LESSONS
91–120

UNIT
4

Simply Good and Beautiful

PRE-ALGEBRA

COURSE BOOK 4

MATH
8

The Good and the Beautiful Curriculum

COURSE BOOK 4

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LESSONS 91–120

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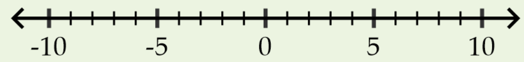
Graphing Linear Inequalities

SUPPLIES: colored pencils

WARM-UP

Solve and graph the inequality.

$4t - 8 \geq 9t - 23$



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LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

Equations and inequalities have similar properties. Recall that a linear function forms a straight line when graphed. A linear *inequality* is a linear function written with a <, >, ≤, or ≥. Just as with linear equations, linear inequalities contain only variables to the first power and constants. Some examples of linear inequalities in different forms are shown below.

Slope-Intercept Form:	$y < 3x - 12$	$y \le -\frac{2}{3}x + 8$	$y > x$
Standard Form:	$3x - 5y > 15$	$2x + 18y \le -48$	$x + y < 22$

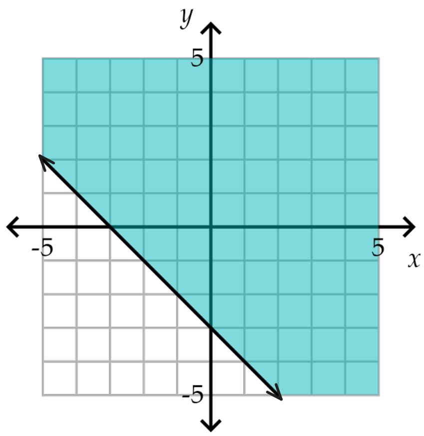
Graphing a linear inequality is similar to graphing a linear equation. Once the inequality is in slope-intercept form, plot the *y*-intercept and use the slope to plot additional points. However, a linear inequality is not always drawn with a solid line.

* Use a dashed line for inequalities with < or > symbols.
  A dashed line shows that points on the line itself are *not* part of the solution set.
* Use a solid line for inequalities with ≤ or ≥ symbols.
  A solid line shows that points on the line itself *are* part of the solution set.

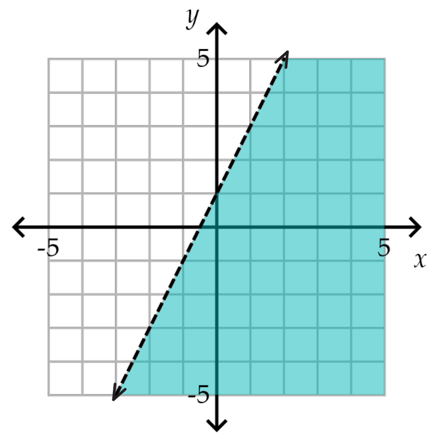
A linear inequality has infinite solutions. Recall that when graphing an inequality on a number line, the number line is shaded to show all possible solutions. Similarly, when graphing a *linear* inequality, the coordinate plane is shaded to show all possible solutions. The coordinate plane is shaded on one side of the line depending on where the solution set lies. In general, when inequalities are in slope-intercept form, a < or ≤ symbol indicates that the solution set is "below" the line, and a > or ≥ symbol indicates that the solution set is "above" the line.

Two examples of linear inequalities and their graphs are shown below.

$y \ge -x - 3$
- solid line
- shaded "above" the line

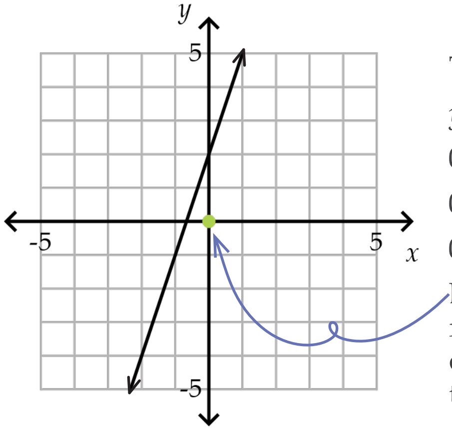


$y < 2x + 1$
- dashed line
- shaded "below" the line



To determine which side of the line to shade on, substitute an ordered pair into the inequality and see if a true statement results. If the ordered pair satisfies the inequality, shade on the side of the line where the ordered pair is located. If the ordered pair does not satisfy the inequality, shade on the other side of the line. Any ordered pair other than one located on the line itself may be used to determine where to shade.

The line for the inequality $y \le 3x + 2$ is shown below. To determine where to shade, the ordered pair $(0,0)$ is substituted into the inequality to see if a true statement results.



Test $(0,0)$:

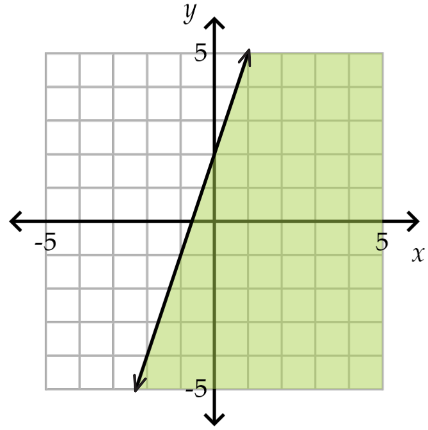
$y \le 3x + 2$

$0 \overset{?}{\le} 3(0) + 2$

$0 \overset{?}{\le} 0 + 2$

$0 \overset{?}{\le} 2$ ✓

Because a true statement results, shade on this side of the line. $(0,0)$ is part of the solution.



Example 1: Graph the inequality $y < 2x + 5$.

Slope: 2 y-intercept: $(0,5)$ Use a dashed line.

Test $(0,0)$ to determine shading:

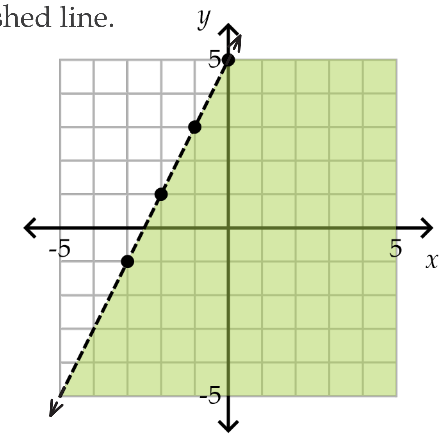
$y < 2x + 5$

$0 \overset{?}{<} 2(0) + 5$

$0 \overset{?}{<} 0 + 5$

$0 \overset{?}{<} 5$ ✓

Shade on the side of the line where $(0,0)$ is located.



Example 2: Graph the inequality $y \ge -\frac{2}{3}x$.

Slope: $-\frac{2}{3}$ y-intercept: $(0,0)$ Use a solid line.

Because $(0,0)$ is on the line, it cannot be used as a test point. Use a point with numbers that are easy to work with.

