Table of Contents

UNIT 1 | LESSON 1

Place Value and Estimation

WARM-UP

a. 60

b. 24

c. 51

PRACTICE

1. The digit to the right of the hundreds place (7) is greater than 5, so round up.

   1400

2. $6.3\overline{81} = 6.3818181...$

   The digit to the right of the hundred thousandths place (8) is greater than 5, so round up.

   6.38182

3. 3

4. $(3 \bullet 100{,}000) + (2 \bullet 1000) + (4 \bullet 100)$

   $= 300{,}000 + 2000 + 400$

   $= 302{,}400$

5. 12,000.006

6. 405,213

7. 18,324 rounds to 18,000.

   $18{,}000 \div 9 = 2000$

8. Numbers with a whole number part of 1 are smallest. Compare the tenths place in these numbers: 1.23 1.203 1.03

   1.03 is the smallest. Then compare the hundredths place in the other two numbers. 1.203 is smaller than 1.23.

   2.31 is greater than numbers with a whole number part of 1 but less than numbers with a whole number part of 12.

   For numbers with a whole number part of 12, look at the tenths place. 12 12.3

   12 has a 0 in the tenths place, so it is smaller than 12.3.

   Least to greatest:

   1.03, 1.203, 1.23, 2.31, 12, 12.3

   Riddle answer: OWL-GEBRA

REVIEW

More than one answer is possible for each problem. A possible solution is given.

1. $6 \bullet 4 - 2$
$= 24 - 2$
$= 22$

2. $\frac{7+5}{4}$
$= \frac{12}{4}$
$= 3$

3. $5 + 5$
$= 10$

4. $7 + 3 + 1$
$= 10 + 1$
$= 11$

5. $\frac{8 \bullet 2 - 6}{5}$
$= \frac{16-6}{5}$
$= \frac{10}{5}$
$= 2$

6. $\frac{8-1}{7} \bullet 7 \bullet 2$
$= \frac{7}{7} \bullet 7 \bullet 2$
$= 1 \bullet 7 \bullet 2$
$= 7 \bullet 2$
$= 14$

7. $8 \div 2 + 2$
$= 4 + 2$
$= 6$

Decimals and Fractions

WARM-UP

a. $22 \bullet 4 = 88$

$21.843 \bullet 3.93 \approx 88$

b. $56 \div 8 = 7$

$55.5 \div 8.24 \approx 7$

c. $11 \bullet 5 = 55$

$11.058 \bullet 5.321 \approx 55$

PRACTICE

1. a. $0.65 = \frac{65}{100} = \frac{13}{20}$

   b. $12.32 = 12\frac{32}{100} = 12\frac{8}{25}$

   c. $10.98 = 10\frac{98}{100} = 10\frac{49}{50}$

   d. $6.4 = 6\frac{4}{10} = 6\frac{2}{5}$

2. a. $\begin{array}{r} 0.6 \\ 5\overline{)3.0} \\ \underline{-\ 30} \\ 0 \end{array}$ $\quad \frac{3}{5} = 0.6$

   b. $\begin{array}{r} 0.466... \\ 15\overline{)7.000...} \\ \underline{-\ 60} \\ 100 \\ \underline{-\ 90} \\ 100 \\ \underline{-\ 90} \\ 10 \end{array}$ $\quad \frac{7}{15} = 0.4\overline{6}$

3. a. same denominator, $5 > 2$

   $\frac{5}{3} > \frac{2}{3}$

   b. same numerator, $7 > 6$

   $\frac{5}{7} < \frac{5}{6}$

   c. Half of 9 is 4.5.

   $\frac{5}{9} > \frac{1}{2}$

   Half of 13 is 6.5.

   $\frac{6}{13} < \frac{1}{2}$

   $\frac{5}{9} > \frac{6}{13}$

   d. Half of 22 is 11.

   $\frac{11}{22} = \frac{1}{2}$

   Half of 16 is 8.

   $\frac{5}{16} < \frac{1}{2}$

   $\frac{11}{22} > \frac{5}{16}$

4. a. Half of 4 is 2.

$$\frac{3}{4} > \frac{1}{2}$$

Half of 5 is 2.5.

$$\frac{2}{5} < \frac{1}{2}$$

$$\frac{4}{3} = 1\frac{1}{3}$$

Greatest to least:

$$\frac{4}{3}, \frac{3}{4}, \frac{1}{2}, \frac{2}{5}$$

b. same numerator (smaller denominators mean larger fractions)

Greatest to least:

$$\frac{7}{9}, \frac{7}{11}, \frac{7}{12}, \frac{7}{13}$$

5.

$\frac{2}{5}$	A 1.375	B $2\frac{4}{9}$
C 0.476	D 1.45	$\frac{119}{250}$
$2.\overline{4}$	$1\frac{9}{20}$	$1\frac{3}{8}$
E $1\frac{3}{11}$	$1.\overline{27}$	0.4

Detailed work is shown below. Work may vary. Students may have converted decimal values to fractions or fractions to decimals to find equivalent numbers.

A $1.375 = 1\frac{375}{1000} = 1\frac{15}{40} = 1\frac{3}{8}$

B
$$\begin{array}{r} 0.44\ldots \\ 9\overline{)4.00\ldots} \\ \underline{-\ 36} \\ 40 \\ \underline{-\ 36} \\ 4 \end{array}$$

C $0.476 = \frac{476}{1000} = \frac{119}{250}$

D $1.45 = 1\frac{45}{100} = 1\frac{9}{20}$

E
$$\begin{array}{r} 0.2727\ldots \\ 11\overline{)3.0000\ldots} \\ \underline{-\ 22} \\ 80 \\ \underline{-\ 77} \\ 30 \\ \underline{-\ 22} \\ 80 \\ \underline{-\ 77} \\ 30 \end{array}$$

REVIEW

1. $$\begin{array}{r} 83 \\ 13\overline{)1079} \\ \underline{-104} \\ 39 \\ \underline{-39} \\ 0 \end{array}$$

83 packages

2. a. Minutes practiced each week:

$$\begin{array}{r} \overset{2}{4}5 \\ \underline{\times \quad 5} \\ 225 \end{array}$$

225 minutes

Minutes practiced in a year:

$$\begin{array}{r} \overset{1}{2}\overset{\overset{2}{1}}{2}5 \\ \underline{\times \quad 52} \\ 450 \\ \underline{+11250} \\ 11700 \end{array}$$

11,700 minutes

b. $$\begin{array}{r} 195 \\ 60\overline{)11700} \\ \underline{-60} \\ 570 \\ \underline{-540} \\ 300 \\ \underline{-300} \\ 0 \end{array}$$

195 hours

3. a. 75,300

b. 75,300

c. 80,000

4. $$\begin{array}{r} \overset{1}{2}4 \\ \underline{\times \quad 30} \\ 720 \end{array}$$

720 hours

UNIT 1 | LESSON 14

Exponent Rules: Part 2

WARM-UP

a. $a^3 \bullet a^{12}$

$= a^{3+12}$

$= a^{15}$

b. $x^{17} \div x^9$

$= x^{17-9}$

$= x^8$

PRACTICE

Students do not need to complete every problem. Problems are to be completed until five in a row (vertically, horizontally, or diagonally) are crossed off in the chart. Work for every problem is shown below.

1. $(ab)^5 = a^5b^5$

2. $\left(\frac{a}{b}\right)^{11} = \frac{a^{11}}{b^{11}}$

3. $(3^2 \bullet 2)^2 = (3^2)^2 \bullet 2^2 = 3^4 \bullet 2^2 = 81 \bullet 4 = 324$

4. $(2a^{11})^4 = 2^4(a^{11})^4 = 16a^{44}$

5. $(a^9b^4)^5 = (a^9)^5(b^4)^5 = a^{45}b^{20}$

6. $\left(\frac{2}{-3}\right)^4 = \frac{2^4}{(-3)^4} = \frac{16}{81}$

7. $\left(\frac{b^9}{f^2}\right)^6 = \frac{(b^9)^6}{(f^2)^6} = \frac{b^{54}}{f^{12}}$

8. $(bf)^3 = b^3f^3$

9. $\left(\frac{3}{2^3}\right)^4 = \frac{3^4}{(2^3)^4} = \frac{3^4}{2^{12}} = \frac{81}{4096}$

10. $(4^3 \bullet 3)^2 = (4^3)^2 \bullet 3^2 = 4^6 \bullet 3^2 = 4096 \bullet 9 = 36{,}864$

11. $(f^6)^7 = f^{42}$

12. $\left(\frac{f^5}{b^3}\right)^4 = \frac{(f^5)^4}{(b^3)^4} = \frac{f^{20}}{b^{12}}$

13. $(2^2f^3b^5)^7 = (2^2)^7(f^3)^7(b^5)^7 = 2^{14}f^{21}b^{35} = 16{,}384f^{21}b^{35}$

14. $(2^4)^5 = 2^{20} = 1{,}048{,}576$

15. $\left(\frac{a^9b^3}{f^4}\right)^5 = \frac{(a^9)^5(b^3)^5}{(f^4)^5} = \frac{a^{45}b^{15}}{f^{20}}$

16. $(a^3)^5 = a^{15}$

17. $\left(\frac{2^3}{3^2}\right)^2 = \frac{(2^3)^2}{(3^2)^2} = \frac{2^6}{3^4} = \frac{64}{81}$

18. $(6^2 \bullet 4^2)^2 = (6^2)^2 \bullet (4^2)^2 = 6^4 \bullet 4^4 = 1296 \bullet 256 = 331{,}776$

19. $\left(\frac{4a}{-5}\right)^3 = \frac{4^3a^3}{(-5)^3} = \frac{64a^3}{-125} = -\frac{64a^3}{125}$

20. $\left(\frac{b^5f^2}{a^7}\right)^7 = \frac{\left(b^5\right)^7\left(f^2\right)^7}{\left(a^7\right)^7} = \frac{b^{35}f^{14}}{a^{49}}$

21. $\left(b^5a^4\right)^3 = \left(b^5\right)^3\left(a^4\right)^3 = b^{15}a^{12}$

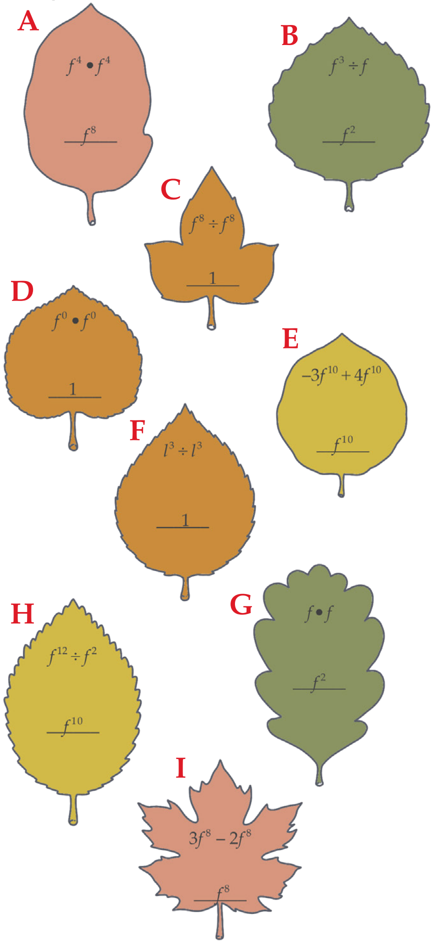
22. $\left(a^5\right)^8 = a^{40}$

23. $\left(\frac{6}{2^4}\right)^2 = \frac{6^2}{\left(2^4\right)^2} = \frac{6^2}{2^8} = \frac{36}{256} = \frac{9}{64}$

24. $\left(2^2 \bullet 3^2\right)^3 = \left(2^2\right)^3 \bullet \left(3^2\right)^3 = 2^6 \bullet 3^6 = 64 \bullet 729 = 46{,}656$

REVIEW

1. Colors will vary. Matching leaves should be in matching colors.



Detailed work for each leaf is shown below.

A $f^4 \bullet f^4 = f^{4+4} = f^8$

B $f^3 \div f = f^{3-1} = f^2$

C $f^8 \div f^8 = f^{8-8} = f^0 = 1$

D $f^0 \bullet f^0 = 1 \bullet 1 = 1$

E $-3f^{10} + 4f^{10} = 1f^{10} = f^{10}$

F $l^3 \div l^3 = l^{3-3} = l^0 = 1$

G $f \bullet f = f^{1+1} = f^2$

H $f^{12} \div f^2 = f^{12-2} = f^{10}$

I $3f^8 - 2f^8 = 1f^8 = f^8$

2. a. $-8^2 = -(8 \bullet 8) = -64$

b. $0.7^2 = 0.7 \bullet 0.7 = 0.49$

c. -1

3.
$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{-2\ 4} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

$3\frac{3}{8} = 3.375$

4. a. $10

b. $4.287 ≈ $4.29

c. $2.82

d. $6.10

Logic Lesson 1

Logic puzzles can be approached in many different ways. The solutions here may not represent all possible methods or answers.

Pie Partners

Amount of leftover pie:

$$\frac{1}{4}+\frac{1}{3}+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+1=3\frac{1}{2}\text{ pies}$$

$$3\frac{1}{2}\div 2=1\frac{3}{4}$$

Each person should get a total of $1\frac{3}{4}$ pies.

More than one solution is possible.
One solution is shown below.
The total pies for each person should be $1\frac{3}{4}$ pies.

Blake: chocolate, blueberry, and apple

Nellie: pumpkin, cherry, and banana cream

Kettle Corn

More than one solution is possible. One solution is shown below.

40 pennies: 40 cents
8 nickels: 40 cents
2 dimes: 20 cents

40 cents + 40 cents + 20 cents = $1

40 pennies, 8 nickels, and 2 dimes

Helpful Hint

The size doubles every 10 minutes, so if it was 100% complete in 50 minutes, it was half the size, or 50% complete, 10 minutes before that (after 40 minutes). It was then half the size, or 25% complete, 10 minutes before that (after 30 minutes).

30 minutes

Humdinger Hayrides

If adding 5 passengers to the smaller amount and taking 5 passengers from the larger amount makes them equal, then the difference between the two amounts is 10. Taking 5 from 25 and adding it to 35 results in 20 passengers and 40 passengers. Forty is twice 20.

Ree: 35 passengers

Piper: 25 passengers

Day Debate

Since only one of the statements is true, list all of the possible days for each statement. The answer is the day of the week that is on only one list because if it appeared on more than one list, both statements would be true. The possible days for each person's statement are listed below:

Hank: Thursday, Friday, Saturday

Joe: Tuesday

Cathy: Wednesday

Olive: Monday, Tuesday, Wednesday, Friday, Saturday

Max: Monday

Monday, Tuesday, Wednesday, Friday, and Saturday are each listed twice, but Thursday is only listed once.

