LESSONS
31–60

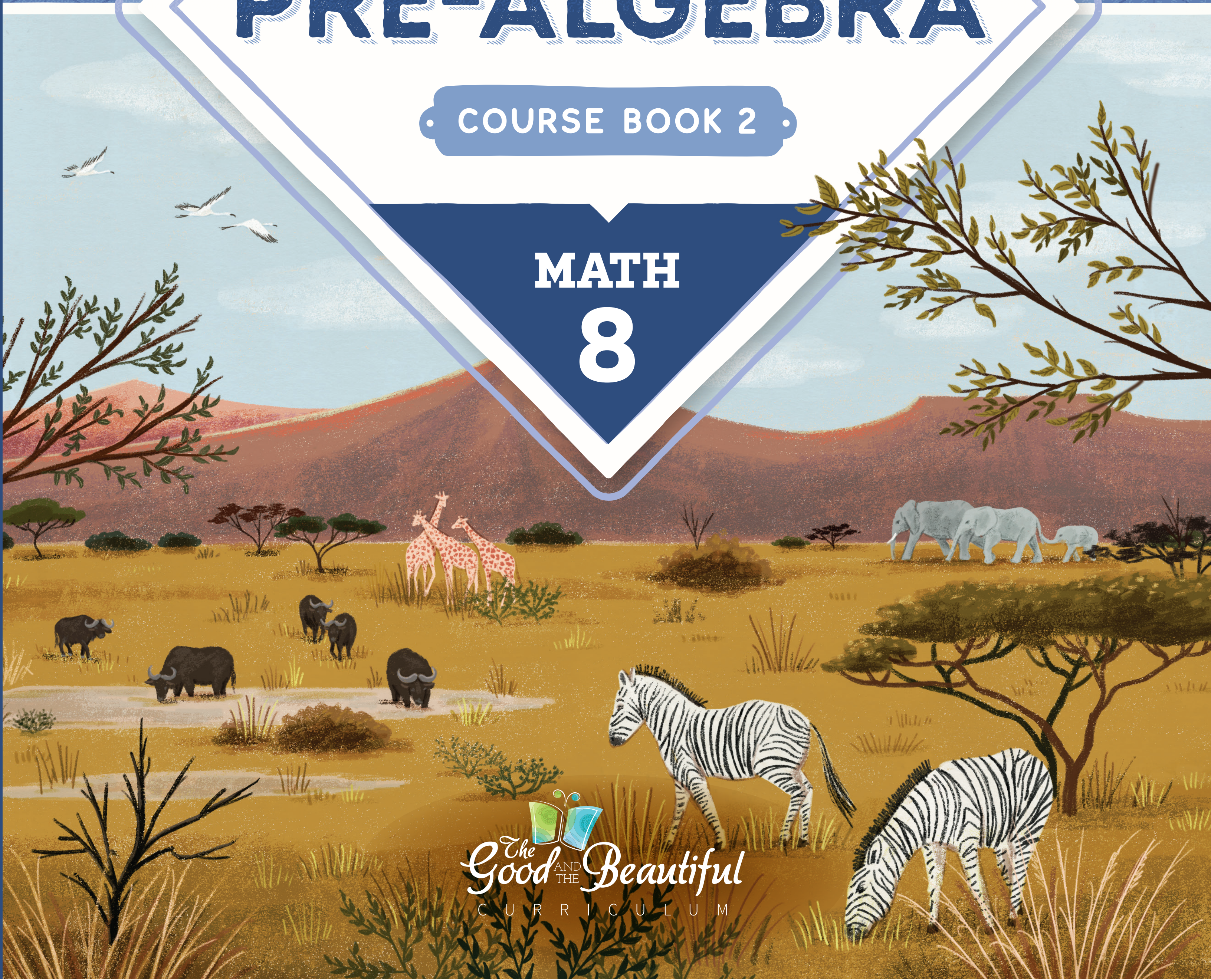
UNIT 2

Simply Good and Beautiful

PRE-ALGEBRA

COURSE BOOK 2

MATH 8



The Good and the Beautiful Curriculum

COURSE BOOK 2

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LESSONS 31–60

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UNIT 2 | LESSON 32

Modeling Real-World Situations with Equations

WARM-UP

Solve each equation.

a. $3x - 14 = 7$

______

b. $5(x + 6) = 40$

______

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

Home School App

VIDEO PRACTICE

LESSON OVERVIEW

Mathematical calculations are frequently needed in everyday situations. Often, by analyzing the situation, an equation that fits the situation can be written. It is helpful when writing an equation to think of and write two separate expressions that represent the same amount. Then put an equal sign between the expressions to make an equation. Several real-world situations are detailed below.

Example 1: A box of chalk comes with t pieces. The Larsen family used three boxes plus nine pieces of chalk from another box. The Hopper family used four boxes and had two pieces of chalk left over. The Hoppers and the Larsens used the same number of pieces of chalk. How many pieces of chalk come in a box?

Write an expression to represent how much chalk each family used.

Larsen Family: $3t + 9$ Hopper Family: $4t - 2$

Since both families used the same number of chalk pieces, these expressions are equivalent. Write an equation by setting the expressions equal to each other.

$3t + 9 = 4t - 2$ Subtract $3t$ from each side.

$3t + 9 - 3t = 4t - 2 - 3t$

$9 = t - 2$ Add 2 to each side.

$9 + 2 = t - 2 + 2$

$11 = t$ Each box has 11 pieces of chalk.

Example 2: Nathan and Preston are on the swim team, and each of them swims l laps as part of his warm-up. Preston swims the backstroke, and Nathan swims the butterfly. Nathan takes six minutes to complete one lap, and he stretches for two minutes. Preston takes three minutes to complete one lap, and he stretches for eight minutes. Preston and Nathan warm up for the same amount of time. How many laps do they each swim?

Write an expression to represent the amount of time each boy spent warming up.

Nathan: $6l + 2$ Preston: $3l + 8$

Since both boys spend the same amount of time warming up, these expressions are equivalent. Write an equation by setting the expressions equal to each other.

$6l + 2 = 3l + 8$ Subtract $3l$ from each side.

$6l + 2 - 3l = 3l + 8 - 3l$

$3l + 2 = 8$ Subtract 2 from each side.

$3l + 2 - 2 = 8 - 2$

$3l = 6$ Divide both sides by 3.

$\frac{3l}{3} = \frac{6}{3}$

$l = 2$ They each swim 2 laps.

Example 3: Christine and Holly went to a fair and bought homemade stickers. Each sticker cost x dollars. Christine bought 15 stickers and split the cost equally with her four siblings. Holly went to the fair three times, and every time she bought 11 stickers. Each time she went to the fair, she received a $5 discount on her stickers. Christine and Holly spent the same amount on stickers. How much did each sticker cost?

Write an expression to represent how much money each girl spent on stickers.
Note: Since Christine split the cost equally with four siblings, the cost is split among five people.

Christine: $\frac{15x}{5}$ Holly: $3(11x - 5)$

Since the girls spent the same amount on stickers, these expressions are equivalent. Write an equation by setting the expressions equal to each other.

$\frac{15x}{5} = 3(11x - 5)$ On the left, simplify the expression. On the right, distribute the 3.

$3x = 33x - 15$ Subtract $3x$ from both sides.

$3x - 3x = 33x - 15 - 3x$

$0 = 30x - 15$ Add 15 to both sides.

$0 + 15 = 30x - 15 + 15$

$15 = 30x$ Divide both sides by 30.

$\frac{15}{30} = \frac{30x}{30}$

$\frac{1}{2} = x$ Each sticker cost $\frac{1}{2}$ of a dollar, or $0.50.

Example 4: Peter enjoys 3D printing with his dad. Together they design and print mini race cars to donate to local preschools. In one week Peter and his dad printed z cars and separated those cars plus eight others equally into 13 donation boxes. The next week, Peter and his dad printed z more cars. Unfortunately, four cars were printed incorrectly. The cars that were printed successfully were put into another donation box. If each donation box ended up with the same number of cars, how many cars did Peter and his dad make each week?