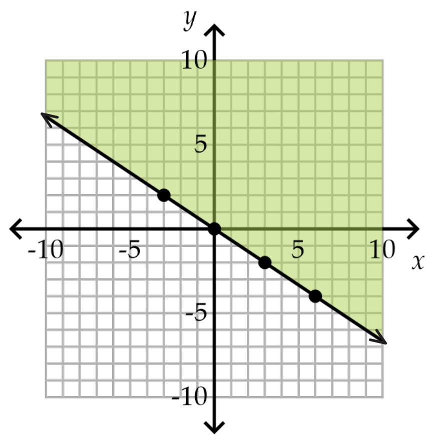
Test $(3,0)$:

$y \ge -\frac{2}{3}x$

$0 \overset{?}{\ge} -\frac{2}{3}(3)$

$0 \overset{?}{\ge} -2$ ✓

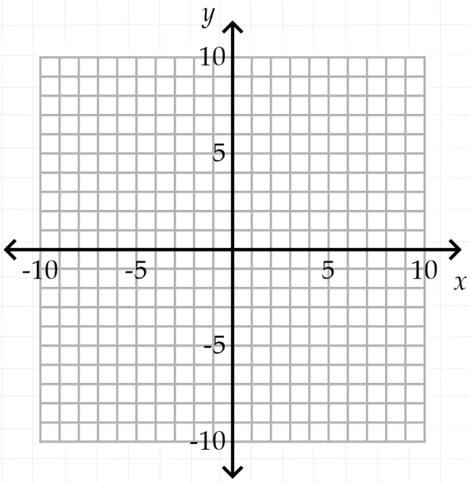
Shade on the side of the line where $(3,0)$ is located.



PRACTICE

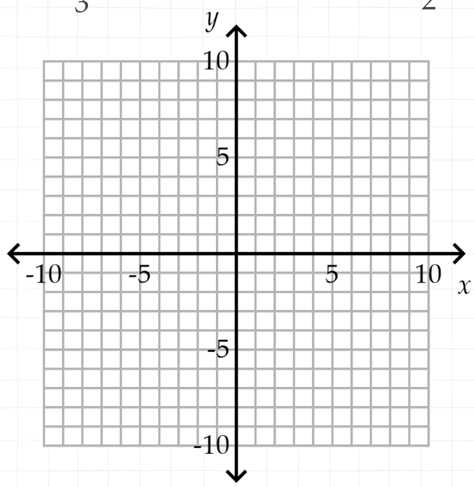
1. Graph the inequalities. Use the indicated colors to shade.

Red: $y > 2x - 6$ Yellow: $y < \frac{3}{4}x$



3. Graph the inequalities. Use the indicated colors to shade.

Blue: $y \leq -\frac{5}{3}x + 3$ Yellow: $y > \frac{1}{2}x - 1$



2. Fill in the table to see if each ordered pair satisfies each inequality. Write "yes" or "no" in each box. Use the graph from Problem 1 to check your answers.

✦ Hint: If a point satisfies both inequalities, it will be shaded orange.

Ordered pair	$y > 2x - 6$	$y < \frac{3}{4}x$
$(0,4)$		
$(0,-4)$		
$(4,0)$		
$(-4,0)$		

4. Fill in the table to see if each ordered pair satisfies each inequality. Write "yes" or "no" in each box. Use the graph from Problem 3 to check your answers.

✦ Hint: If a point satisfies both inequalities, it will be shaded green.

Ordered pair	$y \leq -\frac{5}{3}x + 3$	$y > \frac{1}{2}x - 1$
$(0,4)$		
$(0,-4)$		
$(4,0)$		
$(-4,0)$		

REVIEW

1. Solve the inequality in Part A and Part B.

a. $-15 - 2x \le 24$ L87

______

b. $\frac{5}{3}x + 18 > 2x + 22$ L91

______

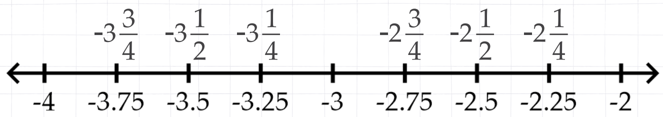
c. Color the circle next to each value blue if it satisfies the inequality in Part A, red if it satisfies the inequality in Part B, and purple if it satisfies both inequalities. L87

- ○ $x = 0$
- ○ $x = -10$
- ○ $x = -30$
- ○ $x = -16$

2. 252 is $\frac{21}{4}$ of what number? L55

______

3. Complete each problem mentally. Use the number line model for help. Write each answer as an integer.



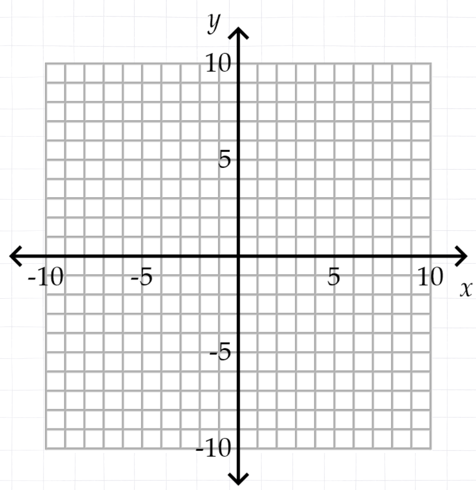
a. $-2.25 + 0.25$ ______

b. $-2\frac{1}{4} - \frac{3}{4}$ ______

c. $-2\frac{1}{2} - 1\frac{1}{2}$ ______

4. Use the x- and y-intercepts to graph the equation. Then use the graph to find the slope of the line. L46

$$9x - 6y = -18$$



Slope: ______

Polynomials

WARM-UP

Find the coordinates of the image if the preimage is dilated from the origin by a factor of 3.

Preimage	$(3, -2)$	$(7, -2)$	$(7, -4)$	$(3, -4)$
Image				

LESSON

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VIDEO PRACTICE

LESSON OVERVIEW

Operations can be performed on expressions with multiple terms and variables as well as on individual numbers. Recall that an ***expression*** is a number, variable, or combination of numbers and variables joined by operations. A ***term*** is one part of an expression, which may be a number, a variable, or a product of numbers and variables. The type of expression discussed in today's lesson is a polynomial. A ***polynomial*** is an expression with variables, coefficients, and constants in which terms are combined by addition, subtraction, or multiplication. Below are some examples of polynomials:

$-3x+5$ $\quad$ $23a-15b^5+ab$ $\quad$ $y^2+12xy-3x^2$ $\quad$ $fg^3h^5+11g^5h$

Polynomials can be classified by the number of terms they have. The table below shows the names of polynomials with one, two, and three terms.

Number of terms	Type of polynomial	Examples
1	monomial	$3x^4$ $\quad$ $-ab^2$ $\quad$ $23g$
2	binomial	x^2+y^5 $\quad$ $-18ac+6c^5$ $\quad$ $14+v^5$
3	trinomial	x^2+xy-y^2 $\quad$ $2a^5-4a^4b+a^3b^2$

Note: Polynomials with more than three terms are just called polynomials.

Like terms are terms with the same variables raised to the same power. Below are some examples of like terms.

x^2 and $-3x^2$
Both terms contain x^2.

ab^2c^5 and $10c^5b^2a$
Both terms contain ab^2c^5.

$24gh$ and $-gh$
Both terms contain gh.

A polynomial can be simplified by combining like terms. To combine like terms, add or subtract the coefficients. The variables and exponents on each variable remain the same. When all like terms have been combined, the polynomial is simplified.

Example 1: Simplify the polynomial.

$34x+2x^2-15x-5x^2$

$=34x+2x^2-15x-5x^2$

$=34x-15x+2x^2-5x^2$

$=19x-3x^2$

The orange terms can be combined by adding 34 and –15. The blue terms can be combined by adding 2 and –5.

Note: These are not like terms. The polynomial is simplified.