Thursday

Speedy Shucking

For the first 3 minutes, Dan is the only one shucking corn. At 4 ears per minute, he shucks 4 • 3 = 12 ears of corn in 3 minutes. The two shucked 75 ears. 75 – 12 = 63, so there are 63 ears of corn left when Delaney starts shucking. Delaney shucks 5 ears per minute, so together, Dan and Delaney shuck 5 + 4 = 9 ears per minute. That means they'll finish shucking in 63 ÷ 9 = 7 minutes. Dan will have shucked for a total of 10 minutes, so he will have shucked 10 • 4 = 40 ears of corn. Delaney will have shucked 7 • 5 = 35 ears of corn.

Dan: 40 ears of corn

Delaney: 35 ears of corn

Family Farm Assignments

Logic puzzles can be completed in different ways. Information that can be gathered from each clue is shown below.

1. Hank, Joe, and Olive do not give hayrides.
2. Joe is not assigned to the petting zoo.
3. Olive and Cathy do not close on Mondays, Wednesdays, or Fridays. That means one girl closes on Tuesdays and one on Thursdays, so none of the boys close on Tuesdays or Thursdays.
4. Since Olive and Cathy do not close on Fridays, they are not assigned to the pumpkin patch. Olive doesn't give hayrides, so she doesn't close on Thursdays. The only day left for Olive to close is Tuesdays. That means Cathy closes on Thursdays.
5. Since boys were assigned to the petting zoo and the maze, Olive was not assigned to either of those. That leaves concession stands as Olive's assignment and means that concession stands and closing on Tuesday go together.
6. If Joe doesn't close on Fridays, then he isn't assigned to the pumpkin patch. That leaves the maze as the only possible assignment for Joe.
7. The only person left who could be assigned to the pumpkin patch is Hank, and the only assignment left for Max is the petting zoo. That means Hank closes on Fridays, Max closes on Wednesdays, and Joe closes on Mondays.

		Assigned Area					Closing Day				
		Maze	Pumpkin Patch	Concession Stands	Hayrides	Petting Zoo	Monday	Tuesday	Wednesday	Thursday	Friday
Children	Hank	X	✓	X	X	X	X	X	X	X	✓
	Joe	✓	X	X	X	X	✓	X	X	X	X
	Cathy	X	X	X	✓	X	X	X	X	✓	X
	Olive	X	X	✓	X	X	X	✓	X	X	X
	Max	X	X	X	X	✓	X	X	✓	X	X
Closing Day	Monday	✓	X	X	X	X					
	Tuesday	X	X	✓	X	X					
	Wednesday	X	X	X	X	✓					
	Thursday	X	X	X	✓	X					
	Friday	X	✓	X	X	X					

a. Hank is assigned to the pumpkin patch and closes on Fridays.

b. Joe is assigned to the maze and closes on Mondays.

c. Cathy is assigned to the hayrides and closes on Thursdays.

d. Olive is assigned to the concession stands and closes on Tuesdays.

e. Max is assigned to the petting zoo and closes on Wednesdays.

Negative Exponents

WARM-UP

$28 + 14 + 36 = 78$ children

PRACTICE

1.

Power	Fraction with Exponent	Decimal
10^{-7}	$\frac{1}{10^7}$	0.0000001
10^{-3}	$\frac{1}{10^3}$	0.001
10^{-11}	$\frac{1}{10^{11}}$	0.00000000001

2. a. 2^{-7}

$$= \frac{1}{2^7}$$
$$= \frac{1}{2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2}$$
$$= \frac{1}{128}$$

b. 4^{-4}

$$= \frac{1}{4^4}$$
$$= \frac{1}{4 \bullet 4 \bullet 4 \bullet 4}$$
$$= \frac{1}{256}$$

c. 3^{-5}

$$= \frac{1}{3^5}$$
$$= \frac{1}{3 \bullet 3 \bullet 3 \bullet 3 \bullet 3}$$
$$= \frac{1}{243}$$

d. $(-6)^{-3}$

$$= \frac{1}{(-6)^3}$$
$$= \frac{1}{(-6) \bullet (-6) \bullet (-6)}$$
$$= -\frac{1}{216}$$

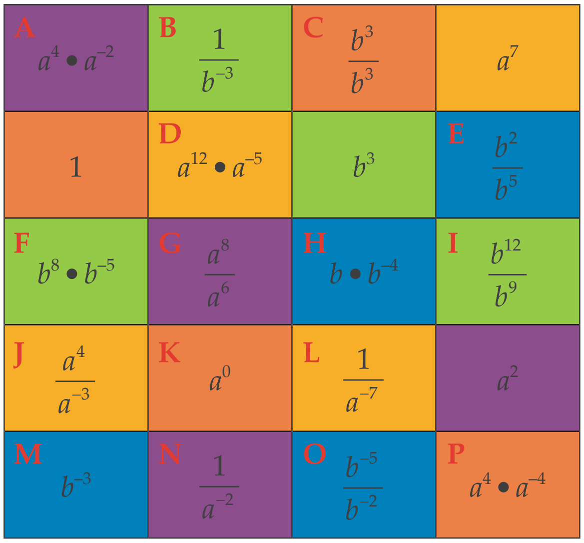
e. $\frac{1}{5^{-3}}$

$$= 5^3$$
$$= 125$$

f. $\frac{1}{(-2)^{-6}}$

$$= (-2)^6$$
$$= 64$$

3.



A $a^4 \bullet a^{-2}$	**B** $\frac{1}{b^{-3}}$	**C** $\frac{b^3}{b^3}$	a^7
1	**D** $a^{12} \bullet a^{-5}$	b^3	**E** $\frac{b^2}{b^5}$
F $b^8 \bullet b^{-5}$	**G** $\frac{a^8}{a^6}$	**H** $b \bullet b^{-4}$	**I** $\frac{b^{12}}{b^9}$
J $\frac{a^4}{a^{-3}}$	**K** a^0	**L** $\frac{1}{a^{-7}}$	a^2
M b^{-3}	**N** $\frac{1}{a^{-2}}$	**O** $\frac{b^{-5}}{b^{-2}}$	**P** $a^4 \bullet a^{-4}$

Detailed work is shown below.

A
$$\begin{aligned} &a^4 \bullet a^{-2} \\ &= a^{4+(-2)} \\ &= a^2 \end{aligned}$$

B
$$\begin{aligned} &\frac{1}{b^{-3}} \\ &= b^3 \end{aligned}$$

C
$$\begin{aligned} &\frac{b^3}{b^3} \\ &= b^{3-3} \\ &= b^0 \\ &= 1 \end{aligned}$$

D
$$\begin{aligned} &a^{12} \bullet a^{-5} \\ &= a^{12+(-5)} \\ &= a^7 \end{aligned}$$

E
$$\begin{aligned} &\frac{b^2}{b^5} \\ &= b^{2-5} \\ &= b^{-3} \\ &= \frac{1}{b^3} \end{aligned}$$

F
$$\begin{aligned} &b^8 \bullet b^{-5} \\ &= b^{8+(-5)} \\ &= b^3 \end{aligned}$$

G
$$\begin{aligned} &\frac{a^8}{a^6} \\ &= a^{8-6} \\ &= a^2 \end{aligned}$$

H
$$\begin{aligned} &b \bullet b^{-4} \\ &= b^{1+(-4)} \\ &= b^{-3} \\ &= \frac{1}{b^3} \end{aligned}$$

I
$$\begin{aligned} &\frac{b^{12}}{b^9} \\ &= b^{12-9} \\ &= b^3 \end{aligned}$$

J
$$\begin{aligned} &\frac{a^4}{a^{-3}} \\ &= a^{4-(-3)} \\ &= a^{4+3} \\ &= a^7 \end{aligned}$$

K $a^0 = 1$

L
$$\begin{aligned} &\frac{1}{a^{-7}} \\ &= a^7 \end{aligned}$$

M
$$\begin{aligned} &b^{-3} \\ &= \frac{1}{b^3} \end{aligned}$$

N
$$\begin{aligned} &\frac{1}{a^{-2}} \\ &= a^2 \end{aligned}$$

O
$$\begin{aligned} &\frac{b^{-5}}{b^{-2}} \\ &= b^{-5-(-2)} \\ &= b^{-5+2} \\ &= b^{-3} \\ &= \frac{1}{b^3} \end{aligned}$$

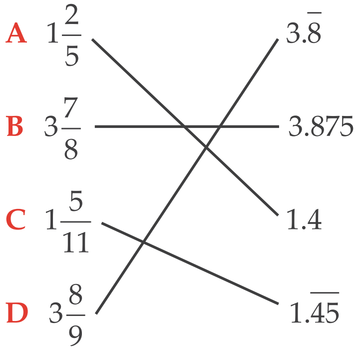
P
$$\begin{aligned} &a^4 \bullet a^{-4} \\ &= a^{4+(-4)} \\ &= a^0 \\ &= 1 \end{aligned}$$

UNIT 1 | LESSON 29

Unit 1 Assessment

1. 5043.82

2.



Detailed work for converting each mixed number to a decimal is shown below.

A $1\frac{2}{5}=1\frac{4}{10}=1.4$

B
$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ -64 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$3\frac{7}{8}=3.875$

C
$$\begin{array}{r} 0.4545\ldots \\ 11\overline{)5.000\ldots} \\ -44 \\ \hline 60 \\ -55 \\ \hline 50 \\ -44 \\ \hline 60 \\ -55 \\ \hline 50 \\ -44 \\ \hline 6 \end{array}$$

$1\frac{5}{11}=1.\overline{45}$

D
$$\begin{array}{r} 0.88\ldots \\ 9\overline{)8.00\ldots} \\ -72 \\ \hline 80 \\ -72 \\ \hline 8 \end{array}$$

$3\frac{8}{9}=3.\overline{8}$

3. $|-15-42|=|-57|=57$

4. a. $5\frac{7}{8}-2\frac{1}{4}$

$=5\frac{7}{8}-2\frac{2}{8}$

$=3\frac{5}{8}$

b.
$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ \ } \\ 7.485 \\ +\ 2.956 \\ \hline 10.441 \end{array}$$

c. $-\frac{8}{5}\bullet 1\frac{3}{4}$

$=-\frac{8}{5}\bullet\frac{7}{4}$

$=-\frac{14}{5}=-2\frac{4}{5}$

d.
$$\begin{array}{r} {\scriptstyle \overset{1}{\cancel{4}}\ 2\ \ } \\ 4.53 \\ \times\ \ 3.8 \\ \hline 3624 \\ +\ 13590 \\ \hline 17214 \end{array}$$

$-3.8\bullet(-4.53)=17.214$

e.
$$\begin{array}{r} 70 \\ 6\overline{)420} \\ -42 \\ \hline 00 \\ -0 \\ \hline 0 \end{array}$$

$-4.2\div 0.06=-70$

f. $$\frac{3\frac{2}{5}}{7\frac{1}{2}}$$

$$= 3\frac{2}{5} \div 7\frac{1}{2}$$

$$= \frac{17}{5} \div \frac{15}{2}$$

$$= \frac{17}{5} \bullet \frac{2}{15}$$

$$= \frac{34}{75}$$

5. a. $536 + 225$

b. $52 \bullet (-43)$

6. a. $b = 123 - 54 = 69$

b. $r = 45 \div (-15) = -3$

7. a. $$\left(\frac{2}{5}\right)^3$$

$$= \frac{2}{5} \bullet \frac{2}{5} \bullet \frac{2}{5}$$

$$= \frac{8}{125}$$

b. -2^5

$$= -(2 \bullet 2 \bullet 2 \bullet 2 \bullet 2)$$

$$= -32$$

c. 1

8. $$\begin{array}{r|l} 2 & \underline{6300} \\ 2 & \underline{3150} \\ 3 & \underline{1575} \\ 3 & \underline{525} \\ 5 & \underline{175} \\ 5 & \underline{35} \\ & 7 \end{array}$$

$6300 = 2^2 \bullet 3^2 \bullet 5^2 \bullet 7$

9. a. $$\frac{2\left(54 - 4^3\right) + 18}{29 - 18 \bullet 3}$$

$$= \frac{2(54 - 64) + 18}{29 - 54}$$

$$= \frac{2(-10) + 18}{-25}$$

$$= \frac{-20 + 18}{-25}$$

$$= \frac{-2}{-25} = \frac{2}{25}$$

b. $75 \bullet 2 \div 5 - 3^3 + (48 - 7)$

$$= 75 \bullet 2 \div 5 - 3^3 + 41$$

$$= 75 \bullet 2 \div 5 - 27 + 41$$

$$= 150 \div 5 - 27 + 41$$

$$= 30 - 27 + 41$$

$$= 3 + 41$$

$$= 44$$

10. $7q + 3q^2 - 4q + 8q^2 - 4q^2$

$$= 7q - 4q + 3q^2 + 8q^2 - 4q^2$$

$$= 3q + 7q^2$$

11. a. $3^2 \bullet 3^5 = 3^{2+5} = 3^7$

b. $a^{15} \div a^8 = a^{15-8} = a^7$

c. $7^{13} \div 7^{11} = 7^{13-11} = 7^2$

d. $z^5 \bullet z^{16} = z^{5+16} = z^{21}$

12. a. $\left(c^2ba^3\right)^5$

$$= \left(c^2\right)^5 b^5 \left(a^3\right)^5$$

$$= c^{10}b^5a^{15}$$

b. $$\left(\frac{r^7}{ts^3}\right)^2$$

$$= \frac{\left(r^7\right)^2}{\left(ts^3\right)^2}$$

$$= \frac{r^{14}}{t^2\left(s^3\right)^2}$$

$$= \frac{r^{14}}{t^2s^6}$$

Modeling Real-World Situations with Equations

WARM-UP

a.
$$\begin{aligned} 3x-14&=7\\ 3x-14+14&=7+14\\ 3x&=21\\ \frac{3x}{3}&=\frac{21}{3}\\ x&=7 \end{aligned}$$

b.
$$\begin{aligned} 5(x+6)&=40\\ \frac{5(x+6)}{5}&=\frac{40}{5}\\ x+6&=8\\ x+6-6&=8-6\\ x&=2 \end{aligned}$$

PRACTICE

1. a. $36d-8$

   b. $44d-10$

   c.
$$\begin{aligned} 44d-10&=36d-8\\ 44d-10-36d&=36d-8-36d\\ 8d-10&=-8\\ 8d-10+10&=-8+10\\ 8d&=2\\ \frac{8d}{8}&=\frac{2}{8}\\ d&=\frac{1}{4} \end{aligned}$$

   $0.25 per cup

2. a. $6b+12$

   b. $8b-4$

   c.
$$\begin{aligned} 6b+12&=8b-4\\ 6b+12-6b&=8b-4-6b\\ 12&=2b-4\\ 12+4&=2b-4+4\\ 16&=2b\\ \frac{16}{2}&=\frac{2b}{2}\\ 8&=b \end{aligned}$$

   8 bottles per package

3. a. $p+3$

   b. $3(p+3)=3p+9$

   c. $p+21$

   d.
$$\begin{aligned} 3p+9&=p+21\\ 3p+9-p&=p+21-p\\ 2p+9&=21\\ 2p+9-9&=21-9\\ 2p&=12\\ \frac{2p}{2}&=\frac{12}{2}\\ p&=6 \end{aligned}$$