Write an expression to represent the number of cars in each donation box each week.

First Week: $\frac{z+8}{13}$ Second Week: $z - 4$

Since each box ended up with the same number of cars, the expressions are equivalent. Write an equation by setting the expressions equal to each other.

$\frac{z+8}{13} = z - 4$ Multiply both sides by 13.

$13 \bullet \frac{z+8}{13} = (z-4) \bullet 13$ Distribute 13 to both terms on the right side.

$z + 8 = 13z - 52$ Subtract z from both sides.

$z + 8 - z = 13z - 52 - z$

$$8 = 12z - 52$$ Add 52 to both sides.

$$8 + 52 = 12z - 52 + 52$$

$$60 = 12z$$ Divide both sides by 12.

$$\frac{60}{12} = \frac{12z}{12}$$

$$5 = z$$ They made 5 cars each week.

PRACTICE

1. Harley and Dennis each set up a lemonade stand in their neighborhood and agree to each charge d dollars per cup of lemonade. Harley spends $10 on supplies and sells 44 cups of lemonade, while Dennis spends $8 on supplies and sells 36 cups of lemonade.

   a. Write an expression that represents Dennis's profit.
   ✦ Hint: Profit is the amount of money made after subtracting expenses.

   ____

   b. Write an expression that represents Harley's profit.

   ____

   c. Write and solve an equation to find how much they charged for each cup of lemonade if Dennis and Harley made the same amount of profit.

   ________ per cup

2. A party package comes with b bottles of bubbles. At a party, the children use six packages of bubbles and then use 12 more bottles that they find on a table. The adults grab eight packages of bubbles but end up having four bottles left over.

   a. Write an expression that shows how many bottles of bubbles the children used.

   ____

   b. Write an expression that shows how many bottles of bubbles the adults used.

   ____

   c. Suppose the children and adults used the same amount of bubbles. Write and solve an equation to find how many bottles are in a package.

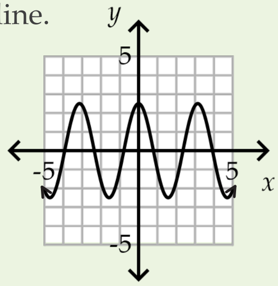
   ________ bottles per package

Linear Functions

WARM-UP

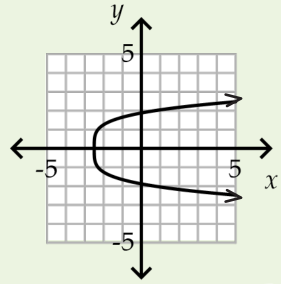
Use the vertical line test to determine if each graph represents a function. Write "yes" or "no" on the line.

a.



____

b.



____

LESSON

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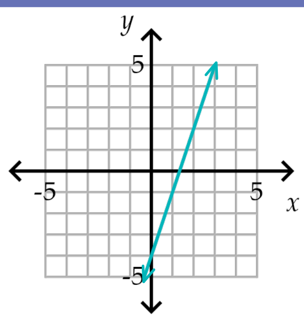
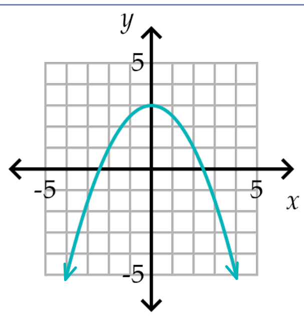
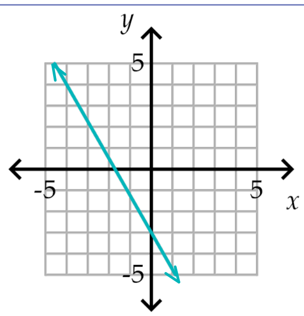
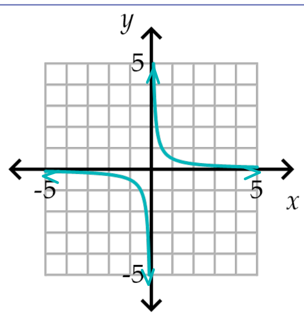
VIDEO PRACTICE

LESSON OVERVIEW

The most basic type of function is a linear function. A ***linear function*** is a function that forms a straight line when graphed.

LINEAR GRAPHS

The table below shows some examples of linear and nonlinear functions. Functions can form many different kinds of graphs that each have their own unique shape. Notice that each linear graph is a straight line.

<table>
<tr><th>Graph of Function</th><th>Linear?</th></tr>
<tr><td>
</td><td>Yes
The graph is a straight line.</td></tr>
<tr><td>
</td><td>No
The graph is a curve.</td></tr>
<tr><td>
</td><td>Yes
The graph is a straight line.</td></tr>
<tr><td>
</td><td>No
The graph contains curves.</td></tr>
</table>

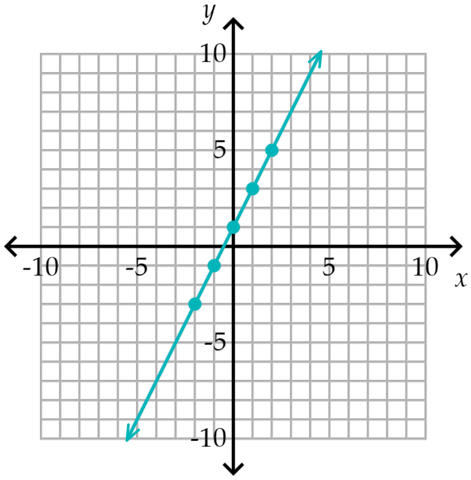
EQUATIONS OF LINEAR FUNCTIONS

Graphing an equation from a table can help determine if an equation is linear or nonlinear. When making a table, any input value that works in the equation can be chosen.

To graph the equation $y = 2x + 1$, a table is created with input values of –2, –1, 0, 1, and 2. The corresponding y-values are found by substituting the x-values into the equation. Then the points formed by the input/output pairs are plotted. The points are connected. It is clear from the graph that the points form a straight line. Therefore, $y = 2x + 1$ is a linear function.

x	y
–2	–3
–1	–1
0	1
1	3
2	5

Tip: Choose input values that are negative, positive, and zero.

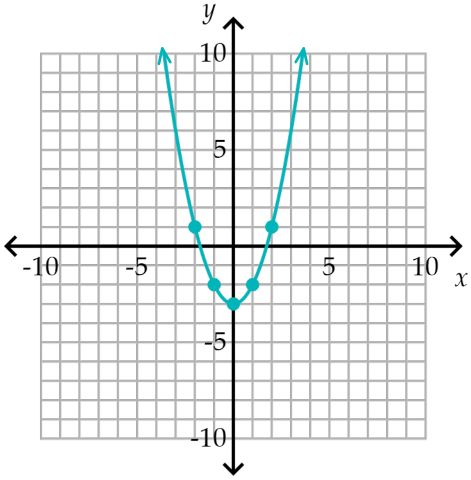


Example 1: Graph the equation $y = x^2 - 3$ to determine if it is linear or nonlinear.

Create a table with input values and find the corresponding output values.

x	y
–2	1
–1	–2
0	–3
1	–2
2	1

Plot and connect the points. The graph is not a straight line, so the equation is nonlinear.



It can be determined if an equation represents a linear or nonlinear function without graphing. A ***linear equation*** is an equation that contains only variables to the first power and constants. It is of the form $y = ax + b$, where a and b are numbers. When determining if an equation is linear or nonlinear without graphing, look at the terms. An exponent of 1 is not typically written, so any other exponent on x means the equation is nonlinear.

The table on the next page shows some linear and nonlinear functions. Notice that every linear function (except horizontal lines) only has x^1 in the equation. Nonlinear functions have x to other powers.