Example 2: Simplify the polynomial.

$125ac^2+55ac^2-64ab+24ab$

$=125ac^2+55ac^2-64ab+24ab$

$=180ac^2-40ab$

The orange terms can be combined by adding 125 and 55. The blue terms can be combined by adding –64 and 24.

ADDING AND SUBTRACTING POLYNOMIALS

To add two polynomials together, combine all like terms. In the example below, the polynomials $34x^2 + 5y - 10$ and $10x^2 - 8x + 3y$ are added together. Terms are first rearranged so like terms are next to each other.

$$34x^2 + 5y - 10 + 10x^2 - 8x + 3y$$
$$= 34x^2 + 10x^2 + 5y + 3y - 10 - 8x$$
$$= 44x^2 + 8y - 10 - 8x$$ This is the simplified answer.

Example 3: Add the polynomials $-11b^4 + 8b^3 - 3bc$ and $10 + 15b^3 + 2b^4$.

Write the polynomials with a plus sign between them.

$-11b^4 + 8b^3 - 3bc + 10 + 15b^3 + 2b^4$ Rearrange the terms.

$= -11b^4 + 2b^4 + 8b^3 + 15b^3 - 3bc + 10$ Combine like terms.

$= -9b^4 + 23b^3 - 3bc + 10$

Example 4: Add the polynomials $3jk - jk^2 + 15j$ and $23j + 5jk + 9j^2k$.

Write the polynomials with a plus sign between them.

$3jk - jk^2 + 15j + 23j + 5jk + 9j^2k$ Rearrange the terms.

$= 3jk + 5jk - jk^2 + 15j + 23j + 9j^2k$ Combine like terms.

$= 8jk - jk^2 + 38j + 9j^2k$

To subtract two polynomials, parentheses are written around the polynomial being subtracted. The minus sign can be thought of as a coefficient of –1. The –1 is distributed to each term in the second polynomial. The coefficient of –1 does not need to be written, but it is shown in the example below for clarity. Terms are then rearranged so like terms are next to each other.

$$45a^2 + 15ab - 30b^2 - (22a^2 - 3ab + 18b^2)$$
$$= 45a^2 + 15ab - 30b^2 - 1(22a^2 - 3ab + 18b^2)$$
$$= 45a^2 + 15ab - 30b^2 - 22a^2 + 3ab - 18b^2$$
$$= 45a^2 - 22a^2 + 15ab + 3ab - 30b^2 - 18b^2$$
$$= 23a^2 + 18ab - 48b^2$$ This is the simplified answer.

Example 5: Subtract $24c^2 + 52cd + 25d^2$ from $3cd - 15d^2 + 5c^2$.

Write the polynomials with a minus sign between them and parentheses around the polynomial being subtracted.

$3cd - 15d^2 + 5c^2 - (24c^2 + 52cd + 25d^2)$ Distribute the minus sign.

$= 3cd - 15d^2 + 5c^2 - 24c^2 - 52cd - 25d^2$ Rearrange the terms.

$= 3cd - 52cd - 15d^2 - 25d^2 + 5c^2 - 24c^2$ Combine like terms.

$= -49cd - 40d^2 - 19c^2$

Example 6: Subtract $-4h^3+8h^2-12h$ from $2h^3+6h-9h^2$.

Write the polynomials with a minus sign between them and parentheses around the polynomial being subtracted.

$2h^3+6h-9h^2-\left(-4h^3+8h^2-12h\right)$ Distribute the minus sign.

$=2h^3+6h-9h^2+4h^3-8h^2+12h$ Rearrange the terms.

$=2h^3+4h^3+6h+12h-9h^2-8h^2$ Combine like terms.

$=6h^3+18h-17h^2$

PRACTICE

Five in a Row

Add or subtract the given polynomials. Problems do not need to be completed in order. Cross out the answer in the table on the next page. Continue completing problems until five in a row (vertically, horizontally, or diagonally) are crossed off. For extra practice, complete all remaining problems.

Polynomials	***A* + *B*** Add the polynomials.	***A* – *B*** Subtract polynomial ***B*** from polynomial ***A***.
A: $2a+3a^2+1$ ***B***: $-a^2-a$		
A: $5ba-b^2$ ***B***: $b+a^2+b^2$		
A: $-4a^2-ab+b^2a$ ***B***: a^2b-3a^2+ba		
A: $-a^3+a^2-a+1$ ***B***: a^3-a^2+a-1		
A: $10b+4b^2-5$ ***B***: $-7b^2-2+3b$		
A: $-2a^3+ab-3ab^2$ ***B***: $2b^2+4a^3+5ab^2$		
A: $b^2a+ab-b^2+3a$ ***B***: $-2a-2ab^2+3ab$		
A: $-a^2-4b+3ab-5$ ***B***: $2ab-3a^2-a^3+1$		

Polynomials	**$A + B$** Add the polynomials.	**$A - B$** Subtract polynomial ***B*** from polynomial ***A***.
A: $4a^2 + 1 - 3a$ ***B***: $-a + a^2 - 4$		
A: $-2b^2a + b^2$ ***B***: $2b^2 + 2a^2 + ab^2$		
A: $3a^2 + 3ab - 5ab^2$ ***B***: $-a^2b + 2a^2 + ba$		
A: $2b^3 - 6b^2 + 4a + 2$ ***B***: $-b^2 + 3b^3 - 2a - 6$		

Note: Answers in the chart below may have terms in a different order.

$a + 2a^2 + 1$	$3ab^2 - 2ab - b^2 + 5a$	$5a^2 - 3 - 4a$	$7b + 11b^2 - 3$	$-b^3 - 5b^2 + 6a + 8$
$5ba + b + a^2$	$-3ab^2 - b^2 - 2a^2$	$-a^2 - 2ab + b^2a - a^2b$	$5a^2 + 4ab - 5ab^2 - a^2b$	$2a^3 + ab + 2ab^2 + 2b^2$
$a^2 + 2ab - 5ab^2 + a^2b$	$-7a^2 + b^2a + a^2b$	Free Space	$2a^2 - 4b + ab - 6 + a^3$	$-6a^3 + ab - 8ab^2 - 2b^2$
$5b^3 - 7b^2 + 2a - 4$	$5ba - 2b^2 - b - a^2$	$-ab^2 + 4ab - b^2 + a$	$3a + 4a^2 + 1$	$-4a^2 - 4b + 5ab - 4 - a^3$
$-2a^3 + 2a^2 - 2a + 2$	$13b - 3b^2 - 7$	$3a^2 + 5 - 2a$	$-ab^2 + 3b^2 + 2a^2$	0

Logic Lesson 4

SWIM-BIKE-RUN-MATH

A triathlon is a race that includes three events: swimming, cycling, and running. Men's and women's triathlons have been part of the summer Olympic Games since the year 2000. Some of the distances in competitive triathlons are called the sprint, standard / Olympic, long course, and ultra. All of these logic puzzles are related to triathlons. This lesson has no video or review problems.

A calculator may be used for this entire lesson.

ONE HUNDRED FIVE

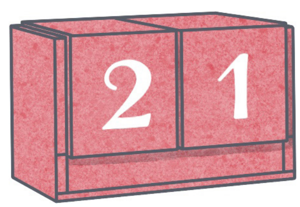
One type of triathlon includes a 2.4-mile swim, a 112-mile bike ride, and a 26.2-mile run (a full marathon). In 2023, Zimbabwean Sean Conway set a world record when he completed 105 of these triathlons in 105 consecutive days!