   $6 per pair

4. a. $7(6p+2.5)=42p+17.5$

   b.
$$\begin{aligned} &55(p-0.34)+10.98\\ &=55p-18.7+10.98\\ &=55p-7.72 \end{aligned}$$

c.

$$\begin{aligned}42p+17.5&=55p-7.72\\42p+17.5-42p&=55p-7.72-42p\\17.5&=13p-7.72\\17.5+7.72&=13p-7.72+7.72\\25.22&=13p\\\frac{25.22}{13}&=\frac{13p}{13}\\1.94&=p\\p&=1.94\end{aligned}$$

$1.94 per pastry

REVIEW

1. $\left(4\sqrt{18}\bullet 11\right)\bullet\sqrt{2}$

$$\begin{aligned}&=44\sqrt{18}\bullet\sqrt{2}\\&=44\sqrt{18\bullet 2}\\&=44\sqrt{36}\\&=44\bullet 6\\&=264\end{aligned}$$

2. a. $8.2+9.5=17.7$

$$\begin{aligned}&\left(8.2\times10^{-13}\right)+\left(9.5\times10^{-13}\right)\\&=17.7\times10^{-13}\\&=1.77\times10^{-12}\end{aligned}$$

b. $3.7\times10^{7}=37\times10^{6}$

$37-6.01=30.99$

$$\begin{aligned}&\left(3.7\times10^{7}\right)-\left(6.01\times10^{6}\right)\\&=30.99\times10^{6}\\&=3.099\times10^{7}\end{aligned}$$

3. a. $\left(\frac{c^9}{5^2}\right)^4$

$$\begin{aligned}&=\frac{\left(c^9\right)^4}{\left(5^2\right)^4}\\&=\frac{c^{36}}{5^8}\\&=\frac{c^{36}}{390{,}625}\end{aligned}$$

b. $\left(b^{12}e^{11}\right)^{11}$

$$\begin{aligned}&=\left(b^{12}\right)^{11}\left(e^{11}\right)^{11}\\&=b^{132}e^{121}\end{aligned}$$

4. a. $\frac{9}{60}=\frac{3}{20}$

b. $\frac{3}{20}=\frac{15}{100}$

0.15

c. $\frac{7}{25}=\frac{28}{100}$

0.28

UNIT 2 | LESSON 34

The Coordinate Plane

WARM-UP

$$F=\frac{9}{5}C+32$$

$$F-32=\frac{9}{5}C+32-32$$

$$F-32=\frac{9}{5}C$$

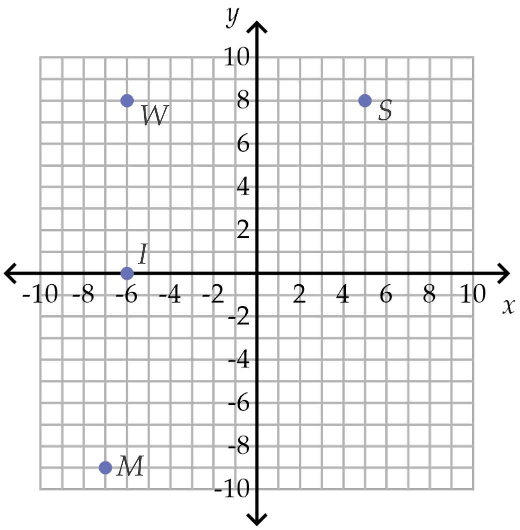
$$\frac{5}{9}\bullet\left(F-32\right)=\frac{9}{5}C\bullet\frac{5}{9}$$

$$\frac{5}{9}\left(F-32\right)=C$$

PRACTICE

1. F: $\left(-3,8\right)$ Quadrant: II
   R: $\left(0,7\right)$ Quadrant: N/A
   O: $\left(6,-4\right)$ Quadrant: IV
   G: $\left(-9,-5\right)$ Quadrant: III

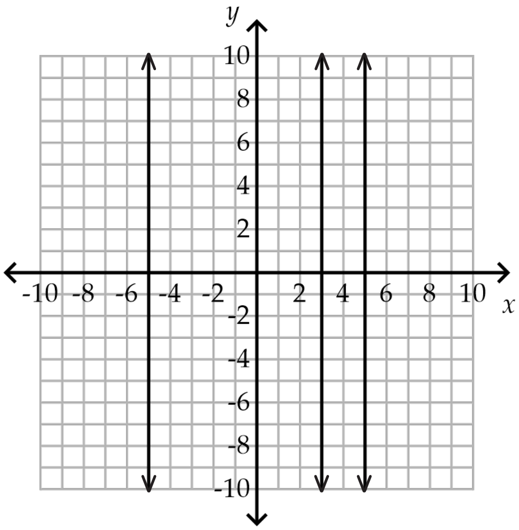
2.



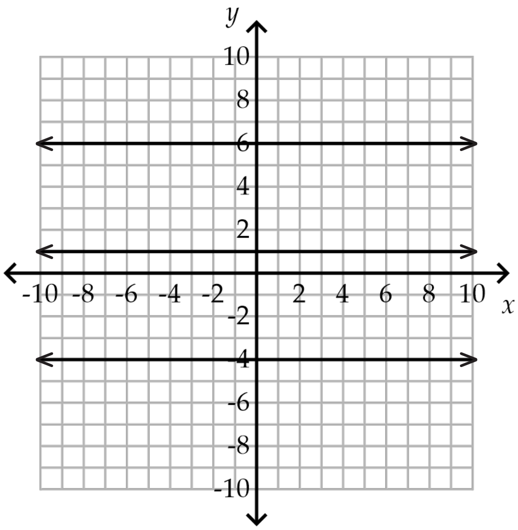
3. a. x
   b. y
   c. axis
   d. $\left(0,0\right)$

4. a. $x=-3$
   b. $y=-9$

5.



6.



REVIEW

1.

$$D = \frac{m}{V}$$

$$V \bullet D = \frac{m}{V} \bullet V$$

$$VD = m$$

$$\frac{VD}{D} = \frac{m}{D}$$

$$V = \frac{m}{D}$$

2.

$$\frac{\frac{2}{3}(-9r-60)}{2} = -2$$

$$2 \bullet \frac{\frac{2}{3}(-9r-60)}{2} = -2 \bullet 2$$

$$\frac{2}{3}(-9r-60) = -4$$

$$-6r - 40 = -4$$

$$-6r - 40 + 40 = -4 + 40$$

$$-6r = 36$$

$$\frac{-6r}{-6} = \frac{36}{-6}$$

$$r = -6$$

3. a.

2⎣154	2⎣330
7⎣77	3⎣165
11	5⎣55
	11

$154 = 2 \bullet 7 \bullet 11$

$330 = 2 \bullet 3 \bullet 5 \bullet 11$

GCF: $2 \bullet 11 = 22$

b. $$\frac{154 \div 22}{330 \div 22} = \frac{7}{15}$$

4. 5,879,000,000,000

5. 9 AM to 3 PM is 6 hours.

$$6 \div \frac{3}{4}$$

$$= 6 \bullet \frac{4}{3}$$

$$= 8$$

8 classes

Slope-Intercept Form

WARM-UP

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3-(-6)}{4-(-3)}$$

$$= \frac{9}{7}$$

PRACTICE

1. a. x-intercept: $(4,0)$ y-intercept: $(0,-2)$

   b. x-intercept: $(1,0)$ y-intercept: $(0,5)$

2. a. $m = 1$

   y-intercept: $\left(0, -\frac{1}{2}\right)$ $b = -\frac{1}{2}$

   b. $m = \frac{5}{4}$

   y-intercept: $(0,3)$ $b = 3$

   c. $m = 3$

   y-intercept: $(0,4)$ $b = 4$

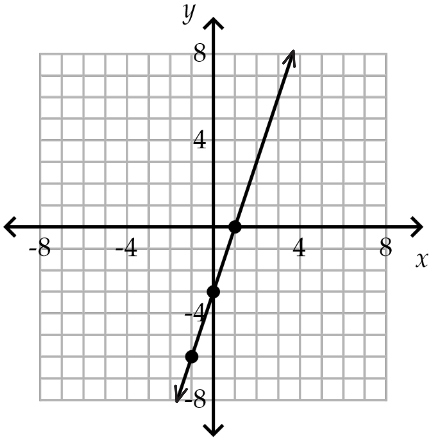
   d. $m = -5$

   y-intercept: $(0,-2)$ $b = -2$

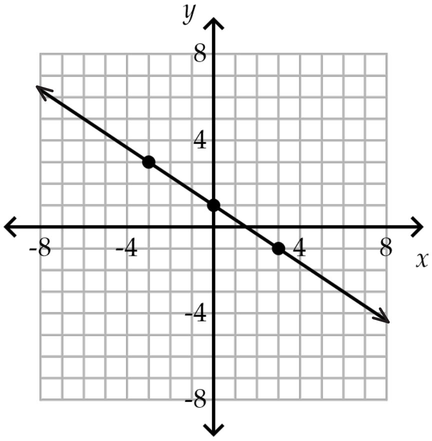
   e. $m = 1$

   y-intercept: $(0,0)$ $b = 0$

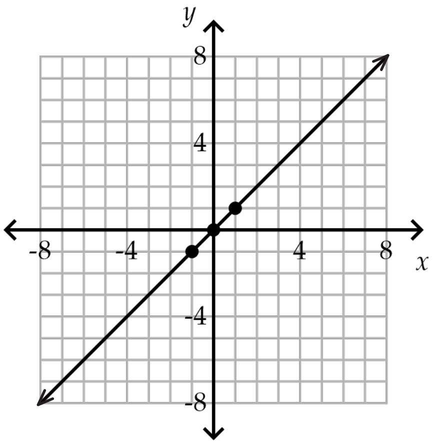
3. a.



b.



c.



4. a. $m=\frac{4}{5}$ $b=-3$

Equation: $y=\frac{4}{5}x-3$

b. $m=-\frac{2}{7}$ $b=0$

Equation: $y=-\frac{2}{7}x$

5. a. $y=3x+4$ [✓ $y=-3x-4$]

$y=-3x+4$ $y=3x-4$

b. [✓ $y=x+2$] $y=-x-2$

$y=-x+2$ $y=x-2$

REVIEW

1. Rule: Multiply the input by $\frac{1}{10}$, or divide the input by 10.

Equation: $y=\frac{1}{10}x$ or $y=x\div 10$

2. $$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{-2-14}{-17-(-49)} \\ &= \frac{-16}{32} \\ &= -\frac{1}{2} \end{aligned}$$

3. $$\begin{aligned} &\left(4.888\times10^{21}\right)\div\left(9.4\times10^{11}\right) \\ &= \frac{4.888\times10^{21}}{9.4\times10^{11}} \\ &= \frac{4.888}{9.4}\times\frac{10^{21}}{10^{11}} \\ &= 0.52\times10^{21-11} \\ &= 0.52\times10^{10} \\ &= 5.2\times10^{9} \end{aligned}$$

4. a. $\sqrt{181}$ is between the whole numbers 13 and 14 but is closer to 13.

b. $\sqrt{181}\approx 13.45$

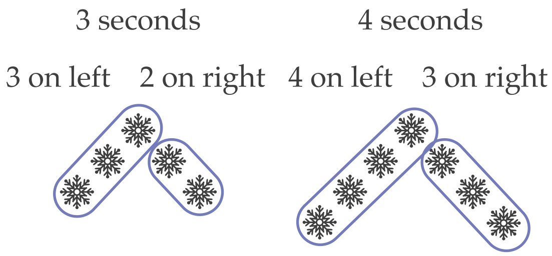
5. $0.555\ldots\approx 0.56$

Logic Lesson 2

Logic puzzles can be approached in many different ways. The solutions here may not represent all possible methods or answers.

1. One approach is to look at the snowflakes as accumulating on the left side and right side of the pile, with the top snowflake counted on the left side.

As shown below, the number of snowflakes on the left side (including the top snowflake) is the same as the number of seconds, and the number of snowflakes on the right side is one less than the number of seconds.



a. 5 seconds: $5+4=9$ snowflakes

b. 25 seconds: $25+24=49$ snowflakes

c. 100 seconds: $100+99=199$ snowflakes

d. x seconds: $x+(x-1)=2x-1$ snowflakes

2. Fran and her father should take path D.

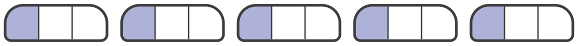
If Path A is correct, then both Mr. Jones's and Mr. Huber's statements are correct. Therefore, Path A cannot be correct.

If Path B is correct, then both Mrs. Cunningham's and Mr. Huber's statements are correct. Therefore, Path B cannot be correct.

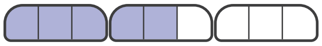
If Path C is correct, then Mrs. Cunningham, Mrs. Smith, and Mr. Huber all gave correct information. Therefore, Path C cannot be correct.

If Path D is correct, then only Mr. Jones gave correct information.

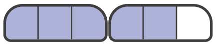
3. The man with 2 loaves of cornbread should receive 1 coin(s), and the man with 3 loaves should receive 4 coin(s).



Each person received 5 thirds.



Traveler A had 3 loaves. If he kept 5 thirds, he shared 4 thirds.



Traveler B had 2 loaves. If he kept 5 thirds, he shared 1 third.

Traveler A shared 4 times as much as Traveler B. Therefore, he should receive 4 times as many coins. $1 \bullet 4 = 4$, and $1+4=5$.

4. a. The sum of the digits 1–9 is 45. Since there are three rows, the sum of each row must be $45 \div 3$, which is 15. The same is true for each column and each diagonal. More than one solution is possible as long as the sum of each row, column, and diagonal is 15. An example solution and a possible approach are given.

2	7	6
9	5	1
4	3	8

One approach is to list the ways to get a sum of 15 using exactly three of the numbers between 1 and 9. These include:

$1+5+9$
$2+5+8$
$3+5+7$
$1+6+8$
$2+6+7$
$4+5+6$
$2+4+9$
$3+4+8$

Since 5 appears in four of the ways, it must go in the middle because the middle square is included in four different sums. Since 2, 4, 6, and 8 each appear in three solutions, they must go in the corners because each corner is included in three different sums. Since 5 is already in the center, 2 and 8 must go in opposite corners to make 10, and 6 and 4 must also go in opposite corners to make 10. The remaining squares can be filled in with the numbers that result in a sum of 15.

b. $\boxed{3} \bullet 2 - \sqrt[3]{\boxed{8}} = 4$

$\boxed{7} - \sqrt{3\boxed{6}} = \boxed{1}$

$\boxed{4} + \sqrt{\boxed{9}} - \boxed{2} = \boxed{5}$

First Equation:

The only perfect cubes from 1 to 9 are 1 and 8. If the number 1 is under the cube root, then 2 times a number minus 1 must equal 4. Since 5 minus 4 is 1, this cannot work because 2 times any remaining number is not 5. Therefore, the number under the cube root is 8. The cube root of 8 is 2, so the number in the other box must be 3 because 6 minus 2 is 4.