Linear Equations	Nonlinear Equations
$y = -x + 6$	$y = \frac{4}{x}$ Note: The power of *x* in this equation is x^{-1}.
$y = 2x - 1$	$y - 3 = x^4 + 9$
$y = x$ Note: This is of the form $y = ax + b$ where a is 1 and b is 0.	$y = 4x^2 + x - 12$ Note: There is an x^1 in this equation, but the x^2 means the equation is nonlinear.
$y - 3 = 2(x - 5)$ Note: This can be simplified to $y = ax + b$ by distributing the 2 and adding 3 to both sides.	$y = 6x^3$
$y = 5x$	$y = x^2$

Note on Horizontal and Vertical Lines

- Equations of the form $y = b$ are horizontal lines. Even though there is no x in the equation, horizontal lines are linear functions.
- Equations of the form $x = a$ are vertical lines and have x to the first power. However, vertical lines are not functions because they do not pass the vertical line test. There are multiple outputs for the input value.

Example 2: Determine if the equation $y = \frac{1}{2}(x - 2)$ is linear or nonlinear without graphing.

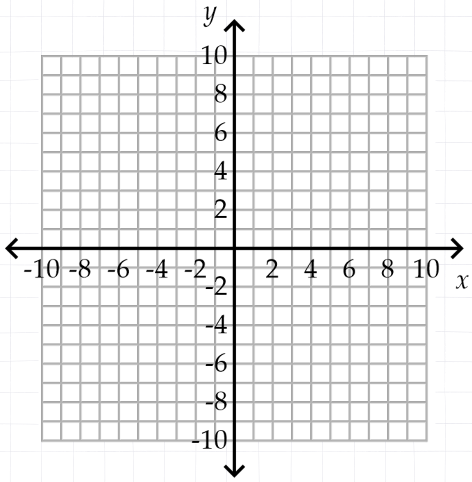
The power of x is 1 because no exponent is shown. This is a linear equation.

PRACTICE

Fill in the tables and graph each equation. Then determine if the equation represents a linear function.

1. $y = 3x - 1$

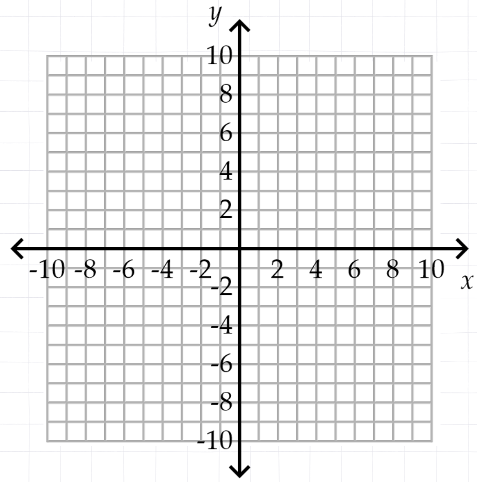
x	y
–2	
–1	
0	
1	
2	



Is it linear? ________

3. $y = 5(x + 2)$

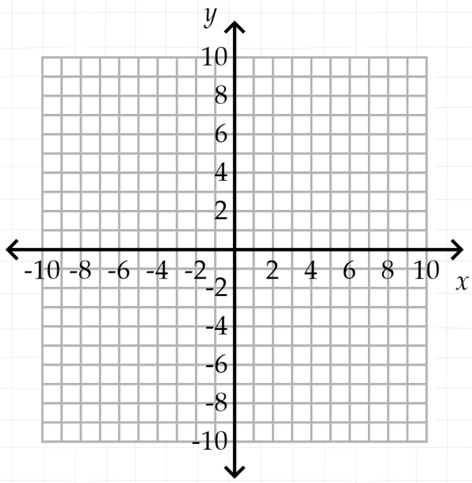
x	y
–4	
–3	
–2	
–1	
0	



Is it linear? ________

2. $y = \left(\frac{x}{5}\right)^2$

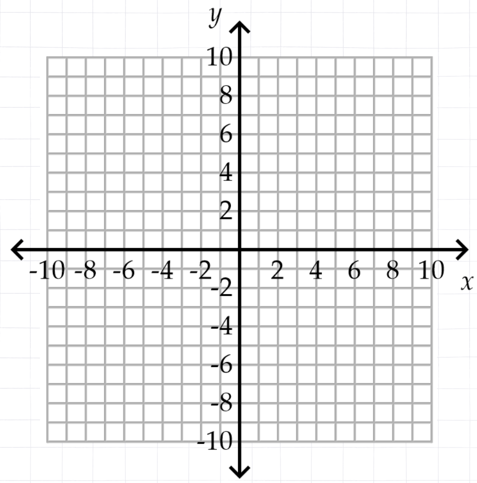
x	y
–10	
–5	
0	
5	
10	



Is it linear? ________

4. $y = 5$

x	y
–2	
–1	
0	
1	
2	

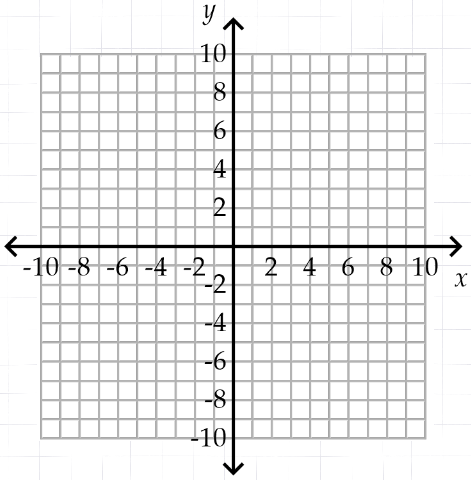


Is it linear? ________

REVIEW

1. a. Complete the table and graph the relation $y = x^2 - 5$. L37

x	y
–3	
–1	
0	
2	
3	



b. Is the relation a function? ______

2. Given the table, find the rule and write the equation for the relation. L36

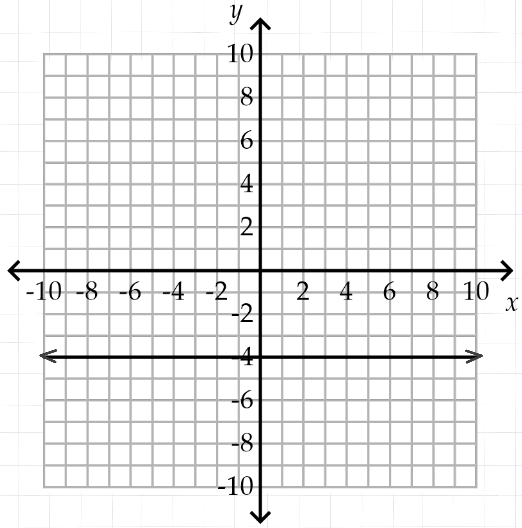
x	y
–4	–2
–2	–1
0	0
2	1
4	2

Rule: ______________________________

Equation: ______________________________

3. Write the equation of the line that is graphed below. L34

✦ Hint: All horizontal lines are of the form $y = b$.



Equation: ______________________________

4. Solve the equation. L31

$$17 - 5(x + 8) = -83$$

____

5. Mindy is making handmade bows for her family's Christmas packages. Each bow takes $1\frac{1}{3}$ feet of ribbon. How many bows can she make from a 60-foot spool of ribbon? L6

______ bows

Logic Lesson 2

THE WONDER OF WINTER

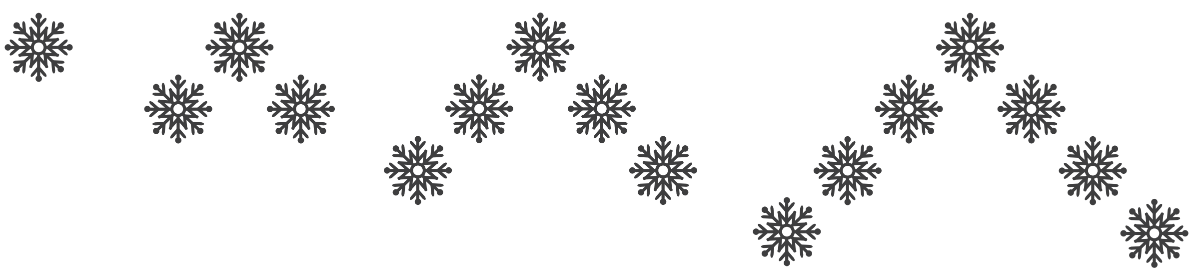
SUPPLIES: scissors

Each year, at least one septillion (1×10^{24}) snowflakes fall from the sky! Snow can be fun, beautiful, peaceful, tolerable, messy, or even annoying. Without ever naming it explicitly, Emily Dickinson shared her perspective about snow in a poem known by its first line: "It Sifts from Leaden Sieves." The logic puzzles in this lesson will refer to some of the lines from Dickinson's poem. This lesson has no video or review problems.