Find three consecutive numbers that add to 105.

______, ______, and ______

BIKE-SHOP BLOCKS

A triathlete takes her bike to a local shop for a tune-up before her next triathlon. While waiting, she notices a calendar made from wooden blocks like the one shown below. What numbers must be painted on each cube in order to show all the possible dates in a month? Assume 9 can be made by turning 6 upside down and that the blocks can switch places.



Numbers on one block: ____________________

Numbers on the other block: _______________

FAMILY FEAST

After finishing a sprint triathlon, two fathers and two sons eat breakfast at a diner. The bill totals exactly $33 including tax and a tip, and they split the cost evenly. Each person pays a whole-number amount of money (no cents). Explain how this could be true.

____

____

____

____

FASTEST FINISHERS

The top three finishers of a long-course triathlon congratulate each other after crossing the finish line just minutes apart. While they know the order in which they finished the race, they are not yet able to view their times for each event. As they discuss the race, they make the following statements:

Jack: "John was the fastest cyclist today."

Don: "That's possible, but John was definitely the fastest runner today."

John: "No, I wasn't the fastest at cycling or running today."

A race organizer overheard the conversation, and as he viewed the race data on his computer, he said, "Interesting. Each of you was the fastest at one event, but the fastest swimmer is wrong, and the fastest cyclist is right."

Which triathlete finished each event in the shortest amount of time?

The fastest swimmer was ______________.

The fastest cyclist was ______________.

The fastest runner was ______________.

CAREERS AND COACHES

Three friends have three different careers, and each of them also volunteers as a coach for one of the events in a triathlon. Use the statements below to figure out which two roles (career and volunteer coach) each person has.

✦ Hint: Once you know something for certain, put a ✓ in that square and fill in the rest of the row and column of that 3 x 3 box with Xs.

1. The physician and the running coach like to go fishing with Christian.
2. The engineer walks Flora's dog when she is out of town.
3. The physician is not the cycling coach.
4. Paula is not the physician.
5. Paula finished before the teacher in the last triathlon in which they both competed.

		Career			Volunteer Coach		
		Physician	Teacher	Engineer	Swimming	Cycling	Running
Friends	Paula						
	Flora						
	Christian						
Volunteer Coach	Swimming						
	Cycling						
	Running						

a. Paula is the ____________ and coaches ____________.

b. Flora is the ____________ and coaches ____________.

c. Christian is the ____________ and coaches ____________.

TRIATHLONS, TRIANGLES, AND TIMERS

Sisters Tallie and Tandee are training together to compete in a triathlon, but today is a rest day in their training program. Finding it difficult to talk about something other than their upcoming race, Tallie has an idea. "Tandee, let's not forget that we both love puzzles and riddles as much as we love swimming, biking, and running. To distract ourselves from thinking about the triathlon, let's have a competition with triangles!" Tandee agrees, and Tallie sketches the image at the right. "The winner is whoever finds the most triangles in 15 minutes," Tallie says.

"Wait!" replies Tandee. "Don't start a timer on your phone. We have a 7-minute and an 11-minute sand hourglass timer in the kitchen. Before we start working on the triangles problem, let's figure out how to keep track of 15 minutes using the sand timers."

First figure out how to use a 7-minute and an 11-minute sand hourglass timer to measure 15 minutes. Write an explanation below.

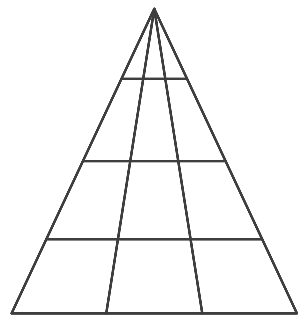
____

____

____

____

____



How many triangles are in the sketch? For a challenge, set a timer for 15 minutes to see if you can find all the triangles in that time.

✦ Hint: Sketch the figure multiple times in the space above or on a separate sheet of paper. Use colored pencils to color the triangles in an organized way.

______ triangles

Factoring

WARM-UP

Divide $12x^8 + 15x^3$ by $3x^2$.

____

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

VIDEO PRACTICE

LESSON OVERVIEW

LEAST COMMON MULTIPLE OF MONOMIALS

Recall that the least common multiple (LCM) of two numbers is the smallest positive number that is a multiple of both numbers. The least common multiple of 3 and 4 is 12. 12 is the smallest number that is a multiple of 3 and a multiple of 4. The least common multiple of monomials can be found as well. To find the LCM of monomials, it can help to write the factorization of each monomial. Then find the product of all factors from each monomial, but do not repeat factors that have already been included from another factorization.

To find the LCM of $9f^2g^3$ and $12fg^2$, first write the factorization of each monomial. Note: The prime factorization is written for coefficients.

$$9f^2g^3 = 3 \bullet 3 \bullet f \bullet f \bullet g \bullet g \bullet g \qquad 12fg^2 = 2 \bullet 2 \bullet 3 \bullet f \bullet g \bullet g$$

The LCM must include all factors from both monomials but should not include any extras. The purple factors below are the factors that both monomials have in common. They are only written once.

$$\text{LCM: } 2 \bullet 2 \bullet 3 \bullet 3 \bullet f \bullet f \bullet g \bullet g \bullet g = 36f^2g^3$$

The LCM of $9f^2g^3$ and $12fg^2$ is $36f^2g^3$. Notice that f^2 and g^3 are the variables with the highest exponents in the original monomials. Because every factor needs to be part of the LCM, the highest power of each variable will be the factor to use in the LCM. Instead of writing the factorization for each monomial, find the LCM of the coefficients and multiply by the highest power of each variable.

Example 1: Find the LCM of m^2n^5 and m^3n.

Highest power of each variable in the original monomials: m^3, n^5

Multiply the highest power of each variable.

LCM: m^3n^5

Example 2: Find the LCM of $15x^3y^2$ and $9x^5y^4$.

Prime factors of the coefficients: $15 = 3 \bullet 5$ $\quad 9 = 3 \bullet 3$

LCM of the coefficients: $3 \bullet 3 \bullet 5 = 45$

Highest power of each variable in the original monomials: x^5, y^4

Multiply the LCM of the coefficients by the highest power of each variable.

LCM: $45x^5y^4$

Example 3: Find the LCM of $5a^5b^3c$, $20b^7c^4$, and $30c^2d$.

Prime factors of the coefficients: $5 = 5$ $\quad 20 = 2 \bullet 2 \bullet 5$ $\quad 30 = 2 \bullet 3 \bullet 5$

LCM of the coefficients: $2 \bullet 2 \bullet 3 \bullet 5 = 60$

Highest power of each variable in the original monomials: a^5, b^7, c^4, d

Multiply the LCM of the coefficients by the highest power of each variable.

LCM: $60a^5b^7c^4d$

PRACTICE

1. Complete Parts A–D to find the least common multiple and greatest common factor of the two monomials.

   a. Write the factorization of each monomial.

   $36a^2bc^5 =$ ______________________________

   $15a^5b^4c^2 =$ ______________________________

   b. Find the LCM of $36a^2bc^5$ and $15a^5b^4c^2$.

   ____

   c. In the factorizations from Part A, circle common factors.

   d. Find the GCF of $36a^2bc^5$ and $15a^5b^4c^2$.

   ____

2. Complete Parts A–D to find the least common multiple and greatest common factor of the two monomials.

   a. Write the factorization of each monomial.