Second Equation:

The only perfect square in the thirties is 36, so a 6 must go under the square root. The remaining numbers are 1, 2, 4, 5, 7, and 9. Since the square root of 36 is 6, one remaining number minus 6 has to equal another remaining number. The only remaining numbers that have a difference of 6 are 7 and 1.

Third Equation:

Now the remaining numbers are 2, 4, 5, and 9. The only perfect squares in this list are 4 and 9. Try 9 first. The square root of 9 is 3. The only way to make a true statement is to add 4 and 3 to get 7 and subtract 2 to get 5.

UNIT 2 | LESSON 50

Solving Equations with Radicals

WARM-UP

a. 12

b. 5

c. 20

PRACTICE

A $\sqrt{x+3}=6$ $x=33$	**B** $2\sqrt{t}=14$ $t=49$	**C** $\sqrt{r+10}=4$ $r=6$	**D** $\sqrt{a}+10=4$ $a=$ no solution
E $3\sqrt{g-1}=21$ $g=50$	**F** $5\sqrt[3]{w-7}=-5$ $w=6$	**G** $\frac{\sqrt{b+15}}{2}=4$ $b=49$	**H** $\sqrt[3]{7-4f}=-5$ $f=33$
I $\frac{5+2\sqrt{m}}{3}=1$ $m=$ no solution	**J** $\sqrt[3]{2d+5}=3$ $d=11$	**K** $5-\sqrt[3]{3-z}=7$ $z=11$	**L** $10-\sqrt{2h}=0$ $h=50$

A

$$\sqrt{x+3}=6$$
$$\left(\sqrt{x+3}\right)^2=6^2$$
$$x+3=36$$
$$x+3-3=36-3$$
$$x=33$$

Check:

$$\sqrt{33+3}\overset{?}{=}6$$
$$\sqrt{36}\overset{?}{=}6$$
$$6=6\ ✓$$

B

$$2\sqrt{t}=14$$
$$\frac{2\sqrt{t}}{2}=\frac{14}{2}$$
$$\sqrt{t}=7$$
$$\left(\sqrt{t}\right)^2=7^2$$
$$t=49$$

Check:

$$2\sqrt{49}\overset{?}{=}14$$
$$2\bullet 7\overset{?}{=}14$$
$$14=14\ ✓$$

C

$$\sqrt{r+10}=4$$
$$\left(\sqrt{r+10}\right)^2=4^2$$
$$r+10=16$$
$$r+10-10=16-10$$
$$r=6$$

Check:

$$\sqrt{6+10}\overset{?}{=}4$$
$$\sqrt{16}\overset{?}{=}4$$
$$4=4\ ✓$$

D

$$\sqrt{a}+10=4$$
$$\sqrt{a}+10-10=4-10$$
$$\sqrt{a}=-6$$
$$\left(\sqrt{a}\right)^2=(-6)^2$$
$$a=36$$

Check:

$$\sqrt{36}+10\overset{?}{=}4$$
$$6+10\overset{?}{=}4$$
$$16\neq 4$$

no solution

E

$$3\sqrt{g-1}=21$$
$$\frac{3\sqrt{g-1}}{3}=\frac{21}{3}$$
$$\sqrt{g-1}=7$$
$$\left(\sqrt{g-1}\right)^2=7^2$$
$$g-1=49$$
$$g-1+1=49+1$$
$$g=50$$

Check:

$$3\sqrt{50-1}\overset{?}{=}21$$
$$3\sqrt{49}\overset{?}{=}21$$
$$3\bullet 7\overset{?}{=}21$$
$$21=21\ ✓$$

F

$$5\sqrt[3]{w-7}=-5$$
$$\frac{5\sqrt[3]{w-7}}{5}=\frac{-5}{5}$$
$$\sqrt[3]{w-7}=-1$$
$$\left(\sqrt[3]{w-7}\right)^3=(-1)^3$$
$$w-7=-1$$
$$w-7+7=-1+7$$
$$w=6$$

Check:

$$5\sqrt[3]{6-7}\overset{?}{=}-5$$
$$5\sqrt[3]{-1}\overset{?}{=}-5$$
$$5(-1)\overset{?}{=}-5$$
$$-5=-5\ \checkmark$$

G

$$\frac{\sqrt{b+15}}{2}=4$$
$$2\bullet\frac{\sqrt{b+15}}{2}=4\bullet 2$$
$$\sqrt{b+15}=8$$
$$\left(\sqrt{b+15}\right)^2=8^2$$
$$b+15=64$$
$$b+15-15=64-15$$
$$b=49$$

Check:

$$\frac{\sqrt{49+15}}{2}\overset{?}{=}4$$
$$\frac{\sqrt{64}}{2}\overset{?}{=}4$$
$$\frac{8}{2}\overset{?}{=}4$$
$$4=4\ \checkmark$$

H

$$\sqrt[3]{7-4f}=-5$$
$$\left(\sqrt[3]{7-4f}\right)^3=(-5)^3$$
$$7-4f=-125$$
$$7-4f-7=-125-7$$
$$-4f=-132$$
$$\frac{-4f}{-4}=\frac{-132}{-4}$$
$$f=33$$

Check:

$$\sqrt[3]{7-4(33)}\overset{?}{=}-5$$
$$\sqrt[3]{7-132}\overset{?}{=}-5$$
$$\sqrt[3]{-125}\overset{?}{=}-5$$
$$-5=-5\ \checkmark$$

I

$$\frac{5+2\sqrt{m}}{3}=1$$
$$3\bullet\frac{5+2\sqrt{m}}{3}=1\bullet 3$$
$$5+2\sqrt{m}=3$$
$$5+2\sqrt{m}-5=3-5$$
$$2\sqrt{m}=-2$$
$$\frac{2\sqrt{m}}{2}=\frac{-2}{2}$$
$$\sqrt{m}=-1$$
$$\sqrt{m}^{2}=(-1)^2$$
$$m=1$$

Check:

$$\frac{5+2\sqrt{1}}{3}\overset{?}{=}1$$
$$\frac{5+2\bullet 1}{3}\overset{?}{=}1$$
$$\frac{5+2}{3}\overset{?}{=}1$$
$$\frac{7}{3}\neq 1$$

no solution

J

$$\sqrt[3]{2d+5}=3$$
$$\left(\sqrt[3]{2d+5}\right)^3=3^3$$
$$2d+5=27$$
$$2d+5-5=27-5$$
$$2d=22$$
$$\frac{2d}{2}=\frac{22}{2}$$
$$d=11$$

Check:

$$\sqrt[3]{2\bullet 11+5}\overset{?}{=}3$$
$$\sqrt[3]{22+5}\overset{?}{=}3$$
$$\sqrt[3]{27}\overset{?}{=}3$$
$$3=3\ \checkmark$$

K

$$5-\sqrt[3]{3-z}=7$$
$$5-\sqrt[3]{3-z}-5=7-5$$
$$-\sqrt[3]{3-z}=2$$
$$\frac{-\sqrt[3]{3-z}}{-1}=\frac{2}{-1}$$
$$\sqrt[3]{3-z}=-2$$
$$(3-z)^3=(-2)^3$$
$$3-z=-8$$
$$3-z-3=-8-3$$
$$-z=-11$$
$$\frac{-z}{-1}=\frac{-11}{-1}$$
$$z=11$$

Check:

$$5-\sqrt[3]{3-11}\overset{?}{=}7$$
$$5-\sqrt[3]{-8}\overset{?}{=}7$$
$$5-(-2)\overset{?}{=}7$$
$$7=7\ \checkmark$$

L

$$10-\sqrt{2h}=0$$
$$10-\sqrt{2h}-10=0-10$$
$$-\sqrt{2h}=-10$$
$$\frac{-\sqrt{2h}}{-1}=\frac{-10}{-1}$$
$$\sqrt{2h}=10$$
$$\left(\sqrt{2h}\right)^2=10^2$$
$$2h=100$$
$$\frac{2h}{2}=\frac{100}{2}$$
$$h=50$$

Check:

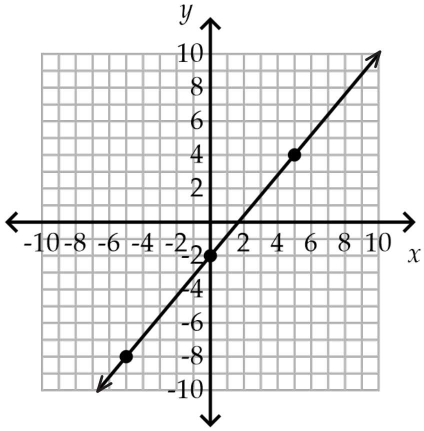
$$10-\sqrt{2\bullet 50}\overset{?}{=}0$$
$$10-\sqrt{100}\overset{?}{=}0$$
$$10-10\overset{?}{=}0$$
$$0=0\ \checkmark$$

REVIEW

1. a.
$$\begin{aligned} -6x+5y&=-10\\ -6x+5y+6x&=-10+6x\\ 5y&=-10+6x\\ \frac{5y}{5}&=\frac{-10}{5}+\frac{6x}{5}\\ y&=\frac{6}{5}x-2 \end{aligned}$$

Slope-intercept form: $y=\frac{6}{5}x-2$

Slope: $\frac{6}{5}$ y-intercept: $(0,-2)$

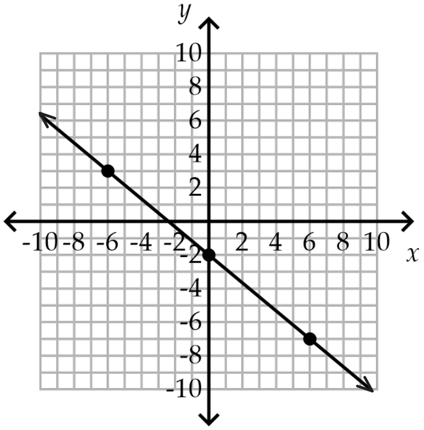


b.
$$\begin{aligned} m&=\frac{3-(-12)}{-6-12}\\ &=\frac{15}{-18}\\ &=-\frac{5}{6} \end{aligned}$$

$$\begin{aligned} y&=mx+b\\ -12&=-\frac{5}{6}(12)+b\\ -12&=-10+b\\ -12+10&=-10+b+10\\ -2&=b \end{aligned}$$

Equation: $y=-\frac{5}{6}x-2$

Slope: $-\frac{5}{6}$ y-intercept: $(0,-2)$



c. The lines are perpendicular because their slopes are opposite reciprocals.

2. 1×10^{-9}

3. a. $96\div8=12$

$12\bullet5=60$

b. $108\div12=9$

$9\bullet7=63$

c. $330\div11=30$

$30\bullet5=150$

d. $250\div50=5$

$5\bullet9=45$

Unit 2 Review

1. $2700 + 726t = 5400$

2. a.

$$\begin{aligned} 3x+1 &= 9-x \\ 3x+1+x &= 9-x+x \\ 4x+1 &= 9 \\ 4x+1-1 &= 9-1 \\ 4x &= 8 \\ \frac{4x}{4} &= \frac{8}{4} \\ x &= 2 \end{aligned}$$

b.

$$\begin{aligned} \frac{(7+19x)}{5} &= 3+4x \\ 5\bullet\frac{(7+19x)}{5} &= (3+4x)\bullet 5 \\ 7+19x &= 15+20x \\ 7+19x-19x &= 15+20x-19x \\ 7 &= 15+x \\ 7-15 &= 15+x-15 \\ -8 &= x \end{aligned}$$

3.

$$\begin{aligned} P &= \frac{3}{2}A+5 \\ P-5 &= \frac{3}{2}A+5-5 \\ P-5 &= \frac{3}{2}A \\ \frac{2}{3}\bullet(P-5) &= \frac{3}{2}A\bullet\frac{2}{3} \\ \frac{2}{3}(P-5) &= A \end{aligned}$$

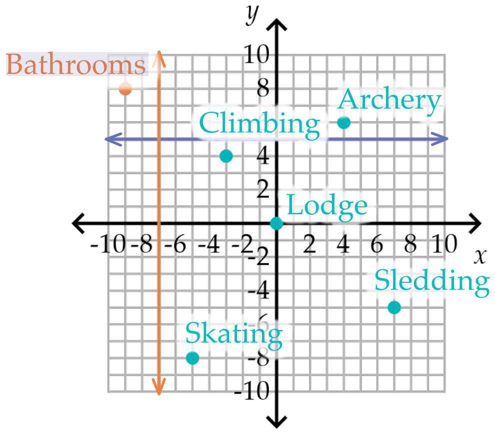
4. a. Archery: I Climbing: II

Sledding: IV Skating: III

b. Skating: $(-5,-8)$ Lodge: $(0,0)$

c. Purple: $y=5$ Orange: $x=-7$

d.



5. a. Domain: $\{13, 14, 15\}$

Range: {Cabin 1, Cabin 2, Cabin 3}

b. no

The input of 13 has two outputs.

c. Independent variable: age

Dependent variable: cabin number

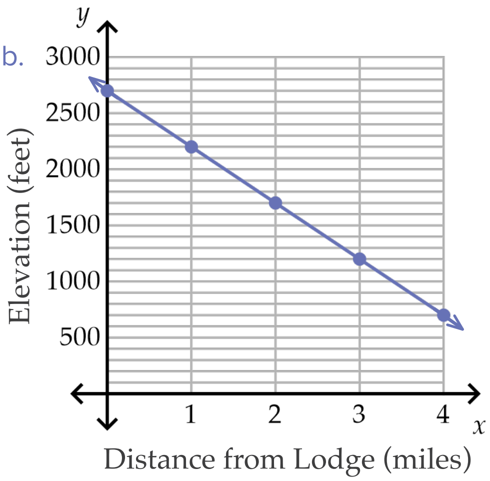
6. Rule: Multiply the input by $\frac{2}{5}$.

Equation: $y = \frac{2}{5}x$

7. a.

x	y	
0	2700	$-500 \bullet 0 + 2700 = 0 + 2700 = 2700$
1	2200	$-500 \bullet 1 + 2700 = -500 + 2700 = 2200$
2	1700	$-500 \bullet 2 + 2700 = -1000 + 2700 = 1700$
3	1200	$-500 \bullet 3 + 2700 = -1500 + 2700 = 1200$
4	700	$-500 \bullet 4 + 2700 = -2000 + 2700 = 700$

b.