It sifts from Leaden Sieves -
It powders all the Wood.
It fills with Alabaster Wool
The Wrinkles of the Road -

1. Dickinson related snow falling from gray clouds and dusting the landscape in white to flour or sugar falling from a metal sieve and powdering the objects on which it lands. As snow accumulates, it covers tracks and grooves ("wrinkles") in the landscape. Suppose some snowflakes accumulated in the forest (powdering the wood) in the pattern below. Assuming the pattern continues, answer each question below.

✦ Hint: Rather than drawing more cases, think of how the number of snowflakes on the left side of each pile (including the top snowflake) corresponds to the number of seconds. Then see how the number of snowflakes in the right side of each pile (the remaining snowflakes) corresponds to the number of seconds.



1 second | 2 seconds | 3 seconds | 4 seconds

a. How many snowflakes will accumulate after 5 seconds? ______ snowflakes

b. How many snowflakes will accumulate after 25 seconds? ______ snowflakes

c. How many snowflakes will accumulate after 100 seconds? ______ snowflakes

d. How many snowflakes will accumulate after x seconds?

✦ Hint: Write and simplify an expression that uses x.

______________ snowflakes

It reaches to the Fence -
It wraps it Rail by Rail
Till it is lost in Fleeces -
It deals Celestial Vail

To Stump, and Stack - and Stem -
A Summer's empty Room -
Acres of Joints, where Harvests were,
Recordless, but for them -

2. As snow piles up, it seems to envelop objects, such as fences, starting at ground level and working upward. On farms, it hides the evidence of the previous harvest except for any stumps, stacks (of hay, for example), and stems that might stick up above the snow.

Suppose that Fran and her dad walked from their farmhouse to the nearest town to buy supplies. While they were there, it snowed enough to cover their tracks, and Fran is not sure of the way home. While her dad visits the tack store, Fran asks some of their neighbors, who are shopping at the mercantile, which of four paths leads back to her family's farm. The paths are referred to as A, B, C, and D. The neighbors' responses are recorded below.

Mrs. Cunningham: It's either B or C.
Mr. Jones: Mrs. Cunningham is completely wrong. It's either A or D.
Mrs. Smith: I'm certain it's C.
Mr. Huber: All I know is it isn't D.

Exasperated, Fran went to find her dad and told him what the neighbors had said. "Don't fret, Franny," he replied. "Fortunately, I know the way home because, unfortunately, only one of the people you asked gave you correct information!"

If only one neighbor was correct, which path should Fran and her father take?

✦ Hint: Assume one path is correct. If that path is correct, determine which statements could be true. If more than one statement could be true, the path is not correct.

Fran and her father should take path ______.

It makes an even Face
Of Mountain, and of Plain -
Unbroken Forehead from the East
Unto the East again -

3. Deep snow covering rugged terrain can make the landscape appear much more even than it actually is. Suppose that many years ago, a wise traveler sought shelter on a winter night to warm himself and not risk losing his way on the deceiving terrain in the dark. He stopped at a countryside inn where two other men were sheltered for the night. One of the men had two small loaves of cornbread, and the other had three. The hungry traveler offered five coins to be divided fairly between the two men if they would share their cornbread with him. The three of them shared the five loaves equally. How many coins should the traveler give to each of the men?

✦ Hint: Draw a picture of five loaves of bread. Split each loaf into three sections and shade the sections that one traveler would receive. Then determine how much of their loaves each traveler shared, and split their coins accordingly.

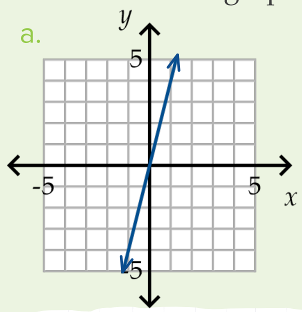
The man with two loaves of cornbread should receive ______ coin(s),
and the man with three loaves should receive ______ coin(s).

Graphing from Standard Form

WARM-UP

Determine if each graph represents a proportional relationship. Circle the correct answer.

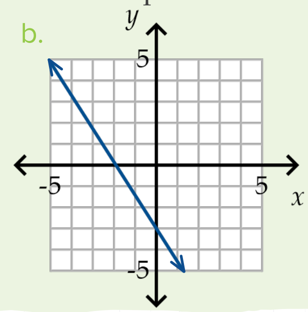
a.



proportional

not proportional

b.



proportional

not proportional

LESSON

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VIDEO PRACTICE

LESSON OVERVIEW

Recall that an x-intercept is the point on a graph where the line crosses the x-axis, and a y-intercept is the point where the line crosses the y-axis. At each intercept, the other variable is zero. For example, at the x-intercept, the value of y is zero, and at the y-intercept, the value of x is zero. Because of this, working with and using intercepts is a simple, but quite useful, process.

Two forms of linear equations that have been studied in this course are slope-intercept form and point-slope form. A third form of a linear equation is often used when working with intercepts. ***Standard form*** is a linear equation of the form $Ax + By = C$, where A, B, and C are integers.

Some equations written in standard form are shown below along with the values of A, B, and C for each equation.

$4x+5y=12$	$-3x+15y=30$	$x-y=-2$
$A=4,\ B=5,\ C=12$	$A=-3,\ B=15,\ C=30$	$A=1,\ B=-1,\ C=-2$

Linear equations written in standard form can be easily graphed by identifying the x- and y-intercepts. The coordinates of any x-intercept can be written as $(x,0)$. Substituting zero for y and solving the equation for x will yield the x-coordinate of the x-intercept. Likewise, the coordinates of any y-intercept can be written as $(0,y)$. Substituting zero for x and solving the equation for y will yield the y-coordinate of the y-intercept.

Given the equation $3x+6y=12$, the x-intercept and y-intercept are found below.

x-intercept:

Substitute zero for y and solve for x.

$$3x+6(0)=12$$
$$3x=12$$
$$\frac{3x}{3}=\frac{12}{3}$$
$$x=4$$

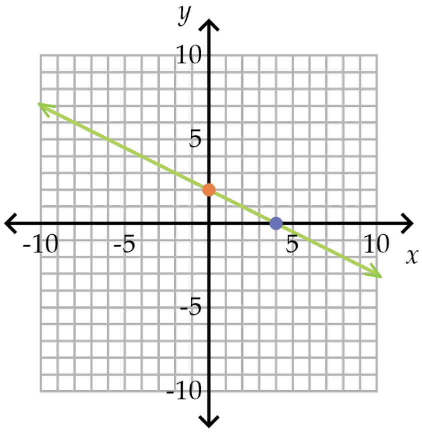
The x-intercept is $(4,0)$.

y-intercept:

Substitute zero for x and solve for y.

$$3(0)+6y=12$$
$$6y=12$$
$$\frac{6y}{6}=\frac{12}{6}$$
$$y=2$$

The y-intercept is $(0,2)$.



Because only two points are needed to draw a line, the x- and y-intercepts can be plotted on a graph to create the line for the linear equation. To the left is the graph of the line $3x+6y=12$ using the intercepts $(4,0)$ and $(0,2)$.