   $12t^3uv^5 =$ ______________________________

   $5tu^4v^3 =$ ______________________________

   b. Find the LCM of $12t^3uv^5$ and $5tu^4v^3$.

   ____

   c. In the factorizations from Part A, circle common factors.

   d. Find the GCF of $12t^3uv^5$ and $5tu^4v^3$.

   ____

3. Use the highest or lowest power of the exponents to find the least common multiple (LCM) and greatest common factor (GCF) of the monomials.

Monomials	LCM	GCF
$16h^3i^4j$ $22h^5j^5$		
$8m^5n^3$ $50m^7n^2$		
$14p^4qr^7$ $52p^3q^{15}r^8$		

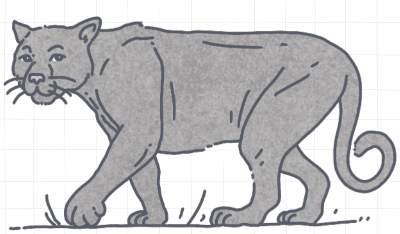
4. Use the GCF to factor each polynomial.

a. $3a^3b^5 + 6a^5b^2$

b. $12cd^5 - 20c^2d^3$

c. $45e^4f^5g^2 + 63e^4f^2g^2$

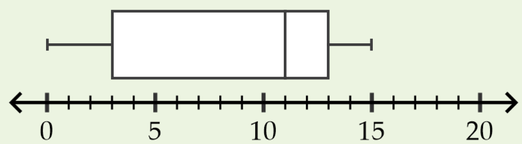
d. $91h^2i^3j^4 - 13hi^2j^3$



Scatter Plots

WARM-UP

Use the box plot to find the IQR of the data.



IQR: ________

LESSON

Use the app to watch the video lesson. Complete problems when instructed during the video in the Video Practice section. Optionally, read the Lesson Overview in place of the video or after the video if more instruction is needed.

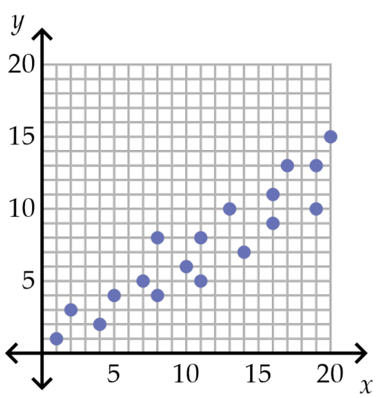
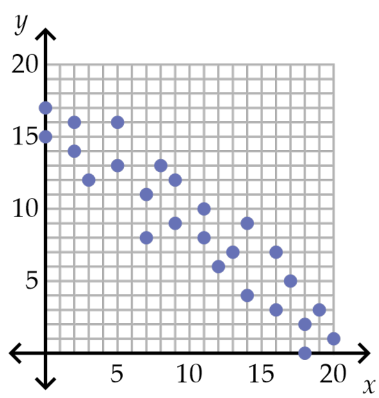
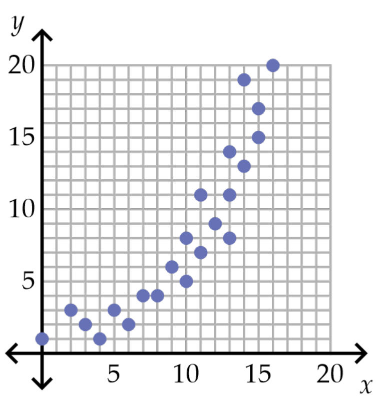
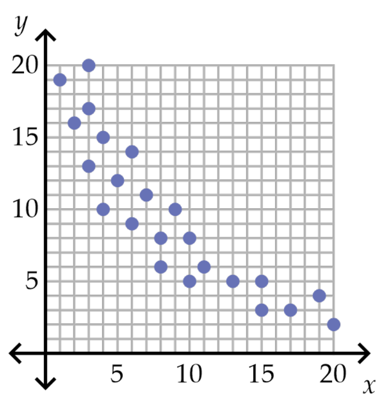
VIDEO PRACTICE

LESSON OVERVIEW

A ***scatter plot*** is a graph on the coordinate plane with one point for each data value. There may be more than one output (y-value) for a given input (x-value). Scatter plots are a visual way to organize data to see trends, patterns, and relationships, as well as to make predictions and inferences about the data.

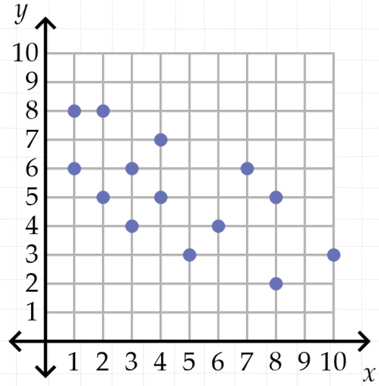
TYPES OF CORRELATION

A ***correlation*** is a relationship between two variables. Correlation can be seen on a scatter plot and can be described in a number of ways. The table below shows four scatter plots with different types of correlation. Each graph shows a different type of relationship between the x-values and the y-values.

Types of Correlation	Positive: Data points generally increase from left to right. As x increases, y increases.	Negative: Data points generally decrease from left to right. As x increases, y decreases.
Linear: Data points generally form a line.	positive linear correlation 	negative linear correlation 
Nonlinear: Data points generally form a curve	positive nonlinear correlation 	negative nonlinear correlation 

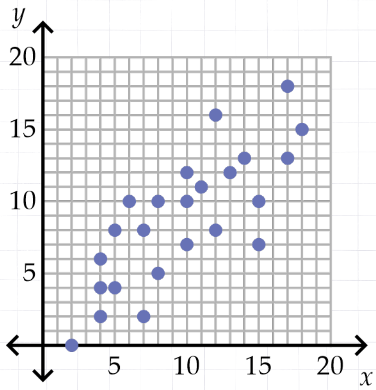
PRACTICE

Seven scatter plots are shown on this page. Each scatter plot represents an answer in Section I and in Section II. Read each problem and find the graph that is the correct answer. Write the letter shown below the graph for Section I or Section II on the line. For example, if the answer to Problem 1 in Section I is the graph at the right, write a V on the line. If the answer to Problem 1 in Section II is this graph, write an L on the line. When finished, fill in the letters on the lines at the end of the practice to answer the riddle.

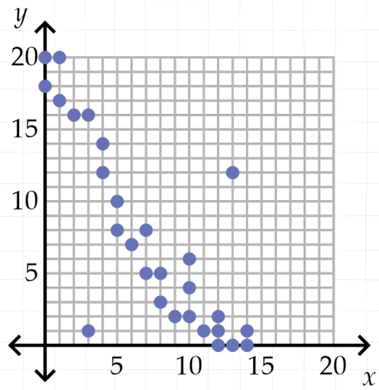


Section I: V   Section II: L

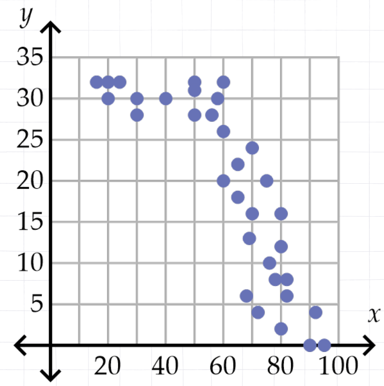
Riddle: Why should you never tell a scatter plot a secret?