8. a. Function? yes

Linear? yes

Proportional? yes

b. Function? yes

Linear? no

Proportional? no

9. [✓ $y=3x+5$] $y=x^2-1$ [✓ $1+y=\frac{1}{2}x$]

[✓ $2x-5y=4$] $2x^2-5y^2=4$ $y=|x|$

10. a.
$$\begin{aligned} m&=\frac{y_2-y_1}{x_2-x_1}\\ &=\frac{5-7}{7-1}\\ &=\frac{-2}{6}\\ &=-\frac{1}{3} \end{aligned}$$

b.
$$\begin{aligned} y&=mx+b\\ 7&=-\frac{1}{3}(1)+b\\ 7&=-\frac{1}{3}+b\\ 7+\frac{1}{3}&=-\frac{1}{3}+b+\frac{1}{3}\\ 7\frac{1}{3}&=b \end{aligned}$$

$$y=-\frac{1}{3}x+7\frac{1}{3}$$

11. a. $m=\frac{\text{rise}}{\text{run}}=\frac{40}{5}=8$

b. $(0,30)$

c. $y=8x+30$

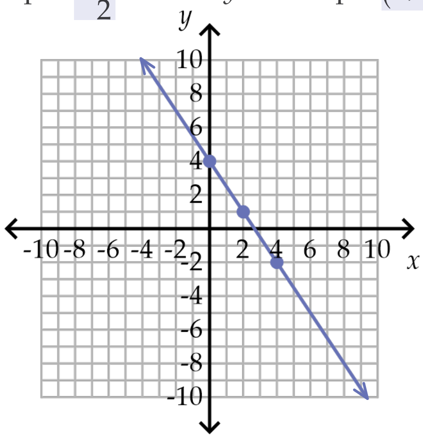
12. a. $y-(-3)=5(x-4)$

$y+3=5(x-4)$

b. $y-(-3)=-\frac{1}{5}(x-4)$

$y+3=-\frac{1}{5}(x-4)$

13. Slope: $-\frac{3}{2}$ y-intercept: $(0,4)$



14. a.
$$\begin{aligned} 3\sqrt{a}+1&=13\\ 3\sqrt{a}+1-1&=13-1\\ 3\sqrt{a}&=12\\ \frac{3\sqrt{a}}{3}&=\frac{12}{3}\\ \sqrt{a}&=4\\ \left(\sqrt{a}\right)^2&=4^2\\ a&=16 \end{aligned}$$

Check:
$$\begin{aligned} 3\sqrt{16}+1&\overset{?}{=}13\\ 3(4)+1&\overset{?}{=}13\\ 12+1&\overset{?}{=}13\\ 13&=13\ \checkmark \end{aligned}$$

b.
$$\begin{aligned} 1-\sqrt[3]{b}&=-3\\ 1-\sqrt[3]{b}-1&=-3-1\\ -\sqrt[3]{b}&=-4\\ \frac{-\sqrt[3]{b}}{-1}&=\frac{-4}{-1}\\ \sqrt[3]{b}&=4\\ \left(\sqrt[3]{b}\right)^3&=4^3\\ b&=64 \end{aligned}$$

Check:
$$\begin{aligned} 1-\sqrt[3]{64}&\overset{?}{=}-3\\ 1-4&\overset{?}{=}-3\\ -3&=-3\ \checkmark \end{aligned}$$

c.
$$\begin{aligned} (2c-4)^2&=100\\ \sqrt{(2c-4)^2}&=\pm\sqrt{100}\\ 2c-4&=\pm10 \end{aligned}$$

$\swarrow$ $\searrow$

$$\begin{aligned} 2c-4&=10\\ 2c-4+4&=10+4\\ 2c&=14\\ \frac{2c}{2}&=\frac{14}{2}\\ c&=7 \end{aligned}$$

$$\begin{aligned} 2c-4&=-10\\ 2c-4+4&=-10+4\\ 2c&=-6\\ \frac{2c}{2}&=\frac{-6}{2}\\ c&=-3 \end{aligned}$$

The number of snowballs Clarice made cannot be negative.

Unit 2 Assessment

1. a. $$\begin{aligned} 3x+18-2x &= 9x-30 \\ x+18 &= 9x-30 \\ x+18-x &= 9x-30-x \\ 18 &= 8x-30 \\ 18+30 &= 8x-30+30 \\ 48 &= 8x \\ \frac{48}{8} &= \frac{8x}{8} \\ 6 &= x \end{aligned}$$

b. $$\begin{aligned} 5(a-12) &= 3a+25 \\ 5a-60 &= 3a+25 \\ 5a-60-3a &= 3a+25-3a \\ 2a-60 &= 25 \\ 2a-60+60 &= 25+60 \\ 2a &= 85 \\ \frac{2a}{2} &= \frac{85}{2} \\ a &= 42.5 \end{aligned}$$

c. $$\begin{aligned} \frac{36-p}{4} &= p-1 \\ 4\bullet\frac{36-p}{4} &= (p-1)\bullet 4 \\ 36-p &= 4p-4 \\ 36-p+p &= 4p-4+p \\ 36 &= 5p-4 \\ 36+4 &= 5p-4+4 \\ 40 &= 5p \\ \frac{40}{5} &= \frac{5p}{5} \\ 8 &= p \end{aligned}$$

2. a. $2m+10$

b. $8m-2$

c. $$\begin{aligned} 2m+10 &= 8m-2 \\ 2m+10-2m &= 8m-2-2m \\ 10 &= 6m-2 \\ 10+2 &= 6m-2+2 \\ 12 &= 6m \\ \frac{12}{6} &= \frac{6m}{6} \\ 2 &= m \end{aligned}$$

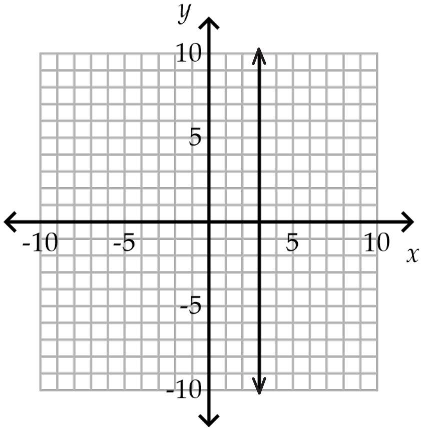
$2 per extra chore

3. a. $$\begin{aligned} A &= lw \\ \frac{A}{l} &= \frac{lw}{l} \\ \frac{A}{l} &= w \end{aligned}$$

b. $$\begin{aligned} w &= \frac{A}{l} \\ w &= \frac{27}{9} \\ w &= 3 \end{aligned}$$

3 ft

4.

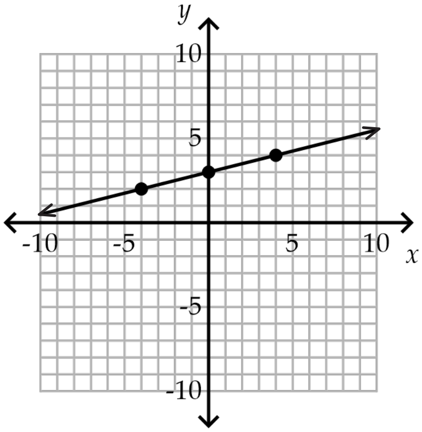


5. $\{(0,a), (0,f), (1,b), (2,d), (3,c), (4,b), (5,f)\}$

6. Rule: Multiply the input by $\frac{1}{2}$.

Equation: $y=\frac{1}{2}x$

7.



linear

8. a.

Change in x	x	y	Change in y
	–5	–2.5	
+3			+1.5
	–2	–1	
+3			+1.5
	1	0.5	
+3			+1.5
	4	2	
+3			+1.5
	7	3.5	

b. yes

c. yes

9.

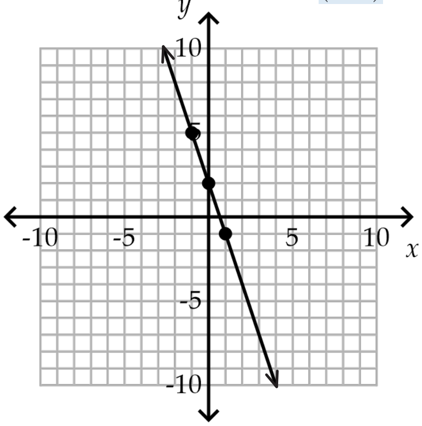
$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{-3-11}{-5-3}$$

$$=\frac{-14}{-8}$$

$$=\frac{14}{8}=\frac{7}{4}$$

10. Slope: –3 y-intercept: $(0,2)$



11. Point-slope form: $y+1=\frac{3}{2}(x-4)$

$$y+1=\frac{3}{2}(x-4)$$

$$y+1=\frac{3}{2}x-6$$

$$y+1-1=\frac{3}{2}x-6-1$$

$$y=\frac{3}{2}x-7$$

Slope-intercept form: $y=\frac{3}{2}x-7$

12. $m=\dfrac{-4-(-7)}{0-(-4)}=\dfrac{3}{4}$

$$y=\frac{3}{4}x+b$$

$$-4=\frac{3}{4}(0)+b$$

$$-4=b$$

$$y=\frac{3}{4}x-4$$

13. Cost: \$3/pound

Equation: $y=3x$

14. a.

$$4x+3y=24$$

$$4x+3(0)=24$$

$$4x=24$$

$$\frac{4x}{4}=\frac{24}{4}$$

$$x=6$$

$$4x+3y=24$$

$$4(0)+3y=24$$

$$3y=24$$

$$\frac{3y}{3}=\frac{24}{3}$$

$$y=8$$

x-intercept: $(6,0)$ y-intercept: $(0,8)$

b.

$$-x+y=-36$$

$$-x+0=-36$$

$$-x=-36$$

$$x=36$$

$$-x+y=-36$$

$$-0+y=-36$$

$$y=-36$$

x-intercept: $(36,0)$ y-intercept: $(0,-36)$

UNIT 2 | LESSON 60

Enrichment: Collatz Conjecture

This is an enrichment lesson. Students are not expected to master content in the enrichment lessons at this level.

1. 6 is even. $6 \div 2 = 3$

3 is odd. $3 \bullet 3 + 1 = 10$

10 is even. $10 \div 2 = 5$

5 is odd. $5 \bullet 3 + 1 = 16$

16 is even. $16 \div 2 = 8$

8 is even. $8 \div 2 = 4$

4 is even. $4 \div 2 = 2$

2 is even. $2 \div 2 = 1$

1 is odd. $1 \bullet 3 + 1 = 4$

Repeat answer: 4

2. 7 is odd. $7 \bullet 3 + 1 = 22$

22 is even. $22 \div 2 = 11$

11 is odd. $11 \bullet 3 + 1 = 34$

34 is even. $34 \div 2 = 17$

17 is odd. $17 \bullet 3 + 1 = 52$

52 is even. $52 \div 2 = 26$

26 is even. $26 \div 2 = 13$

13 is odd. $13 \bullet 3 + 1 = 40$

40 is even. $40 \div 2 = 20$

20 is even. $20 \div 2 = 10$

10 is even. $10 \div 2 = 5$

5 is odd. $5 \bullet 3 + 1 = 16$

16 is even. $16 \div 2 = 8$

8 is even. $8 \div 2 = 4$

4 is even. $4 \div 2 = 2$

2 is even. $2 \div 2 = 1$

1 is odd. $1 \bullet 3 + 1 = 4$

Repeat answer: 4

3. 8 is even. $8 \div 2 = 4$

4 is even. $4 \div 2 = 2$

2 is even. $2 \div 2 = 1$

1 is odd. $1 \bullet 3 + 1 = 4$

Repeat answer: 4

4. The first answer to repeat in these examples is 4.

5. and 6. Answers will vary based on the starting integer chosen. Answers should follow the two rules listed at the beginning of the lesson.

7. Yes, the first repeated answer is still 4.

8. –5 is odd. $-5 \bullet 3 + 1 = -14$

–14 is even. $-14 \div 2 = -7$

–7 is odd. $-7 \bullet 3 + 1 = -20$

–20 is even. $-20 \div 2 = -10$

–10 is even. $-10 \div 2 = -5$

Repeat answer: –5

9. –6 is even. $-6 \div 2 = -3$

–3 is odd. $-3 \bullet 3 + 1 = -8$

–8 is even. $-8 \div 2 = -4$

–4 is even. $-4 \div 2 = -2$

–2 is even. $-2 \div 2 = -1$

–1 is odd. $-1 \bullet 3 + 1 = -2$

Repeat answer: –2

Simple Probability

WARM-UP

$$A = P(1+r)^t$$

$$A = 2000(1+0.0375)^{15}$$

$$A = 3474.17$$

$3474.17

PRACTICE

1. a. $\frac{20}{40} = 0.5 = 50\%$

   b. $\frac{10}{40} = 0.25 = 25\%$

   c. $\frac{8}{40} = 0.2 = 20\%$

   d. Note: The prime numbers from 1 to 40 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and 37.

   $\frac{12}{40} = 0.3 = 30\%$

   e. Note: The two-digit palindromes from 1 to 40 are 11, 22, and 33.

   $\frac{3}{40} = 0.075 = 7.5\%$

2. a. $P(\text{green}) = \frac{2}{11} = 0.181818... \approx 18.18\%$

   $P(\text{yellow}) = \frac{1}{11} = 0.090909... \approx 9.09\%$

   $P(\text{red}) = \frac{3}{11} = 0.272727... \approx 27.27\%$

   $P(\text{blue}) = \frac{4}{11} = 0.363636... \approx 36.36\%$

   $P(\text{purple}) = \frac{1}{11} = 0.090909... \approx 9.09\%$

   b. green: $0.1818 \bullet 550 = 99.99 \approx 100$

   yellow: $0.0909 \bullet 550 = 49.995 \approx 50$

   red: $0.2727 \bullet 550 = 149.985 \approx 150$

   blue: $0.3636 \bullet 550 = 199.98 \approx 200$

   purple: $0.0909 \bullet 550 = 49.995 \approx 50$

   c. $P(\text{green}) = \frac{105}{550} = 0.190909... \approx 19.09\%$

   $P(\text{yellow}) = \frac{61}{550} = 0.110909... \approx 11.09\%$

   $P(\text{red}) = \frac{152}{550} = 0.276363... \approx 27.64\%$

   $P(\text{blue}) = \frac{207}{550} = 0.376363... \approx 37.64\%$

   $P(\text{purple}) = \frac{25}{550} = 0.045454... \approx 4.55\%$

3. a.