Once the line is graphed, the slope can be found using rise over run. The slope of this line is –2 (down 2) over 4 (right 4), which is $-\frac{1}{2}$.

KEY INFORMATION

To find an intercept, substitute zero for the other variable.

Example 1: Use the x- and y-intercepts to graph the equation $2x+3y=-18$. Then find the slope of the line.

Find the x-intercept:

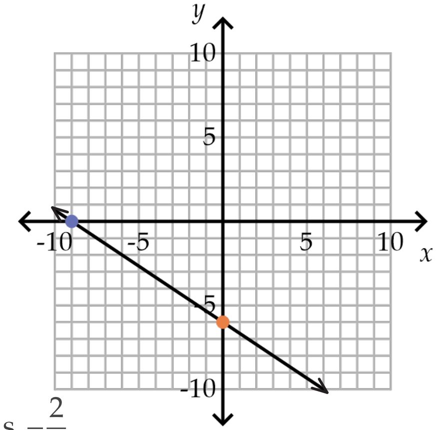
$$2x+3(0)=-18$$
$$2x=-18$$
$$\frac{2x}{2}=\frac{-18}{2}$$
$$x=-9$$

x-intercept: $(-9,0)$

Find the y-intercept:

$$2(0)+3y=-18$$
$$3y=-18$$
$$\frac{3y}{3}=\frac{-18}{3}$$
$$y=-6$$

y-intercept: $(0,-6)$



The slope is –6 (down 6) over 9 (right 9), which is $-\frac{2}{3}$.

Example 2: Use the x- and y-intercepts to graph the equation $5x-10y=20$. Then find the slope of the line.

Find the x-intercept:

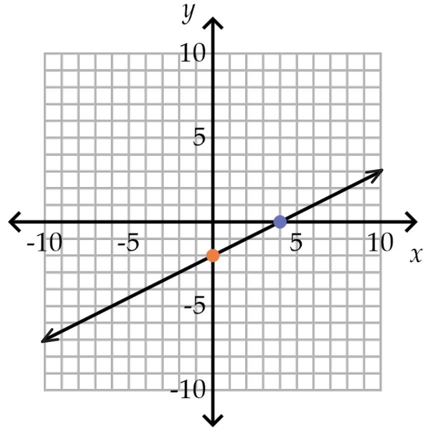
$$5x-10(0)=20$$
$$5x=20$$
$$\frac{5x}{5}=\frac{20}{5}$$
$$x=4$$

x-intercept: $(4,0)$

Find the y-intercept:

$$5(0)-10y=20$$
$$-10y=20$$
$$\frac{-10y}{-10}=\frac{20}{-10}$$
$$y=-2$$

y-intercept: $(0,-2)$



The slope is 2 (up 2) over 4 (right 4), which is $\frac{1}{2}$.

Example 3: Use the x- and y-intercepts to graph the equation $x-y=9$. Then find the slope of the line.

Find the x-intercept:

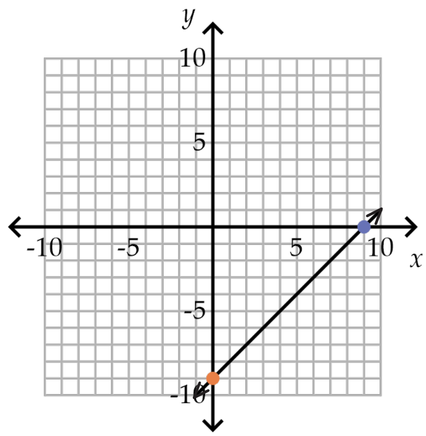
$$x-0=9$$
$$x=9$$

x-intercept: $(9,0)$

Find the y-intercept:

$$0-y=9$$
$$-y=9$$
$$\frac{-y}{-1}=\frac{9}{-1}$$
$$y=-9$$

y-intercept: $(0,-9)$



The slope is 9 (up 9) over 9 (right 9), which is 1.

PRACTICE

Complete each problem below. Find the answer in the table on the next page and cross off the phrase next to it. Once all problems have been completed, write the remaining phrases, from top to bottom, at the bottom of the page to discover a neat fact about God's creation!

1. Find the x-intercept for each equation.

   a. $5x - 4y = 60$ ________

   b. $-x + 2y = 6$ ________

   c. $3x + 5y = -120$ ________

   d. $-7x - 5y = 105$ ________

2. Find the y-intercept for each equation.

   a. $5x - 4y = 60$ ________

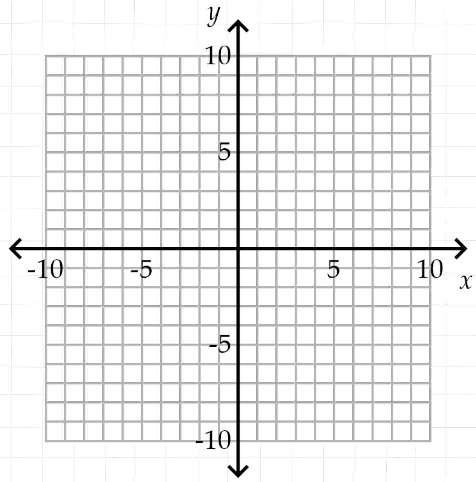
   b. $-x + 2y = 6$ ________

   c. $3x + 5y = -120$ ________

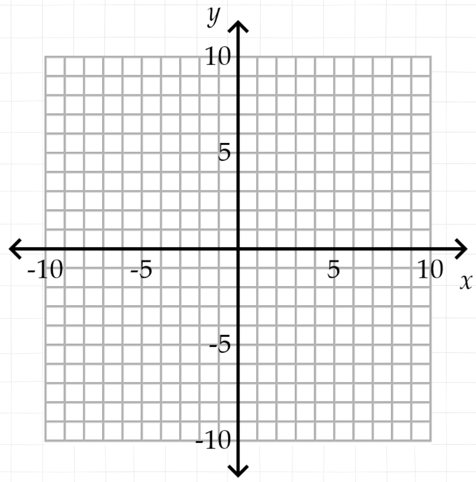
   d. $-7x - 5y = 105$ ________

3. Use the x- and y-intercepts to graph each equation. Then use the graph to find the slope of each line. The slope is the answer to cross off in the table on the next page.

   a. $-3x - 4y = 24$ Slope: ________



   b. $6x - 8y = 24$ Slope: ________



The Pythagorean Theorem

SUPPLIES: scissors

WARM-UP

Solve the equation.

$(8+c)^2 - 15 = 34$

____

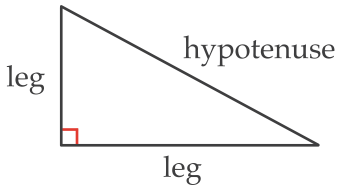
LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

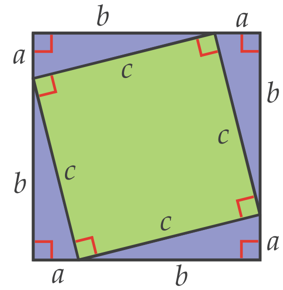
LESSON OVERVIEW

The Pythagorean theorem is named for the ancient Greek philosopher Pythagoras. A theorem is a proven statement in math. While records of reference to the Pythagorean theorem go back thousands of years, this important formula is widely used today in construction and distance calculations.

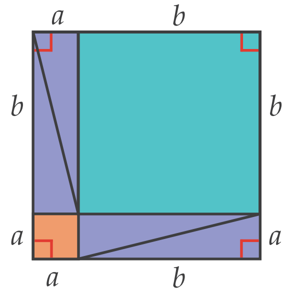


In a right triangle, one angle is a right angle as indicated by the red box. The ***Pythagorean theorem*** states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two side lengths. The ***hypotenuse*** is the longest side of a right triangle. It is the side opposite, or across from, the right angle. The ***legs*** are the two sides of a right triangle that are adjacent to the right angle. Below is the formula for the Pythagorean theorem, where a and b represent the legs and c represents the hypotenuse.