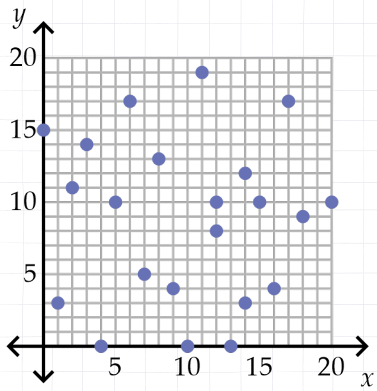
Section I: S   Section II: R



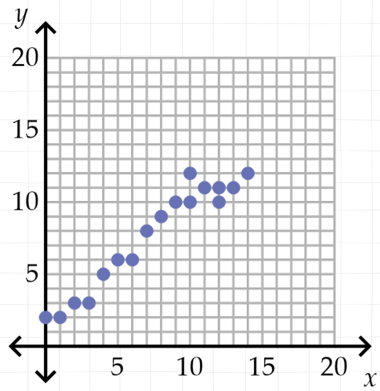
Section I: D   Section II: P



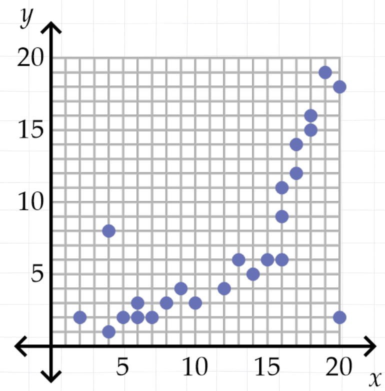
Section I: H   Section II: A



Section I: W   Section II: T



Section I: C   Section II: E



Section I: O   Section II: I

Section I: Match the description of correlation and outliers to the correct scatter plot.

1. strong, positive, linear ______
2. strong, negative, nonlinear ______
3. weak, negative, linear ______
4. positive, nonlinear, some outliers ______
5. strong, negative, some outliers ______
6. weak, positive, no outliers ______
7. no noticeable correlation ______

Section II: A research company compiled data on dental healthcare. Match each scenario with the scatter plot that most likely represents the situation and answer the questions.

8. a. The number of cavities a person has (y) has a weak correlation with how much candy they eat (x).

______

b. What does the correlation in the scatter plot indicate?

____

____

9. a. The number of cavities a person has (y) is strongly correlated with how often they brush their teeth each week (x), but there are notable outliers.

______

b. What do the outliers in the scatter plot indicate?

____

____

10. After childhood, the number of teeth a person has (y) is strongly correlated with age (x), with significant loss occurring at advanced ages.

______

11. The number of cavities children have (x) does not appear to be correlated with the number of cavities their parents have (y).

______

12. A dentist's monthly income in thousands of dollars (y) is positively correlated with the number of daily clients he or she has (x), but the correlation is not linear. There are some outliers.

______

13. a. At a dentist's office, oral hygiene is measured on a scale with higher numbers corresponding to better oral health. The oral hygiene of a person (y) is strongly correlated with frequency of flossing (x).

______

b. What does the correlation in the scatter plot indicate?

____

____

14. The amount of plaque buildup (y) is weakly correlated with the cost of the toothbrush used (x).

______

Riddle Answer:

__ __ (12 11) __ __ __ __ (7 12 14 14) __ __ __ __ __ __ (6 9 8 13 10 5) __ __ (12 11)

__ __ __ (10 14 14) __ __ __ __ (4 3 13 8) __ __ __ (11 2 13) __ __ __ __ __ (9 14 10 1 13)!

REVIEW

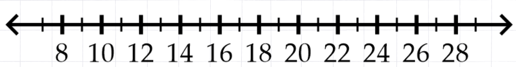
A calculator may be used for this entire review section.

1. The data set below shows the height (in thousands of feet, rounded to the nearest thousand) of the highest peak on each continent. Create a box plot for the data and find the IQR. L111

19, 19, 20, 23, 29, 16, 7

Minimum: _____ Maximum: _____

Q1: _____ Q2: _____ Q3: _____

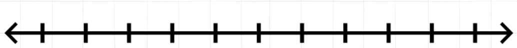


IQR: _____

2. Solve and graph the inequality. L91

$$\frac{3}{4}(8x-24)<\frac{15x-27}{2}$$

____

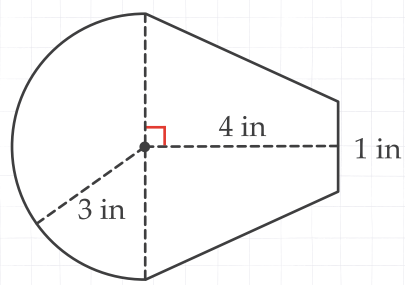


3. Alicia is using an app to learn a foreign language. She set a goal to spend an average of 20 minutes per day, six days a week, practicing on the app. The numbers of minutes she practiced each day from Monday to Friday are listed below. How many minutes does she need to practice on Saturday in order to average 20 minutes per day this week? L110

22, 15, 18, 20, 23

_____ minutes

4. Find the area of the composite figure to the nearest hundredth. L81



____

Unit 4 Review

SUPPLIES: protractor, colored pencils

Complete this Unit Review to prepare for the Unit Assessment. There is no video, lesson, or practice. Because Unit Reviews include practice for an entire unit, they may take longer than regular lessons, and students may decide to take two days to finish.

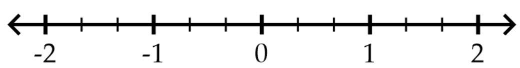
A rainforest is a forest that receives a lot of rainfall, generally over 80 inches a year. Complete the problems in this review. At the end of the review, write the green word on the line that corresponds to the solution to discover some amazing facts about rainforests.

LESSONS 91–92

1. Solve and graph the inequality.

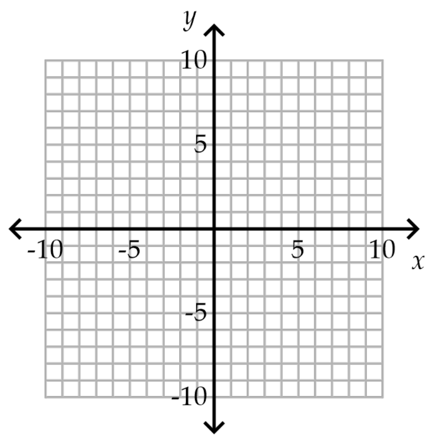
$$3x + 2 \le 4 - 3x$$

$x \le$ ________ surface



2. a. Graph the linear inequality.

$$5x + 2y > 20$$



b. One of the points below satisfies the inequality from Part A. Use the graph to determine which point it is and use that word as the clue.