First die → Second die ↓	⚀	⚁	⚂	⚃	⚄	⚅
⚀	0	1	2	3	4	5
⚁	1	0	1	2	3	4
⚂	2	1	0	1	2	3
⚃	3	2	1	0	1	2
⚄	4	3	2	1	0	1
⚅	5	4	3	2	1	0

b. $P(0) = \frac{6}{36} = 0.1666... \approx 16.67\%$

$P(1) = \frac{10}{36} = 0.2777... \approx 27.78\%$

$P(2) = \frac{8}{36} = 0.2222... \approx 22.22\%$

$P(3) = \frac{6}{36} = 0.1666... \approx 16.67\%$

$P(4) = \frac{4}{36} = 0.1111... \approx 11.11\%$

$P(5) = \frac{2}{36} = 0.0555... \approx 5.56\%$

c. Number of 0s: $0.1667 \bullet 400 = 66.68 \approx 67$

Number of 1s: $0.2778 \bullet 400 = 111.12 \approx 111$

Number of 2s: $0.2222 \bullet 400 = 88.88 \approx 89$

Number of 3s: $0.1667 \bullet 400 = 66.68 \approx 67$

Number of 4s: $0.1111 \bullet 400 = 44.44 \approx 44$

Number of 5s: $0.0556 \bullet 400 = 22.24 \approx 22$

UNIT 3 | LESSON 70

Basic Geometry Terms

WARM-UP

a. 6
b. 5
c. 10

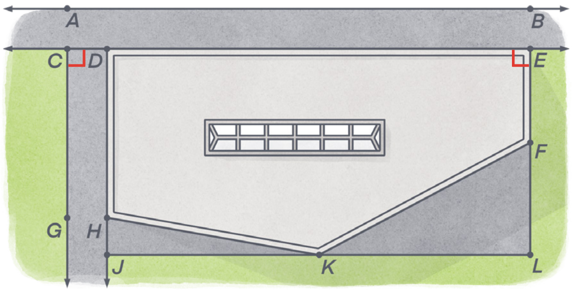
PRACTICE

1. a. $\overleftrightarrow{DE}$
b. $\overrightarrow{CG}$
c. K
d. $\angle DHK$
e. $\overline{FL}$

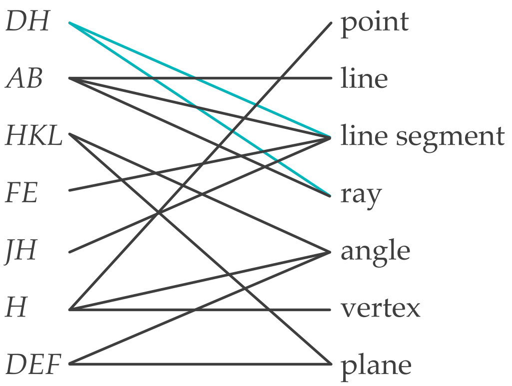
2. The exact angle measures are shown below. Answers within 1–3 degrees of the measure listed are acceptable.

Angle	Classification	Measure
$\angle DEF$	right	90°
$\angle DHK$	obtuse	100°
$\angle HKJ$	acute	10°
$\angle GCE$	right	90°
$\angle LFK$	acute	62°

3.

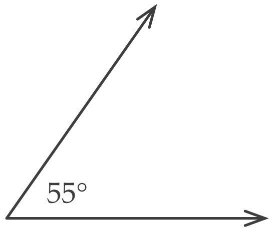


4.

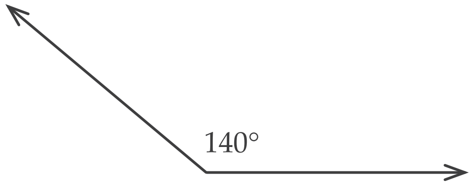


5. a. no
b. ≅
c. no

6. a.



b.



REVIEW

1. $x \bullet 12 = 5$

$$\frac{x \bullet 12}{12} = \frac{5}{12}$$

$$x = 0.41\overline{6} \approx 0.42$$

42%

2. a. $(0,30)$

Phillip already knew 30 digits of pi when he signed up for the competition.

b. Rise: up 80 Run: right 8

$$m = \frac{80}{8} = 10$$

Phillip learned 10 new digits per week.

c. $y = mx + b$

$y = 10x + 30$

d. $(0,5)$

Jasmine already knew five digits of pi when she signed up for the competition.

e. Rise: up 120 Run: right 8

$$m = \frac{120}{8} = 15$$

Jasmine learned 15 new digits per week.

f. $y - 80 = 15(x - 5)$

g. up to week 5

h. after week 5

i. At week 5, Phillip and Jasmine both have 80 digits of pi memorized.

j. January 17

Go back eight weeks from March 14. Two weeks (14 days) before March 14 is February 28. Four weeks (28 days) before February 28 is January 31. Two weeks (14 days) before January 31 is January 17.

UNIT 3 | LESSON 71

Angle Relationships and Transversals

WARM-UP

123°

PRACTICE

1. Detailed work is shown below.

Angle	Complementary	Supplementary
37°	53° **A**	143° **B**
69°	21° **C**	111° **D**
104°	none **E**	76° **F**
192°	none **G**	none **H**

A

90° – 37° = 53°

B

180° – 37° = 143°

C

90° – 69° = 21°

D

180° – 69° = 111°

E

104° is greater than 90°.

F

180° – 104° = 76°

G

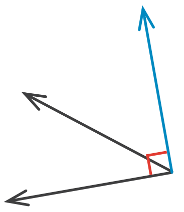
192° is greater than 90°.

H

192° is greater than 180°.

2. The student should have drawn a ray to form a right angle with the angle given. Possible answers are shown in blue.

a.



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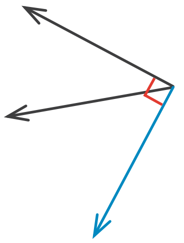
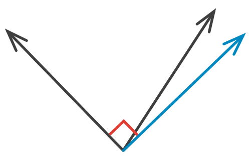


b.



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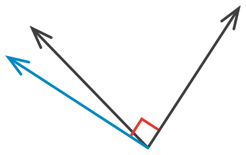
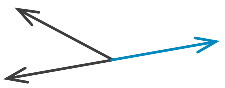
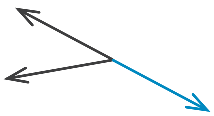


3. The student should have drawn a ray to form a straight line with the angle given. Possible answers are shown in blue.

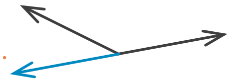
a.



or



b.



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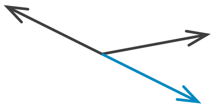


4. a. The angle that corresponds to $\angle a$ is $\angle e$, so it measures 65°.

b. $180° - 65° = 115°$

The interior angle that is adjacent to $\angle a$ is $\angle d$, and it measures 115°.

c. The angle vertical to $\angle a$ is $\angle c$, so it measures 65°.

d. An alternate interior angle for $\angle a$ is $\angle g$, so it measures 65°.

e. $180° - 65° = 115°$

The exterior angle that is adjacent to $\angle a$ is $\angle b$, and it measures 115°.

f. The angle that corresponds to $\angle b$ is $\angle f$, so it measures 115°.

g. The angle that corresponds to $\angle d$ is $\angle h$, so it measures 115°.

5.

Word	Definition
supplementary angles	two angles whose sum is 180°
adjacent angles	two angles that have the same vertex and a common side
alternate exterior angles	nonadjacent exterior angles that are located on opposite sides of the transversal
transversal	line that intersects two or more lines
corresponding angles	angles located in the same position on parallel lines when the parallel lines are cut by a transversal
vertical angles	nonadjacent angles that are opposite each other at the intersection of two lines
complementary angles	two angles whose sum is 90°
alternate interior angles	nonadjacent interior angles that are located on opposite sides of the transversal
parallel lines	lines that never intersect and are always the same distance apart

UNIT 3 | LESSON 75

Logic Lesson 3

Logic puzzles can be approached in many different ways. The solutions here may not represent all possible methods or answers.

1. 8 people

There are four children (two boys and two girls), their parents, and their grandparents (their mother's mother and their father's father). The grandfather is also a father, and the grandmother is also a mother. The parents are also children: the father is a son and the mother a daughter. The grandparents are each in-laws of one of the parents, and the mother is the grandfather's daughter-in-law.

2. More than one solution is possible. One solution for each number is shown below.

a. $13 = \frac{55+5+5}{5}$

b. $14 = 5+5+5-\frac{5}{5}$

c. $15 = \frac{5}{5} \bullet (5+5+5)$

3. The son is 37 years old, and his mother is 73 years old.

Guess-and-check method:

Start with a possible adult age for the son and reverse the digits: 24 and 42. Since 41 is not twice 23, try a new starting age close to the first one. Try 25 and 52. 51 is not double 24, but a pattern can be observed. The second digit in 41 and 51 is odd, so it cannot be twice another number. Whenever a number in the twenties is doubled and then one is subtracted, the second digit will be odd. Therefore, the son is not in his twenties.

Start with a possible adult age for the son in the thirties and reverse the digits: 35 and 53. Since 52 is not twice 34, try a new starting age. Try 36 and 63. Since 62 is not twice 35, try a new starting age. Try 37 and 73. 72 is twice 36.

4. 1st: Somsak's family
2nd: Malee's family
3rd: Arthit's family
4th: Kate's family

If Arthit was wrong, then he was either first or last. This would mean that Malee was also wrong. The organizer said only one person was wrong, so it couldn't have been Arthit.

If Somsak was wrong, then he did finish last. That would mean that Kate was also wrong. Only one person can be wrong, so it couldn't have been Somsak.

If Malee was wrong, then she was not first and Arthit was not second. The other three statements could be correct, and this would be the order:

1st: Somsak (not last)
2nd: Malee (not first)
3rd: Arthit (in the middle but not second)
4th: Kate (last)

5. Logic puzzles can be completed in different ways. Information that can be gathered from each clue is shown below.

Because there are two April 14ths, there are two possible solutions:

	Favorite Day				Favorite Event			
	April 13th	April 14th	April 14th	April 15th	Dancing	Making Garlands	Water Fight	Building Sand Pagodas
Dara (girl)	X	✓	X	X	X	X	X	✓
Mali (girl)	✓	X	X	X	✓	X	X	X
Saksit (boy)	X	X	✓	X	X	✓	X	X
Phet (boy)	X	X	X	✓	X	X	✓	X
Dancing	✓	X	X	X				
Making Garlands	X	X	✓	X				
Water Fight	X	X	X	✓				
Building Sand Pagodas	X	✓	X	X				

	Favorite Day				Favorite Event			
	April 13th	April 14th	April 14th	April 15th	Dancing	Making Garlands	Water Fight	Building Sand Pagodas
Dara (girl)	X	X	✓	X	X	X	X	✓
Mali (girl)	✓	X	X	X	✓	X	X	X
Saksit (boy)	X	✓	X	X	X	✓	X	X
Phet (boy)	X	X	X	✓	X	X	✓	X
Dancing	✓	X	X	X				
Making Garlands	X	✓	X	X				
Water Fight	X	X	X	✓				
Building Sand Pagodas	X	X	✓	X				

Clue #1: Because the event that matches to April 14 cannot be Water Fight or Dancing, the other two events must go with April 14. Put a check mark in an April 14 column for Making Garlands and a check mark in the other April 14 column for Building Sand Pagodas.

Clue #2: Put a check mark in the April 15 column (final day of festival) for Water Fight. That means Dancing goes with April 13. Also, since a boy chose Water Fight, put an X in the row for both girls for Water Fight.

Clue #3: Since a boy chose Making Garlands, put an X in the row for both girls for Making Garlands.

Clue #4: If Dara's favorite day is not April 13 (first day), her favorite event was not Dancing. That means her favorite event was Building Sand Pagodas, which was on April 14.

From Clue #1, a boy and a girl did not choose the Water Fight and Dancing, so the other boy and girl must have chosen those events. From Clue #2, a boy chose Water Fight. Therefore, a girl chose Dancing. Since Dara did not choose Dancing, Mali chose Dancing on April 13.

Clue #5: If Phet did not choose Making Garlands, then Saksit did. Therefore, Saksit chose April 14, and Phet chose the Water Fight and April 15.

a. Dara's favorite day: April 14th
Dara's favorite event: building sand pagodas

b. Mali's favorite day: April 13th
Mali's favorite event: dancing

c. Saksit's favorite day: April 14th
Saksit's favorite event: making garlands

d. Phet's favorite day: April 15th
Phet's favorite event: water fight

Drawings and Constructions

WARM-UP

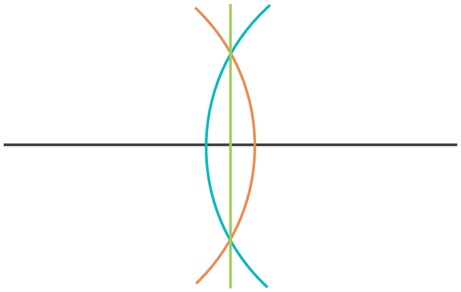
$$\frac{25}{50}=\frac{7}{3x+2}$$
$$25(3x+2)=50\bullet 7$$
$$75x+50=350$$
$$75x+50-50=350-50$$
$$75x=300$$
$$\frac{75x}{75}=\frac{300}{75}$$
$$x=4$$

$$3(4)+2=14$$

14 in

PRACTICE

1. a. Note: Arcs drawn from a compass are shown. The perpendicular bisector is the green line segment.

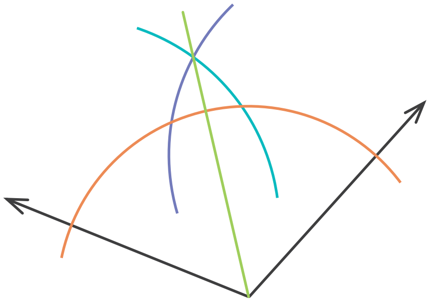


b. Left side: 3 cm   Right side: 3 cm

c. The answer should be within a few degrees of 90.

90°

2. a. Note: Arcs drawn from a compass are shown. The angle bisector is the green line segment.



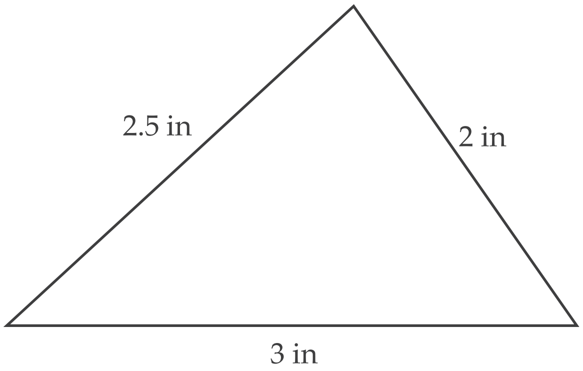
b. The answers should be within a few degrees of 55.

Left angle measure: 55°

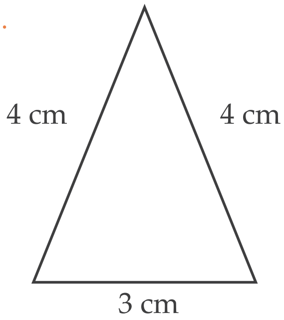
Right angle measure: 55°

3. Note: Triangles may be oriented differently, but they should have the same side lengths.

a.

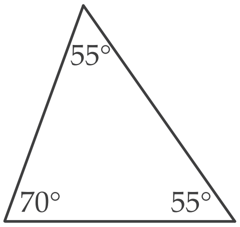


b.

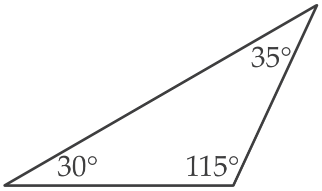


4. Note: Triangles may be oriented differently, but they should have the same angle measures.

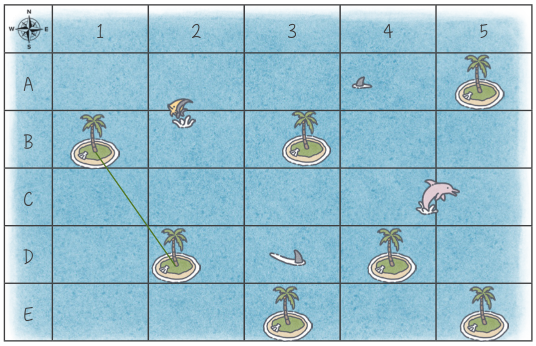
a.