Pythagorean theorem: $a^2 + b^2 = c^2$



The Pythagorean theorem can be proven by considering a square with side lengths of $a + b$ as shown in the first diagram at the right. Notice that the inside of the square is made up of four right triangles (in purple), each with side lengths a, b, and c, and the center is a square (in green) whose area is c^2 because all side lengths are c.



If the purple triangles are rearranged, two smaller squares (in orange and blue) are formed as shown in the second diagram at the right. Notice that one smaller square has side lengths a and the other has side lengths b. The area of the smaller squares is a^2 and b^2, respectively.

Since the total area must be the same in each diagram, the sum of areas of the orange square and blue square must equal the area of the green square. Therefore, $a^2 + b^2 = c^2$.

PYTHAGOREAN TRIPLES

Pythagorean triples are groups of three integers that satisfy the Pythagorean theorem.

The integers 3, 4, and 5 are a Pythagorean triple because the sum of 3^2 and 4^2 is equal to 5^2.

$3^2 + 4^2 = 9 + 16 = 25$ $\qquad$ $5^2 = 25$

The numbers 6, 8, and 10 are also a Pythagorean triple because $6^2 + 8^2 = 10^2$. To determine if three numbers are a Pythagorean triple, add the squares of the two smaller numbers and see if it equals the square of the largest number.

Example 1: Do the numbers 4, 8, and 9 form a Pythagorean triple?

Add the squares of the two smaller numbers. $\quad 4^2 + 8^2 = 16 + 64 = 80$
Find the square of the largest number. $\quad 9^2 = 81$

Since $4^2 + 8^2$ does not equal 9^2, the numbers 4, 8, and 9 do not form a Pythagorean triple.

PRACTICE

1. a. Fill in the blanks to verify that 5, 12, and 13 form a Pythagorean triple.

   $5^2 + 12^2 =$ ______ + ______ = ______

   $13^2 =$ ______

   b. Multiply 5, 12, and 13 each by 3.

   $5 \bullet 3 =$ ______ $12 \bullet 3 =$ ______ $13 \bullet 3 =$ ______

   c. Check if the values found in Part B are also Pythagorean triples. Circle "yes" or "no."

   ______^{2} + ______^{2} = ______ + ______ = ______

   ______^{2} = ______

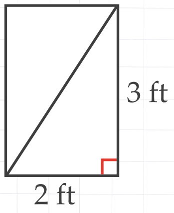
   Pythagorean triple? yes / no

2. Determine if a triangle with the given side lengths is a right triangle. Write "yes" or "no" on the line.

   a. 4 km, 7 km, 8 km ______

   b. 20 in, 21 in, 29 in ______

3. Sarah wants to reinforce the side of a crate that is 3 ft by 2 ft by bracing it with a diagonal beam as shown below. She finds a piece of wood that is 4 ft long. Will that piece work for the brace without her having to cut it?



______

4. Follow the steps to create a proof of the Pythagorean theorem.

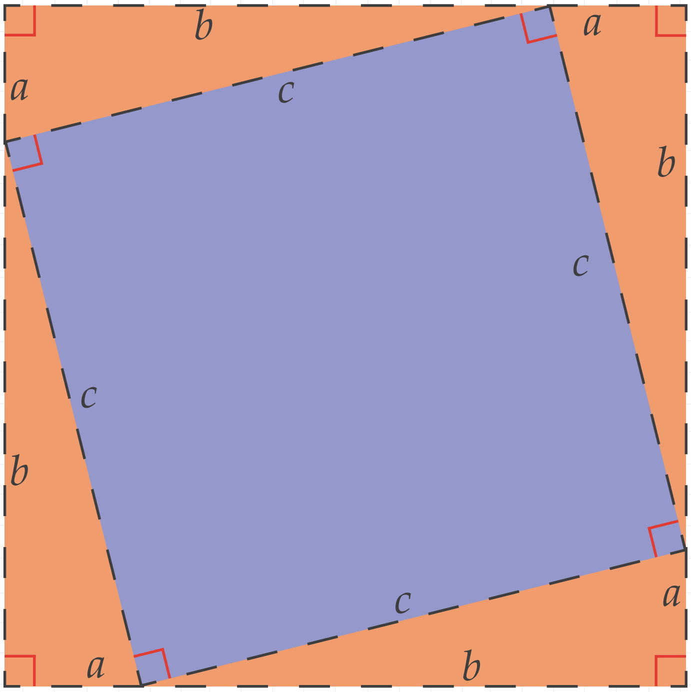
   a. Consider the square on the bottom of this page. The smaller square inside has side length c. What is the area of the purple square?

   ✦ Hint: Do not measure. Write the area using a variable.

   ______

   Note that this is the area of the large square that is NOT covered by the four orange triangles.

   b. Cut on all dashed lines to cut out the four orange triangles and the purple square below.



UNIT 2 | LESSON 56

Fractions, Decimals, and Percents

WARM-UP

Complete the problem and state the meaning of the answer.

45 is what fraction of 75? ________

Meaning: ____

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

Fractions, decimals, and percents are all ways of expressing part of a whole. Knowing how to convert between each representation builds number sense and aids in understanding when encountering numbers in different forms.

CONVERTING BETWEEN FRACTIONS AND DECIMALS

Fraction → Decimal

Convert a fraction to a decimal by dividing the numerator by the denominator.

To convert $\frac{17}{20}$ to a decimal, divide 17 by 20.

$$\begin{array}{r} 0.85 \\ 20\overline{)17.00} \\ -\underline{160} \\ 100 \\ -\underline{100} \\ 0 \end{array}$$

$$\frac{17}{20} = 0.85$$

To convert $4\frac{5}{8}$ to a decimal, divide 5 by 8 and write the whole number, 4, before the decimal point.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -\underline{48} \\ 20 \\ -\underline{16} \\ 40 \\ -\underline{40} \\ 0 \end{array}$$

$$4\frac{5}{8} = 4.625$$

Decimal → Fraction

Convert a decimal to a fraction by writing the decimal digits in the numerator and the place value of the last decimal digit in the denominator. Simplify the fraction.

To convert 0.35 to a fraction, write the decimal digits, 35, over the place value of the 5, which is the hundredths place. Simplify.

$$0.35 = \frac{35}{100} = \frac{7}{20}$$

To convert 8.024 to a fraction, write the decimal digits, 24, over the place value of the 4, which is the thousandths place. Keep the whole number. Simplify the fraction.

$$8.024 = 8\frac{24}{1000} = 8\frac{3}{125}$$

CONVERTING BETWEEN DECIMALS AND PERCENTS

Percent → Decimal

Percent means "per hundred." Convert a percent to a decimal by dividing by 100. Dividing by 100 moves the decimal point two places to the left.

To convert 5% to a decimal, move the decimal point two places to the left.

5.

5% = 0.05

To convert 200% to a decimal, move the decimal point two places to the left.

200.

200% = 2

Decimal → Percent

Convert a decimal to a percent by multiplying by 100. Multiplying by 100 moves the decimal point two places to the right.

To convert 0.85 to a percent, move the decimal point two places to the right.

0.85

0.85 = 85%

To convert 3.25 to a percent, move the decimal point two places to the right.

3.25

3.25 = 325%

Note: 1 is 100%. A number greater than 1 is more than 100%.

CONVERTING BETWEEN PERCENTS AND FRACTIONS

Percent → Fraction

Convert a percent to a fraction by writing the percent over 100 and simplifying.

To convert 55% to a fraction, write 55 over 100 and simplify.

$$55\% = \frac{55}{100} = \frac{11}{20}$$

To convert 120% to a fraction, write 120 over 100 and simplify.

$$120\% = \frac{120}{100} = \frac{6}{5} = 1\frac{1}{5}$$

Note: A percent greater than 100% can be written as an improper fraction or a mixed number in fraction form.