$(0,0)$	$(1,6)$	$(6,1)$
evaporating	absorbing	falling

LESSONS 93–96

3. Find the value of $2(4a + b) - 1$ if $\frac{2a}{3} - 1 = 5$ and $2b = 4b + 12$.

______ ten

4. Solve each equation or system of equations. Use the specified method if one is indicated. Write "none" or "infinitely many" if there is not a unique solution.

a. $5 + 3(2u - 4) = -7 - (-6u)$

________________ canopy

b. $2v + 3 = -3(4 - v) - v$

________________ medicines

c. Substitution
$x = 3y + 1$
$2x - 4y = 6$

________________ inches

d. Elimination
$6x + 2y = 8$
$-9x - 3y = -12$

____

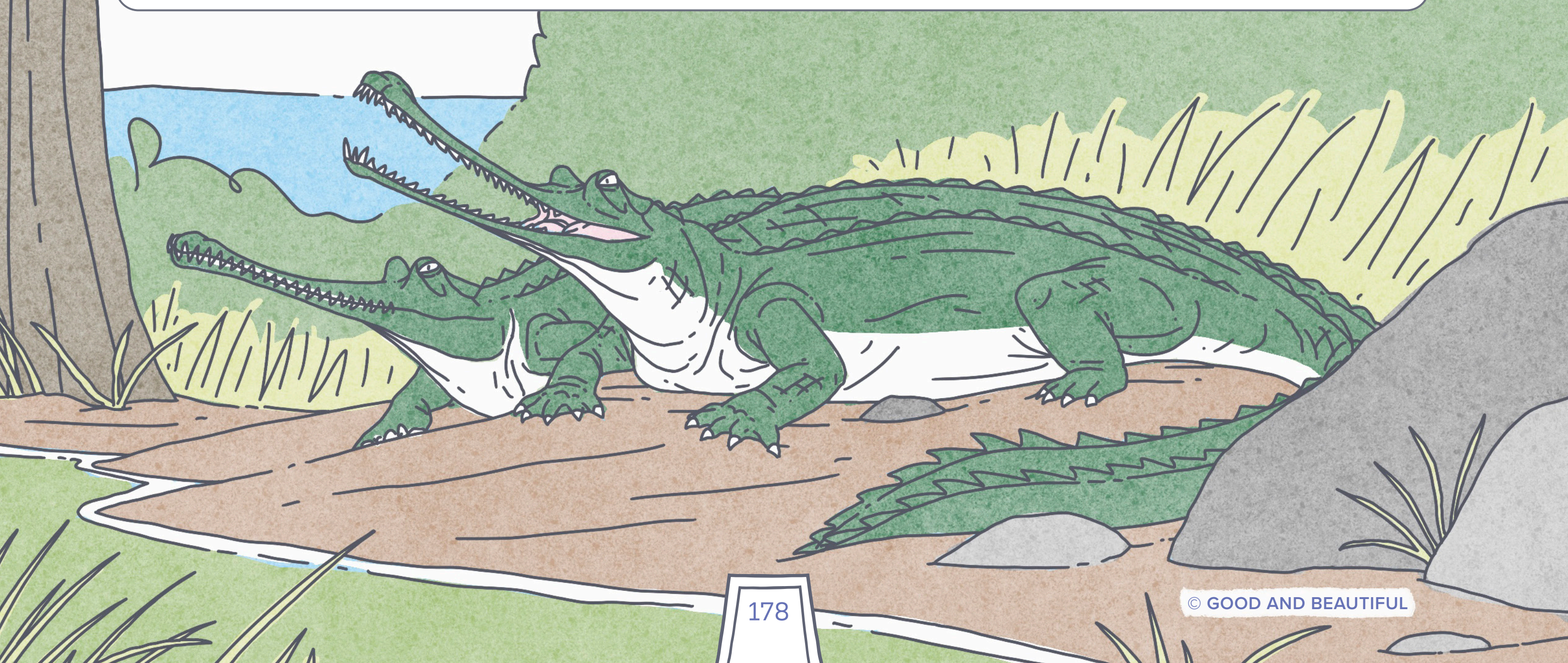
RAINFOREST FACTS:

1. Roughly ____________ of the earth's ____________ is ____________ .
   $y=\frac{5}{4}x+\frac{1}{2}$; $\frac{1}{3}$; 20

2. A ____________ ____________ can take as long as ____________ minutes to fall from the upper ____________ of a ____________ all the way to the ground.
   $(6,1)$; 28 ; 59 ; infinitely many ; 20

3. During years of heavy ____________ , a ____________ might receive as many as 400 ____________ of ____________ in a single year.
   $(x,y)\rightarrow(x+4,y+4)$; 20 ; $(7,2)$; 3

4. About ____________ of modern ____________ originate from ____________ in the ____________ . The ____________ ____________ Institute in the United States estimates that as many as ____________ of ____________ -fighting ____________ originate from ____________ ____________ .
   $(0,5)$; none ; 38 ; 20 ; 10.53% ; 2 ; $2e^2-e-3$; $2kl$; 3.1 ; 20 ; 38



Course Assessment

SUPPLIES: protractor, colored pencils

This assessment covers concepts taught in Pre-Algebra. Problems are designed to assess multiple skills. Read the instructions carefully and do not rush through the problems. You may use the Reference Chart for the assessment. Lesson numbers are given by each problem so you can review lessons for any answers that are incorrect. Calculators may be used where needed on the entire assessment.

1. Simplify the expression. L16, 19–21

a. $\sqrt{49}$ ______

b. $\sqrt[3]{512}$ ______

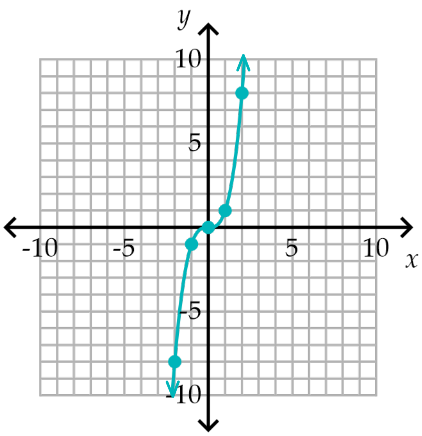
c. 2^{-3} ______

d. $\sqrt{2} \bullet \sqrt{8}$ ______

e. $5\sqrt{3} - 2\sqrt{3}$ ______

f. $\sqrt[3]{\frac{1000 - 2 \bullet 68}{8^2 - 4 \bullet 15}}$ ______

2. Fill in the table with the ordered pairs shown on the graph of the relation. Then determine the rule and equation for the relation. L37



x	y

Rule: ______________________

Equation: ____________

3. a. Fill in the table to find the change in x and y. L39

Change in x	x	y	Change in y
	–2	3	
	–1	4	
	0	5	
	1	6	
	2	7	

b. Is the rate of change constant? ______

c. Does the table represent a linear function?

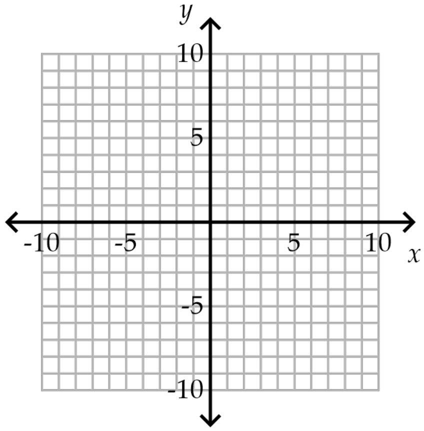
______

4. Three linear equations are given below. Graph each line. Then identify a pair of parallel lines and perpendicular lines. L46–47, 49

A $y = -\frac{1}{2}x + 7$

B $y - 2x = -6$

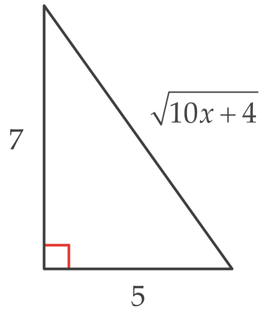
C $y = 2x + 3$



Parallel lines: _____ and _____

Perpendicular lines: ____ and ____

5. Use the Pythagorean theorem to solve for x. L50, 52–53



____

6. Chase bought a car five years ago for \$42,250. Now Chase's car is worth \$32,110. What is the percent decrease in the value of Chase's car after five years? L61