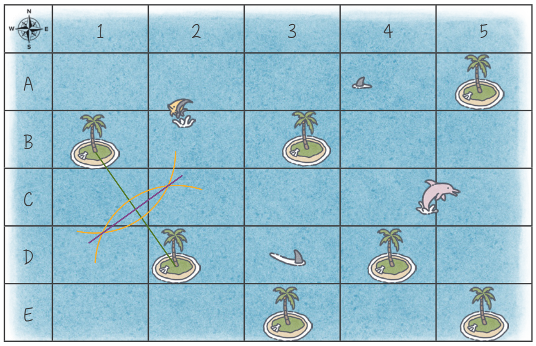
b.



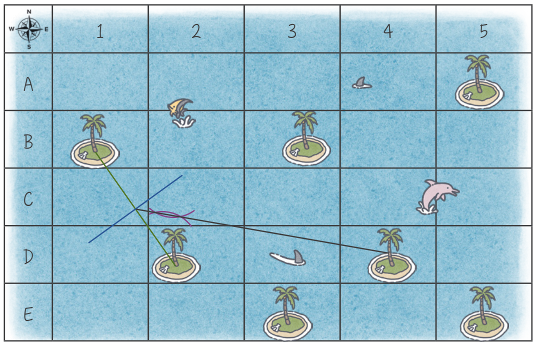
5. a.



b. Note: Arcs drawn from a compass are shown. The perpendicular bisector is the purple line segment.



c. Note: Arcs drawn from a compass are shown. The angle bisector is the black line segment.



d. D4

UNIT 3 | LESSON 88

Unit 3 Review

1. a. $40\% + 100\% = 140\%$

$1.4 \bullet 15 = 21$

b. $8280 - 7200 = 1080$

$$\begin{aligned} x \bullet 7200 &= 1080 \\ \frac{x \bullet 7200}{7200} &= \frac{1080}{7200} \\ x &= \frac{1080}{7200} = 0.15 \end{aligned}$$

15%

c. $100\% - 15\% = 85\%$

$$\begin{aligned} 0.85 \bullet x &= 78.2 \\ \frac{0.85 \bullet x}{0.85} &= \frac{78.2}{0.85} \\ x &= 92 \end{aligned}$$

92 kg

2. a. $I = Prt$

$I = 400 \bullet 0.045 \bullet 10$

$I = 180$

$400 + 180 = 580$

\$580

b.

$$\begin{aligned} A &= P(1+r)^t \\ 2{,}200{,}000 &= P(1+0.07)^{30} \\ 2{,}200{,}000 &= P(1.07)^{30} \\ 2{,}200{,}000 &\approx P(7.61225504) \\ \frac{2{,}200{,}000}{7.61225504} &\approx \frac{P(7.61225504)}{7.61225504} \\ 289{,}007 &\approx P \end{aligned}$$

\$289,000

3. a. $P(\text{biologist or ecologist})$

$$\begin{aligned} &= P(\text{biologist}) + P(\text{ecologist}) \\ &= \frac{56}{175} + \frac{62}{175} \\ &= \frac{118}{175} \approx 0.6743 \end{aligned}$$

67.43%

b. $56 \bullet 62 \bullet 45 = 156{,}240$

156,240

c. $P(\text{no biologist, ecologist, or oceanographer})$

$$\begin{aligned} &= P(\text{other scientist first}) \\ &\bullet P(\text{other scientist second } | \text{other scientist first}) \\ &= \frac{12}{175} \bullet \frac{11}{174} \\ &= \frac{132}{30{,}450} \approx 0.0043 \end{aligned}$$

0.43%

4. a. $56 : 62 = 28 : 31$

b. $\dfrac{45 \text{ oceanographers}}{175 \text{ scientists}} = \dfrac{x \text{ oceanographers}}{245 \text{ scientists}}$

$$\begin{aligned} \frac{45}{175} &= \frac{x}{245} \\ 45 \bullet 245 &= 175x \\ 11{,}025 &= 175x \\ \frac{11{,}025}{175} &= \frac{175x}{175} \\ 63 &= x \end{aligned}$$

5. a. 2 gal 1 c • 24 = 48 gal 24 c

24 c = 1 gal 8 c = 1 gal 2 qt

48 gal + 1 gal 2 qt = 49 gal 2 qt

b. 12,100 yd^2

$$=12{,}100\ \cancel{\text{yd}}\bullet\cancel{\text{yd}}\bullet\frac{1\ \cancel{\text{m}}}{1.1\ \cancel{\text{yd}}}\bullet\frac{1\ \cancel{\text{m}}}{1.1\ \cancel{\text{yd}}}$$

$$\bullet\frac{1\text{ km}}{1000\ \cancel{\text{m}}}\bullet\frac{1\text{ km}}{1000\ \cancel{\text{m}}}$$

$$=\frac{12{,}100\text{ km}\bullet\text{km}}{1.1\bullet1.1\bullet1000\bullet1000}$$

$$=0.01\text{ km}^2$$

6. a. $450\bullet x=1.5$

$$\frac{450\bullet x}{450}=\frac{1.5}{450}$$

$$x=\frac{1.5}{450}=\frac{3}{900}=\frac{1}{300}$$

b. $$x\bullet\frac{1}{300}=8$$

$$300\bullet x\bullet\frac{1}{300}=8\bullet300$$

$$x=2400$$

2400 cm

c. $0.3\bullet300^2=27{,}000$

27,000 m^2

7. $5\bullet250\text{ m}=1250\text{ m}$

8. a. $\overleftrightarrow{AB}$

b. A

c. $\overline{AB}$

9. a. f

b. h

c. Angles a and c are supplementary, so angle c measures 180° – 73° = 107°.

Angles c and g correspond, so angle g measures 107°.

10. a. obtuse, scalene

b. $$110+3x+2x-5=180$$

$$105+5x=180$$

$$105+5x-105=180-105$$

$$5x=75$$

$$\frac{5x}{5}=\frac{75}{5}$$

$$x=15$$

$3x=3\bullet15=45$

$2x-5=2\bullet15-5=30-5=25$

45° and 25°

11. a. concave

b. 5 sides → $3\bullet180°=540°$

c. $$90+5x+(4x-3)+(x+13)+10x=540$$

$$100+20x=540$$

$$100+20x-100=540-100$$

$$20x=440$$

$$\frac{20x}{20}=\frac{440}{20}$$

$$x=22$$

A: 90°

B: $10\bullet22=220\rightarrow$ 220°

C: $22+13=35\rightarrow$ 35°

D: $4\bullet22-3=88-3=85\rightarrow$ 85°

E: $5\bullet22=110\rightarrow$ 110°

12. [✓ AAA] ASA AAS

SAS [✓ SSA] SSS

13. $$\frac{30}{5}=6$$

$$\frac{36}{6}=6$$

$$\frac{50}{8}=6.25$$

no

UNIT 3 | LESSON 89

Unit 3 Assessment

1. $420{,}000 - 350{,}000 = 70{,}000$

$$x \bullet 350{,}000 = 70{,}000$$

$$\frac{x \bullet 350{,}000}{350{,}000} = \frac{70{,}000}{350{,}000}$$

$$x = 0.2$$

20% increase

2. $A = P(1+r)^t$

$$A = 450(1+0.035)^{12}$$

$$A = 450 \bullet 1.035^{12}$$

$$A \approx 450 \bullet 1.511$$

$$A \approx 679.95$$

$679.95

3. a. $120 + 80 + 100 + 150 + 50 = 500$

Pink: $\frac{120}{500} = 0.24 = 24\%$

Yellow: $\frac{80}{500} = 0.16 = 16\%$

Blue: $\frac{100}{500} = 0.2 = 20\%$

Green: $\frac{150}{500} = 0.3 = 30\%$

White: $\frac{50}{500} = 0.1 = 10\%$

b. $n = 50$

Pink: $50 \bullet 0.24 = 12$

Yellow: $50 \bullet 0.16 = 8$

Blue: $50 \bullet 0.2 = 10$

Green: $50 \bullet 0.3 = 15$

White: $50 \bullet 0.1 = 5$

4. $\frac{3}{6} \bullet \frac{1}{5} = \frac{3}{30} = \frac{1}{10}$

5. $3.78 \div 46 \approx 0.08$

$0.08 per cookie

6. $\frac{500 \text{ grams pasta}}{0.5 \text{ onions}} = \frac{x \text{ grams pasta}}{0.75 \text{ onions}}$

$$\frac{500}{0.5} = \frac{x}{0.75}$$

$$500 \bullet 0.75 = 0.5x$$

$$375 = 0.5x$$

$$\frac{375}{0.5} = \frac{0.5x}{0.5}$$

$$750 = x$$

750 grams

7. a. 4 lb • 3 = 12 lb

6 oz • 3 = 18 oz

18 oz = 1 lb 2 oz

12 lb + 1 lb 2 oz = 13 lb 2 oz

b. 2 ft 5 in + 4 ft 8 in = 6 ft 13 in

13 in = 1 ft 1 in

6 ft + 1 ft 1 in = 7 ft 1 in

8. $\frac{40 \text{ lb}}{1} \bullet \frac{1 \text{ kg}}{2.2 \text{ lb}} = \frac{40 \text{ kg}}{2.2} \approx 18.18 \text{ kg}$

9. 320 ft : 4 in → 80 ft : 1 in

10. acute

11. $\angle a$ and $\angle f$

$\angle b$ and $\angle e$

$\angle c$ and $\angle h$

$\angle d$ and $\angle g$

12.
$$\begin{aligned} 6x+1+8x+1+11x+3&=180\\ 25x+5&=180\\ 25x+5-5&=180-5\\ 25x&=175\\ \frac{25x}{25}&=\frac{175}{25}\\ x&=7 \end{aligned}$$

$$m\angle P=\left(11\left(7\right)+3\right)^\circ$$
$$m\angle P=\left(77+3\right)^\circ$$
$$m\angle P=80^\circ$$

13.

Shape	Reason
quadrilateral	It has four sides.
parallelogram	It has two pairs of parallel sides.
rhombus	All sides are congruent.

14. side-angle-side

15.
$$\frac{JG}{NK}=\frac{GH}{KL}$$
$$\frac{2.5}{1}=\frac{4.5}{p}$$
$$2.5p=4.5$$
$$\frac{2.5p}{2.5}=\frac{4.5}{2.5}$$
$$p=1.8$$

16. Perimeter of semicircle:

$C=\pi d$

$C=25\pi$

$C\approx 78.54$

Divide C by 2: $78.54\div 2=39.27$

Hypotenuse of triangle:

$$\begin{aligned} 25^2+25^2&=c^2\\ 625+625&=c^2\\ 1250&=c^2\\ \sqrt{1250}&=\sqrt{c^2}\\ c&\approx 35.36 \end{aligned}$$

$P=39.27\text{ ft}+35.36\text{ ft}+25\text{ ft}=99.63\text{ ft}$

17. a. $\frac{22.5^\circ}{360^\circ}=\frac{1}{16}$

$C=2\pi r$

$C=2\pi\bullet 6.5=13\pi$

$13\pi\bullet\frac{1}{16}=\frac{13}{16}\pi\approx 2.55$

2.55 cm

b. $A=\pi r^2$

$A=\pi\left(6.5\right)^2$

$A=42.25\pi$

$42.25\pi\bullet\frac{1}{16}=\frac{42.25}{16}\pi\approx 8.30$

8.30 cm^2

18. $A_{tri}=\frac{1}{2}bh$

$A_{tri}=\frac{1}{2}\left(0.5\right)\left(0.75\right)$

$A_{tri}\approx 0.188$

0.188 in^2

$A_{semi}=\frac{1}{2}\pi r^2$

$A_{semi}=\frac{1}{2}\pi\left(0.25\right)^2$

$A_{semi}=\frac{1}{2}\pi\left(0.0625\right)$

$A_{semi}=0.03125\pi$

$A_{semi}\approx 0.098$

0.098 in^2

Total Area: $6\left(0.188\text{ in}^2+0.098\text{ in}^2\right)=1.716\text{ in}^2$

19. $SA_{cyl}=\pi r^2+2\pi rh$

$SA_{cyl}=\pi\left(15\right)^2+2\pi\left(15\right)\left(25\right)$

$SA_{cyl}=225\pi+750\pi$

$SA_{cyl}=975\pi$

$SA_{cyl}\approx 3063.05$

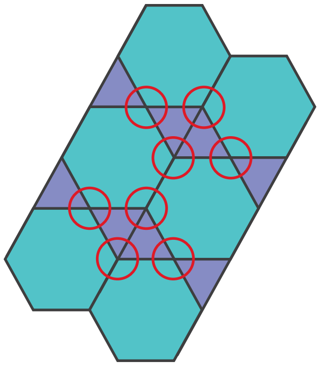
3063.05 ft^2

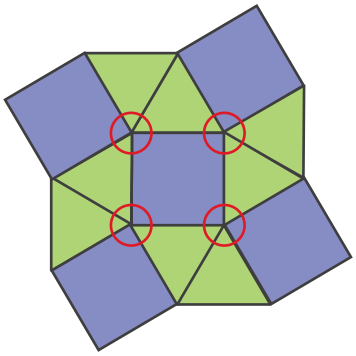
UNIT 3 | LESSON 90

Enrichment: Tessellations

This is an enrichment lesson. Students are not expected to master content in the enrichment lessons at this level.