Fraction → Percent

Convert a fraction to a percent by doing one of the following:

- Make an equivalent fraction with a denominator of 100. The numerator is the percent.
- If the original denominator is not a factor of 100, convert the fraction to a decimal by dividing. Then convert the decimal to a percent by moving the decimal point two places to the right.

To convert $\frac{3}{25}$ to a percent, write an equivalent fraction with 100 in the denominator. The numerator, 12, is the percent.

$$\frac{3}{25} = \frac{12}{100} = 12\%$$

To convert $\frac{1}{8}$ to a percent, divide 1 by 8 because 8 is not a factor of 100. Then move the decimal point two places to the right.

$$\begin{array}{r} 0.125 \\ 8\overline{)1.000} \\ -\underline{8} \\ 20 \\ -\underline{16} \\ 40 \\ -\underline{40} \\ 0 \end{array}$$

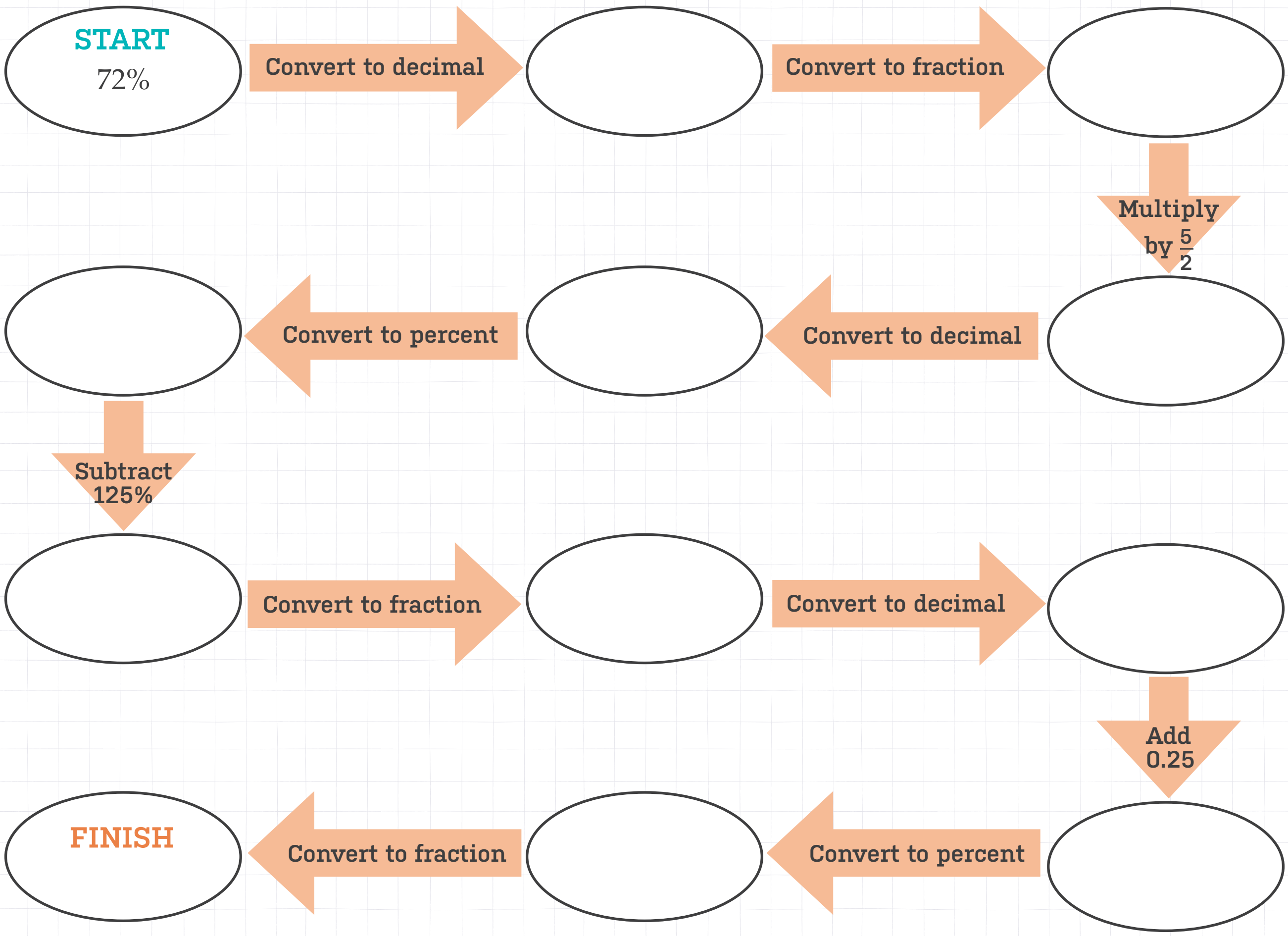
$$\frac{1}{8} = 0.125 = 12.5\%$$

To convert $3\frac{4}{5}$ to a percent, write an equivalent fraction with 100 in the denominator. The whole number, 3, is 300%.

$$3\frac{4}{5} = 3\frac{80}{100} = 300\% + 80\% = 380\%$$

PRACTICE

1. Begin at START. Follow the instructions to reach FINISH.



2. There are 50 plates at a party, 18 of which are red. What percent of the plates are red?

______

3. Of the people at the party, 74% are children. What fraction of the people at the party are children?

______

4. A banner at the party is 0.8 meters high and is printed on paper that is 1 meter high. What fraction of the paper height is not being used for the banner?

______

5. There were 40 people who said they were coming to the party, but 50 people actually attended. What percent of the people who said they were coming actually attended? In other words, 50 is what percent of 40?

______

Unit 2 Review

Complete this Unit Review to prepare for the Unit Assessment. There is no video, lesson, or practice. Because Unit Reviews include practice for an entire unit, they may take longer than regular lessons, and students may decide to take two days to finish.

Hannah is away at Winter Camp with some of her friends from church. Complete the problems related to Hannah's trip as you review material from Unit 2.

LESSON 32

1. On the first day of Winter Camp, Hannah and her friends go for a hike. They start at an elevation of 2700 ft above sea level. Their elevation increases 726 ft every hour. They finish at an elevation that is twice their initial elevation. Write an equation to model the situation, using t for the time they spent hiking, in hours.

____

LESSON 31

2. Solve the following equations.

a. $3x+1=9-x$ b. $\frac{(7+19x)}{5}=3+4x$

$x=$ ____ $x=$ ____

LESSON 33

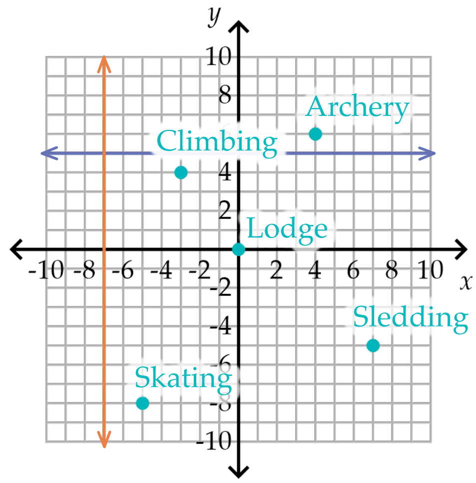
3. Hannah and her friends build a snow fort that has a perimeter given by the equation below, where A is the area of the fort. Solve the equation for A.

$$P=\frac{3}{2}A+5$$

____

LESSON 34

4. Hannah's group is given the following map of the campground.



a. In which quadrant is each activity located?

Archery: ____ Climbing: ____

Sledding: ____ Skating: ____

b. Determine the coordinates of the indicated area.

Skating: ________ Lodge: ________

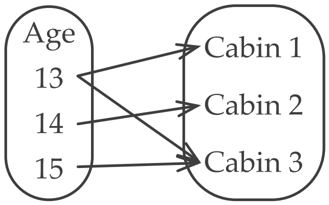
c. The purple and orange lines represent trails through the grounds. Find the equation of each line.