____

7. A bag contains the following tiles: 8 circles, 6 squares, 7 triangles, and 4 stars. L63–64

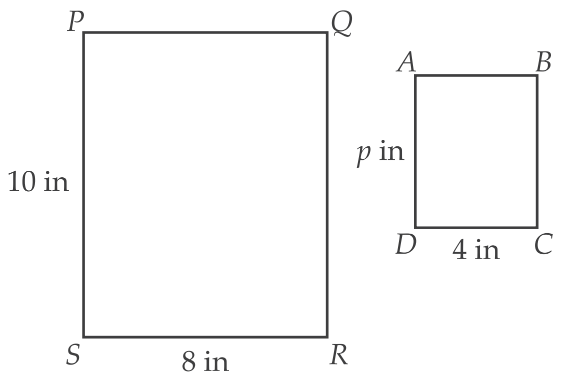
a. Find the probability (as a percent) of drawing a triangle.

______

b. Find the probability (as a percent) of drawing a square, not replacing it, and then drawing a star.

______

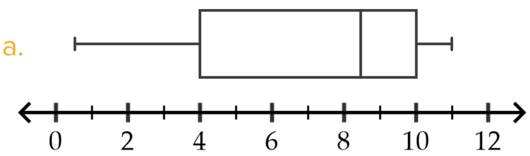
8. Given that $PQRS \sim ABCD$, find the value of p. L66, 74, 76



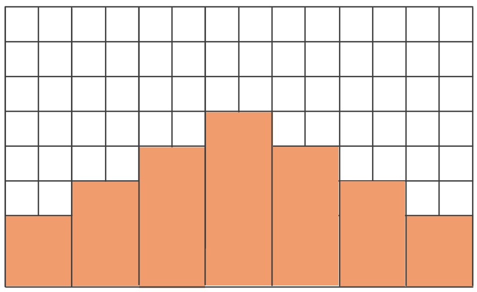
____

24. Determine whether each situation represents a data set that is symmetric, right-skewed, or left-skewed. L114

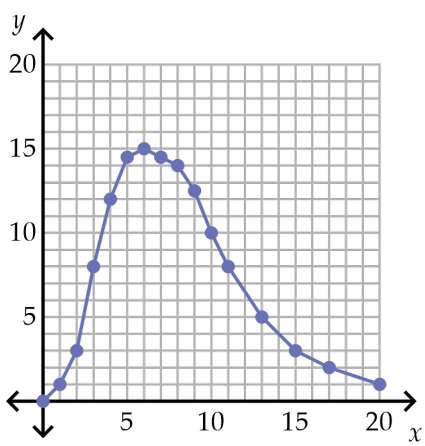
a.



b.



c.



25. Given the two-way table, determine what percent of the total sample are females who picked tennis. L115

	Male	Female	Total
Tennis	140	95	235
Pickleball	110	155	265
Total	250	250	500

26. Determine if the survey questions are biased or unbiased. L116

a. Do you prefer a truck or an SUV?

b. Do you prefer to read a book or watch a movie?

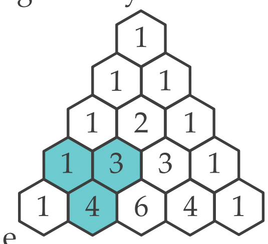
c. Do you prefer to watch the challenging sport of soccer or boring football?

Enrichment: Pascal's Triangle

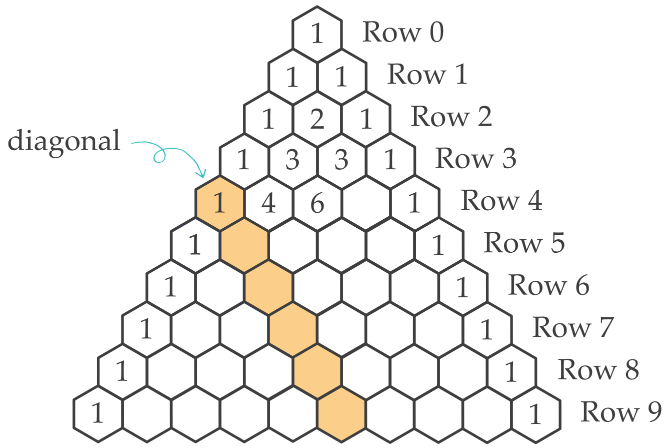
This is an enrichment lesson. Mastery is not expected at this level. There is no video, practice, or review. A calculator may be used for this entire lesson.

Blaise Pascal was a famous French philosopher, writer, inventor, physicist, and mathematician who lived in the 1600s. He made significant contributions to the fields of probability and geometry.

One of Pascal's most famous works is called Pascal's triangle. Pascal's triangle is made up of numbers that form a triangular shape with the digit 1 on the left and right edge of each row. The other values in each row are found by adding the two values directly above it. For example, in part of Pascal's triangle at the right, the number 4 is found by adding the 1 and 3 above it.



Additional rows can be added at the bottom of Pascal's triangle to make the triangle larger and larger. The top row is considered Row 0. A diagonal is highlighted in yellow below. Pascal's triangle has many patterns that can be seen in the rows and diagonals.



Try it!

1. Fill in the missing values of Pascal's triangle above.

2. Spend five minutes studying the numbers in this triangle and their arrangement. Write any observations you notice about patterns in the triangle.

Another interesting characteristic of Pascal's triangle can be found by looking at the following example. Suppose there are three different hamburger toppings to choose from: cheese, lettuce, and tomato. If any combination of toppings can be chosen, how many different burgers can be made? Note that choosing cheese and tomato is the same as choosing tomato and cheese because the order of the toppings does not matter.

0 toppings:	1 burger can be made with 0 toppings. Options: no toppings
1 topping:	3 different burgers can be made with 1 topping. Options: cheese or lettuce or tomato
2 toppings:	3 different burgers can be made with 2 toppings. Options: cheese and lettuce, cheese and tomato, lettuce and tomato
3 toppings:	1 burger can be made with 3 toppings. Options: cheese, lettuce, and tomato

Look at the number of possible burgers with 0, 1, 2, and 3 toppings: 1, 3, 3, 1. This is the third row of Pascal's triangle. The row sum is the number of possible combinations: 8.

Try it!

7. Suppose there are four different hamburger toppings to choose from: cheese, lettuce, tomato, and pickles. If any combination of toppings can be chosen, how many different burgers can be made? Remember that the order of the toppings does not matter.

a. 0 toppings: _____

Options: ____________________________

b. 1 topping: _____

Options: ____________________________

____

c. 2 toppings: _____

Options: ____________________________

____

____

d. 3 toppings: _____

Options: ____________________________

____

____

e. 4 toppings: _____

Options: ____________________________

____

f. Number of possible combinations: ______

The answers to Parts A–E should be the numbers in the fourth row of Pascal's triangle, and the answer to Part F is the sum of the numbers in that row.

The rows of Pascal's triangle give information about *combinations*. In math, combinations refer to the number of ways to choose a certain number of items. In the example with three toppings, there were three ways to choose two of the three toppings. In the example with four toppings, there were six ways to choose two of the four toppings. Pascal's triangle has so many more interesting properties!