Shape (number of sides)	Interior Angle Measure	360° ÷ Interior Angle Measure	Does the shape tessellate?
Equilateral Triangle (3)	$\frac{180^\circ(n-2)}{n}$ $=\frac{180^\circ(3-2)}{3}$ $=60^\circ$	$360^\circ \div 60^\circ = 6$	yes
Square (4)	$\frac{180^\circ(n-2)}{n}$ $=\frac{180^\circ(4-2)}{4}$ $=90^\circ$	$360^\circ \div 90^\circ = 4$	yes
Regular Pentagon (5)	$\frac{180^\circ(n-2)}{n}$ $=\frac{180^\circ(5-2)}{5}$ $=108^\circ$	$360^\circ \div 108^\circ = 3.\overline{3}$	no
Regular Hexagon (6)	$\frac{180^\circ(n-2)}{n}$ $=\frac{180^\circ(6-2)}{6}$ $=120^\circ$	$360^\circ \div 120^\circ = 3$	yes
Regular Heptagon (7)	$\frac{180^\circ(n-2)}{n}$ $=\frac{180^\circ(7-2)}{7}$ $\approx 128.57^\circ$	$360^\circ \div 128.57^\circ \approx 2.8$	no
Regular Octagon (8)	$\frac{180^\circ(n-2)}{n}$ $=\frac{180^\circ(8-2)}{8}$ $=135^\circ$	$360^\circ \div 135^\circ = 2.\overline{6}$	no





Regular hexagon: 120°

Equilateral triangle: 60°

Interior angle sum at intersection:
60° + 60° + 120° + 120° = 360°

Square: 90°

Equilateral triangle: 60°

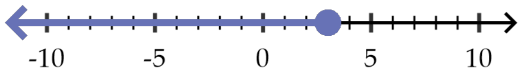
Interior angle sum at intersection:
90° + 90° + 60° + 60° + 60° = 360°

UNIT 4 | LESSON 92

Graphing Linear Inequalities

WARM-UP

$$4t - 8 \geq 9t - 23$$
$$4t - 8 + 8 \geq 9t - 23 + 8$$
$$4t \geq 9t - 15$$
$$4t - 9t \geq 9t - 15 - 9t$$
$$-5t \geq -15$$
$$\frac{-5t}{-5} \geq \frac{-15}{-5}$$
$$t \leq 3$$



PRACTICE

1. Red:

Slope: 2

y-intercept: $(0,-6)$

Test $(0,0)$:

$$y > 2x - 6$$
$$0 \overset{?}{>} 2 \bullet 0 - 6$$
$$0 \overset{?}{>} 0 - 6$$
$$0 \overset{?}{>} -6 \ ✓$$

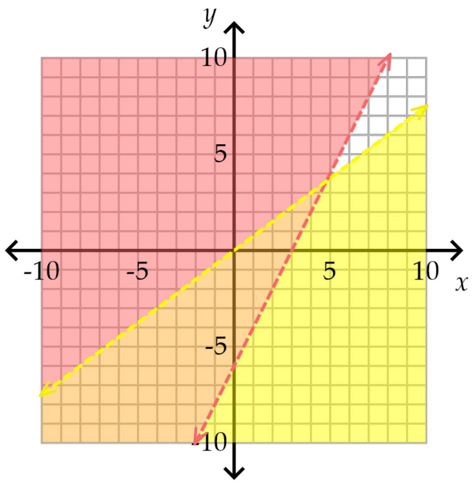
Yellow:

Slope: $\frac{3}{4}$

y-intercept: $(0,0)$

Test $(0,1)$:

$$y < \frac{3}{4}x$$
$$1 \overset{?}{<} \frac{3}{4}(0)$$
$$1 \overset{?}{<} 0 \ ✗$$



2.

Ordered pair	$y > 2x - 6$	$y < \frac{3}{4}x$
$(0,4)$	$y > 2x - 6$ $4 \overset{?}{>} 2(0) - 6$ $4 \overset{?}{>} -6$ ✓ yes	$y < \frac{3}{4}x$ $4 \overset{?}{<} \frac{3}{4}(0)$ $4 \overset{?}{<} 0$ ✗ no
$(0,-4)$	$y > 2x - 6$ $-4 \overset{?}{>} 2(0) - 6$ $-4 \overset{?}{>} -6$ ✓ yes	$y < \frac{3}{4}x$ $-4 \overset{?}{<} \frac{3}{4}(0)$ $-4 \overset{?}{<} 0$ ✓ yes
$(4,0)$	$y > 2x - 6$ $0 \overset{?}{>} 2(4) - 6$ $0 \overset{?}{>} 2$ ✗ no	$y < \frac{3}{4}x$ $0 \overset{?}{<} \frac{3}{4}(4)$ $0 \overset{?}{<} 3$ ✓ yes
$(-4,0)$	$y > 2x - 6$ $0 \overset{?}{>} 2(-4) - 6$ $0 \overset{?}{>} -14$ ✓ yes	$y < \frac{3}{4}x$ $0 \overset{?}{<} \frac{3}{4}(-4)$ $0 \overset{?}{<} -3$ ✗ no

3. Blue:

Slope: $-\frac{5}{3}$

y-intercept: $(0,3)$

Test $(0,0)$:

$$y \le -\frac{5}{3}x + 3$$

$$0 \overset{?}{\le} -\frac{5}{3}(0) + 3$$

$$0 \overset{?}{\le} 3$$ ✓

Yellow:

Slope: $\frac{1}{2}$

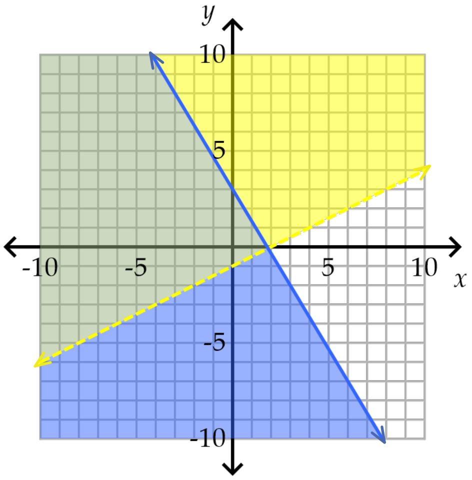
y-intercept: $(0,-1)$

Test $(0,0)$:

$$y > \frac{1}{2}x - 1$$

$$0 \overset{?}{>} \frac{1}{2}(0) - 1$$

$$0 \overset{?}{>} -1$$ ✓



Solving Systems by Substitution

WARM-UP

a. 1700

b. 2.31

c. 0.0005487

PRACTICE

Note: There are many ways to solve systems of equations. Work may vary, but the solutions should be the same.

1. Solve for x:

$$-x-3y=-1$$
$$-x-3y+3y=-1+3y$$
$$-x=-1+3y$$
$$\frac{-x}{-1}=\frac{-1}{-1}+\frac{3y}{-1}$$
$$x=1-3y$$

Substitution:

$$3x+4y=-2$$
$$3(1-3y)+4y=-2$$
$$3-9y+4y=-2$$
$$3-5y=-2$$
$$3-5y-3=-2-3$$
$$-5y=-5$$
$$\frac{-5y}{-5}=\frac{-5}{-5}$$
$$y=1$$

$$x=1-3y$$
$$x=1-3(1)$$
$$x=-2$$

Solution: $(-2,1)$

2. Solve for y:

$$4x+2y=22$$
$$4x+2y-4x=22-4x$$
$$2y=22-4x$$
$$\frac{2y}{2}=\frac{22}{2}-\frac{4x}{2}$$
$$y=11-2x$$

Substitution:

$$2x-3y=-9$$
$$2x-3(11-2x)=-9$$
$$2x-33+6x=-9$$
$$8x-33=-9$$
$$8x-33+33=-9+33$$
$$8x=24$$
$$\frac{8x}{8}=\frac{24}{8}$$
$$x=3$$

$$y=11-2x$$
$$y=11-2(3)$$
$$y=5$$

Solution: $(3,5)$

Substitution:

$$\begin{aligned} -4x+6y&=-52\\ -4\left(13+\frac{3}{2}y\right)+6y&=-52\\ -52-6y+6y&=-52\\ -52&=-52 \end{aligned}$$

Solution: infinitely many solutions

7. Solve for y:

$$\begin{aligned} -5x-y&=0\\ -5x-y+5x&=0+5x\\ -y&=5x\\ \frac{-y}{-1}&=\frac{5x}{-1}\\ y&=-5x \end{aligned}$$

Substitution:

$$\begin{aligned} x+y&=0\\ x+(-5x)&=0\\ -4x&=0\\ \frac{-4x}{-4}&=\frac{0}{-4}\\ x&=0 \end{aligned}$$

$$\begin{aligned} y&=-5x\\ y&=-5(0)\\ y&=0 \end{aligned}$$

Solution: $(0,0)$

8. Solve for y:

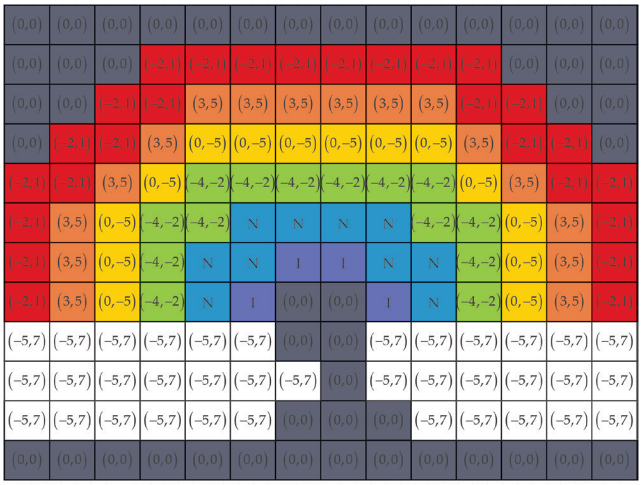
$$\begin{aligned} 2x+y&=-3\\ 2x+y-2x&=-3-2x\\ y&=-3-2x \end{aligned}$$

Substitution:

$$\begin{aligned} 3x+2y&=-1\\ 3x+2(-3-2x)&=-1\\ 3x-6-4x&=-1\\ -x-6&=-1\\ -x-6+6&=-1+6\\ -x&=5\\ \frac{-x}{-1}&=\frac{5}{-1}\\ x&=-5 \end{aligned}$$

$$\begin{aligned} y&=-3-2x\\ y&=-3-2(-5)\\ y&=7 \end{aligned}$$

Solution: $(-5,7)$



(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
(0,0)	(0,0)	(0,0)	(–2,1)	(–2,1)	(–2,1)	(–2,1)	(–2,1)	(–2,1)	(–2,1)	(–2,1)	(0,0)	(0,0)	(0,0)
(0,0)	(0,0)	(–2,1)	(–2,1)	(3,5)	(3,5)	(3,5)	(3,5)	(3,5)	(3,5)	(–2,1)	(–2,1)	(0,0)	(0,0)
(0,0)	(–2,1)	(–2,1)	(3,5)	(0,–5)	(0,–5)	(0,–5)	(0,–5)	(0,–5)	(0,–5)	(3,5)	(–2,1)	(–2,1)	(0,0)
(–2,1)	(–2,1)	(3,5)	(0,–5)	(–4,–2)	(–4,–2)	(–4,–2)	(–4,–2)	(–4,–2)	(–4,–2)	(0,–5)	(3,5)	(–2,1)	(–2,1)
(–2,1)	(3,5)	(0,–5)	(–4,–2)	(–4,–2)	N	N	N	N	(–4,–2)	(–4,–2)	(0,–5)	(3,5)	(–2,1)
(–2,1)	(3,5)	(0,–5)	(–4,–2)	N	N	I	I	N	N	(–4,–2)	(0,–5)	(3,5)	(–2,1)
(–2,1)	(3,5)	(0,–5)	(–4,–2)	N	I	(0,0)	(0,0)	I	N	(–4,–2)	(0,–5)	(3,5)	(–2,1)
(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(0,0)	(0,0)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)
(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(0,0)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)
(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(0,0)	(0,0)	(0,0)	(–5,7)	(–5,7)	(–5,7)	(–5,7)	(–5,7)
(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)

UNIT 4 | LESSON 100

Rotations and Symmetry

WARM-UP

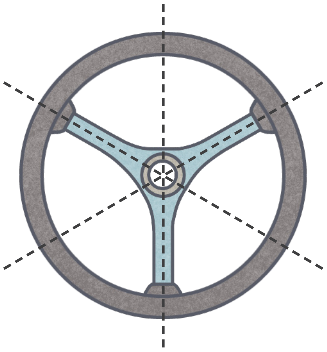
a. 12.5%

b. 0.08%

c. 145%

PRACTICE

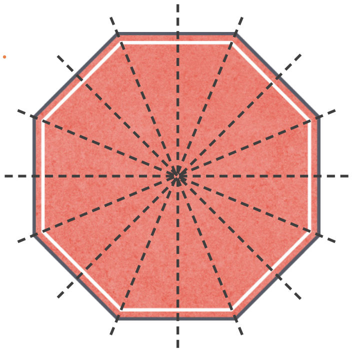
1. a.



Lines of symmetry: 3

Order: 3

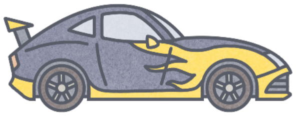
b.



Lines of symmetry: 8

Order: 8

c.

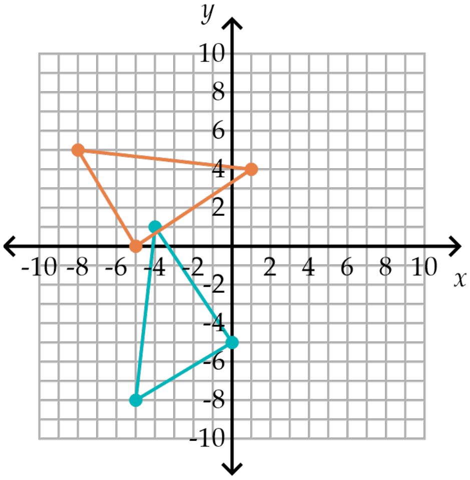


Lines of symmetry: 0

Order: 1

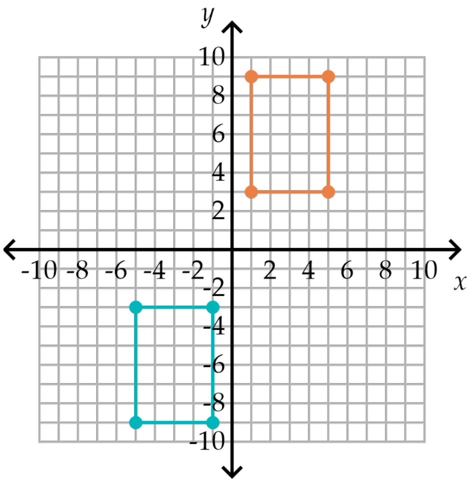
2. a. Rule: $(x,y)\to(-y,x)$

Rotate 90° Counterclockwise			
Preimage	$(-8,5)$	$(-5,0)$	$(1,4)$
Image	$(-5,-8)$	$(0,-5)$	$(-4,1)$



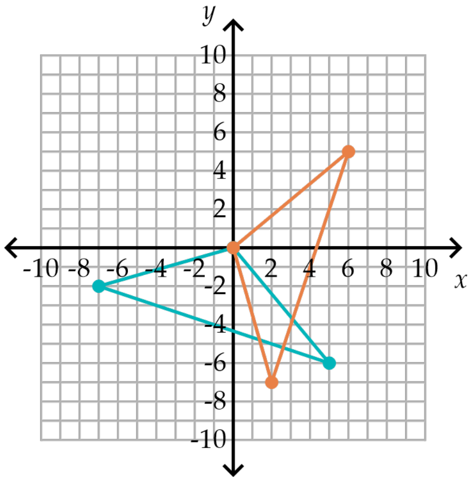
b. Rule: $(x,y) \rightarrow (-x,-y)$

Rotate 180° Clockwise				
Preimage	$(1,9)$	$(5,9)$	$(5,3)$	$(1,3)$
Image	$(-1,-9)$	$(-5,-9)$	$(-5,-3)$	$(-1,-3)$



c. Rule: $(x,y) \rightarrow (y,-x)$

Rotate 90° Clockwise			
Preimage	$(6,5)$	$(0,0)$	$(2,-7)$
Image	$(5,-6)$	$(0,0)$	$(-7,-2)$



d. Rule: $(x,y) \rightarrow (y,-x)$

Rotate 270° Counterclockwise					
Preimage	$(-2,8)$	$(-5,4)$	$(-8,8)$	$(-10,-2)$	$(-1,-2)$
Image	$(8,2)$	$(4,5)$	$(8,8)$	$(-2,10)$	$(-2,1)$