Purple: ________ Orange: ________

d. The camp director said the map is several years old and that bathrooms have been added at the point $(-9,8)$. Plot the bathrooms on the coordinate plane above.

LESSONS 35, 36

5. The following mapping diagram shows how Hannah and her three friends are assigned to cabins, relative to their ages.



a. Determine the domain and range of this relation in set notation.

Domain: ______________________________

Range: ______________________________

b. Is the relation a function? ________

Why or why not? ______________________

____

c. Identify the independent and dependent variables.

Independent variable: _________________

Dependent variable: ___________________

6. The lodge makes hot cocoa based on how many people are staying there each day. The table below shows how many packages of cocoa the lodge makes relative to the number of campers. Find the rule and write the equation for the relation.

Number of Campers	Number of Cocoa Packages
10	4
20	8
30	12
40	16

Rule: ________________________________

Equation: _____________________________

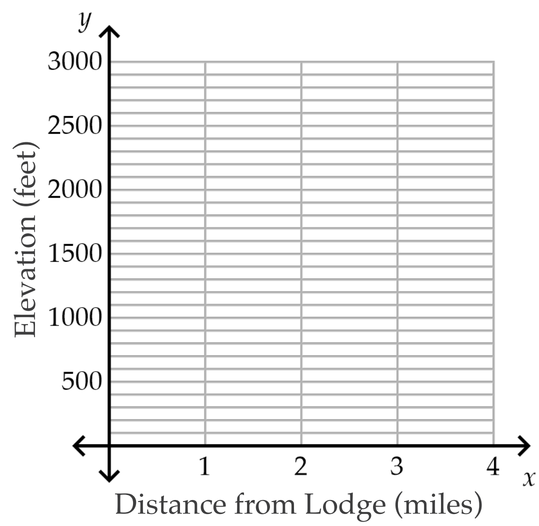
LESSON 37

7. The elevation down the side of the mountain can be modeled by the equation $y = -500x + 2700$, where x is the distance from the lodge (in miles) and y is the elevation (in feet) above sea level.

a. Use the equation to fill in the table.

x	y
0	
1	
2	
3	
4	

b. Use the table in Part A to graph the relation.



Unit 2 Assessment

○ This assessment covers concepts taught in Unit 2. Problems are designed to assess multiple skills. Read the instructions carefully and do not rush through the problems.

○ You may use the Reference Chart for the assessment. Calculators should only be used when noted. Lesson numbers are given by each problem so you can review lessons for any answers that are incorrect.

1. Solve the equations. L31

a. $3x + 18 - 2x = 9x - 30$

____

b. $5(a - 12) = 3a + 25$

____

c. $\frac{36 - p}{4} = p - 1$

____

2. Before a family trip, Logan and his sister earned money to buy souvenirs by doing extra chores. They each earned m dollars for every extra chore they did. Logan did two extra chores. He spent his extra chore money, plus \$10 he had saved, on souvenirs. His sister did eight extra chores and spent \$2 less than the amount she earned. L32

a. Write an expression that shows how much money Logan spent.

____

b. Write an expression that shows how much money his sister spent.

____

c. Suppose that Logan and his sister spent the same amount of money on souvenirs. Write and solve an equation to find how much they earned for each extra chore.

________ per extra chore

3. The formula for the area of a rectangle is $A = lw$.

a. Solve the formula for w.

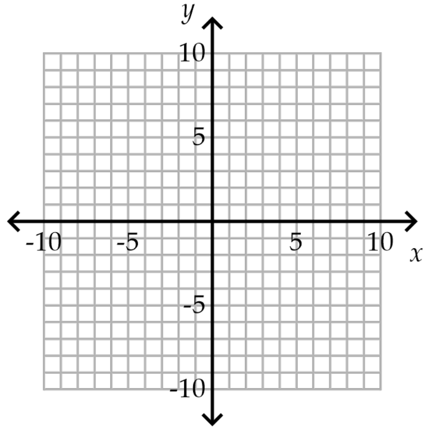
____

b. Find the width of a rectangle with an area of 27 ft^2 and a length of 9 ft. L33

____

10. For the equation $y = -3x + 2$, find the slope and y-intercept and graph the line. L41

Slope: ______ y-intercept: ________



11. Write the equation of the line in point-slope form that has the given slope and passes through the given point. Then convert the equation to slope-intercept form. L42

Slope: $\frac{3}{2}$ Point: $(4, -1)$

Point-slope form: _________________

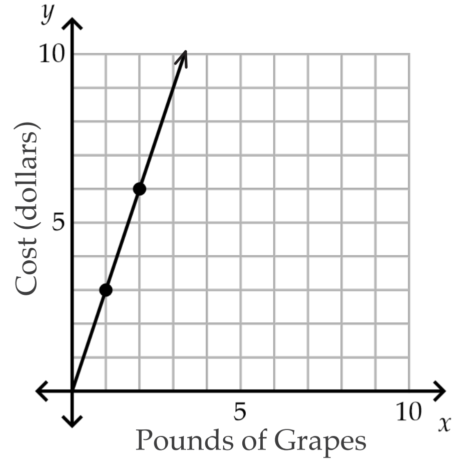
Slope-intercept form: ____________

12. Write the equation of the line that passes through the points below. L43

$(-4, -7)$ and $(0, -4)$

____

13. Use the graph to find the cost per pound of grapes. Write the equation of the line. L44



Cost: __________

Equation: _____________

14. Find the x- and y-intercepts for each equation. L46

a. $4x + 3y = 24$

x-intercept: ______ y-intercept: ______

b. $-x + y = -36$

x-intercept: ______ y-intercept: ______

Enrichment: Collatz Conjecture

This is an enrichment lesson. Mastery is not expected at this level. There is no video, practice, or review. A calculator may be used for this entire lesson.

Consider a number game in which you start with any positive integer and follow just one of two rules at each step of the game. The rules are below.

- ✦ If the number is even, divide it by two.
- ✦ If the number is odd, multiply it by three and add one.

The steps are then repeated continually. Observe what happens when starting with the integer 5.

Starting integer: 5

5 is odd.	$5 \bullet 3 + 1 = 16$
16 is even.	$16 \div 2 = 8$
8 is even.	$8 \div 2 = 4$
4 is even.	$4 \div 2 = 2$
2 is even.	$2 \div 2 = 1$
1 is odd.	$1 \bullet 3 + 1 = 4 \leftarrow$ This is a repeat of a previous answer.

Once a repeat answer is found, the process will just loop. In this example, the answers will cycle as shown below.

$$4 \rightarrow 2 \rightarrow 1$$

Try it!

Begin with the given integer and continue until you reach a repeat answer. Write the first repeat answer on the line.

1. Starting integer: 6

Repeat answer: _____

2. Starting integer: 7

Repeat answer: _____

3. Starting integer: 8

Repeat answer: _____

4. What do you notice about the first repeated answer in the examples above?

____

____

In Problem 8, the cycle of answers becomes this:

$$-5 \rightarrow -14 \rightarrow -7 \rightarrow -20 \rightarrow -10$$

In contrast, Problem 9 has a smaller loop:

$$-2 \rightarrow -1$$

Problem 10 has a much larger loop:

$$-17 \rightarrow -50 \rightarrow -25 \rightarrow -74 \rightarrow -37 \rightarrow -110 \rightarrow -55 \rightarrow -164 \rightarrow -82$$
$$\rightarrow -41 \rightarrow -122 \rightarrow -61 \rightarrow -182 \rightarrow -91 \rightarrow -272 \rightarrow -136 \rightarrow -68 \rightarrow -34$$

When observing negative integers, the Collatz conjecture is clearly not true because at least three different cycles are obtained as shown above. Mathematicians do not yet know if these three negative loops are the only loops that can be obtained when starting with a negative integer or whether there are others. It is fascinating that math problems exist that are simple to understand but that no one actually knows the answers to yet!

Fun fact: Mathematical loops are widely used in coding and computer programming!